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THE ASTROPHYSICAL JOURNAL

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An International Review of Spectroscopy and
Astronomical Physics

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CONTENTS

NUMBER I

PAGE

Photographic Observations of Comet 1911 *c* (Brooks). E. E. Barnard 1

Tertiary Standards with the Plane Grating, The Testing and Selection of Standards. Charles E. St. John and L. W. Ware 14

On the Determination of the Orbital Elements of Eclipsing Variable Stars. II. Henry Norris Russell 54

Selective Absorption of Light on the Moon's Surface and Lunar Petrography. R. W. Wood 75

Reviews: Outlines of Applied Optics, P. G. Nutting (A. C. Lunn), 85.

NUMBER II

The Synthetic Development of Radiation Laws for Metals. Edward P. Hyde 89

Elements of the Eclipsing Variables *W Delphini*, *W Ursae Majoris*, and *W Crucis*. Henry Norris Russell 133

On a New Method of Determining the Angular Diameters of Stars by Means of Elliptically Polarized Light. S. Pokrowsky 156

NUMBER III

Yerkes Actinometry. J. A. Parkhurst 169

The Absorption Spectrum of Tellurium Vapor and the Effect of High Temperature upon It. E. J. Evans 228

On Darkening at the Limb in Eclipsing Variables. I. Henry Norris Russell and Harlow Shapley 239

On Convergence Frequency in Spectral Series. Fernando Sanford 255

Reviews: Die optischen Instrumente, Dr. Moritz von Rohr (Stanley C. Reese), 263.

NUMBER IV

PAGE

THE ATTRACTION OF SUN-SPOTS FOR PROMINENCES. Frederick Slocum 265

ELEMENTS OF THE ECLIPSING VARIABLES *W Delphini*, *S Cancri*, *SW Cygni*, and *U Cephei*. Harlow Shapley 269

BAND SPECTRA OF ALUMINIUM, CADMIUM, AND ZINC. Emily E. Howson 286

OBSERVATIONS OF THE SPECTRUM OF *Nova Geminorum* NO. 2. Walter S. Adams and Arnold Kohlschutter 293

THE PRIMARY STANDARD OF LIGHT. Herbert E. Ives 322

THE INFLUENCE OF TEMPERATURE ON THE PHENOMENA OF PHOSPHORESCENCE IN THE ALKALINE EARTH SULPHIDES. Herbert E. Ives and M. Luckiesh 330

NUMBER V

ON THE DEVIATION FROM LAMBERT'S COSINE LAW OF THE EMISSION FROM TUNGSTEN AND CARBON AT GLOWING TEMPERATURES. A. G. Worthing 345

THE INTEGRATED SPECTRUM OF THE MILKY WAY. E. A. Fath . . 362

THE *Algol* VARIABLE *RR Draconis*. Frederick H. Seares . . . 368

ON DARKENING AT THE LIMB IN ECLIPSING VARIABLES. II. Henry Norris Russell and Harlow Shapley 385

REVIEWS: Bijdrage tot de Kennis van Lijnenspectra, T. Van Lohuizen (F. A. Saunders), 409.

THE
ASTROPHYSICAL JOURNAL
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AND ASTRONOMICAL PHYSICS

VOLUME XXXVI JULY 1912 NUMBER 1

PHOTOGRAPHIC OBSERVATIONS OF COMET 1911 *c* (BROOKS)

By E. E. BARNARD

This comet was discovered by Dr. W. R. Brooks at Geneva, N.Y., on July 20, 1911. Perihelion passage occurred on October 27.

At first it was a faint object. Even with photographs of long exposure at this time there was no evidence of a tail. The comet was simply a round nebulous mass, a little brighter in the middle, but with no nucleus. It brightened rapidly, but was slow in developing a tail. By August 20 the comet was faintly visible to the naked eye as a hazy spot of light, but the tail was not seen until about September 16. It became a splendid object in the evening sky in the latter part of September, when its slender tail attained a length of about 25°. When near perihelion in the morning sky it was especially bright and beautiful. It was one of the finest comets of recent years and the duration of naked-eye visibility, about four months, has been exceeded by but few comets. At its best it was but little inferior to Halley's comet. Though a large one, it was not a great comet in the correct sense of the word. The regular parabolic rim outlining the head and tail was not formed. Even at perihelion small streamers at considerable angles to the tail were still present. These small streamers are usually absent in great comets, in which the outline of the head sweeps back to form the boundaries of the tail when the comet makes a close approach to the sun, and all the small side streamers are swept back into the main tail.

THE PHOTOGRAPHS AND THE EXTRA GUIDING TUBE

The photographs were taken with the Bruce photographic telescope, the excellent mounting of which was made by Warner & Swasey. This instrument, as is well known (*Astrophysical Journal*, **21**, 35–48, 1905), is a triple telescope consisting of a visual guiding telescope with a Brashear object-glass of 5 inches aperture, and two portrait lenses, one of 10 inches diameter by Brashear, the other of 6.2 inches by Voigtländer (refigured by Brashear). A Clark 3.4-inch doublet is also attached to the mounting in a wooden camera-box. There is also attached to the instrument a Darlot lantern lens of 1.5-inch diameter, which will give a field 30° wide or more.

In the original condition of the mounting, the guiding eyepiece has a lateral motion of 1° or 2° with which a comet's head may be displaced by that amount from the center of the plate to show more of the tail. Having found that this displacement is not as much as the field of good definition will allow, I decided to attach an additional, light, adjustable tube to the mounting to secure more of a comet's tail. This tube will carry the object-glass and eye-end of the regular guiding telescope. It can be thrown out of the parallel of the main tubes for 6° or 8° in any direction. It will thus permit much more of a comet's tail to be shown by placing the head nearer the edge of the plate—still keeping within the field of fair definition. This tube can be placed in position on the mounting or removed in a few minutes' time. As this arrangement is wanted only when a large comet is present, it is very satisfactory, and the slight trouble of putting it in position is of no consequence. It has proved to be a very great advantage and was used in the present observations and in those of Halley's comet. This tube and its adjustments were made in the shop of the observatory by Mr. Oscar Romare.

All the photographs were made on Lumière Sigma plates.

Following are the approximate positions of the head of the comet on the plates accompanying this paper taken from the *B.D.* charts for the epoch 1855.0.

Date	α	δ
September 22.....	15h 25m	+52° 29′
October 19.....	12 39	+17 45
October 20.....	12 38	+16 20
October 23.....	12 33	+11 35
October 25.....	12 31	+ 8 42
October 27.....	12 30	+ 5 35
October 28.....	12 30	+ 4 10

Up to the middle of October the comet was a great disappointment from a photographic standpoint, for, though bright and conspicuous to the naked eye, its effect upon the sensitive plate was very slow, and long exposures were required to show the tail, which was simply a straight and rather narrow stream of diffused matter, with nothing of interest except occasionally an effort at additional streamers near the head. The photographs were very much alike night after night and it looked as if the comet would not be of any special importance. This condition continued until near perihelion, when it suddenly became an object of the deepest interest and showed a wealth of detail in the structure of the tail that has been equaled by only one comet (Morehouse's) since photography has recorded their spectacular changes. At the same time its photographic action increased very greatly, apparently much more than was due to its actual increase in brightness. Morehouse's comet, which was relatively a faint object, and just dimly visible to the naked eye for a couple of days, was not only remarkable for the great changes that took place in its phenomena, but was also singularly active in its effect on the photographic plate, requiring comparatively short exposures to show the full extent of its tail, which was too faint even to be seen in the telescope. The spectroscope showed the presence of cyanogen strongly in this latter comet, and to this fact was attributed its great photographic activity and the unique features which developed in its tail.

The second, or active, period of Brooks's comet showed similar characteristics of details and photographic effect, but, though cyanogen was present in the head, it was, so far as I can learn, not shown to be present in the tail. It might therefore be suggested that it was not really cyanogen that was responsible for the peculiarities of Morehouse's comet. Many of the phenomena resembled those of Morehouse's comet, and though not so weird in some of its features, it was on some occasions more beautiful than that object, from a photographic standpoint, and of course visually it was infinitely finer.

One fact that is strikingly apparent in the photographs of Brooks's comet is the remarkable reduction in the size of the head as it neared perihelion. On September 18 and 19 the actual diameter on the photograph was 1,200,000 km, while on October 28 it had diminished to 510,000 km.

At first the head was large and round, and the tail where it joined it was a narrow thread—a "neck"—as if the two had no more connection than a mere attachment. This is shown by the measures on September 18, when the head was 54′ in diameter and the "neck" only 6′. As the comet approached the sun, the head diminished in size, while the "neck" correspondingly increased, until finally the two became nearly of the same size, the outlines of the head more or less merging into and forming the outlines of the tail.

This phenomenon always occurs in the case of comets that make any near approach to the sun and is doubtless due to the increased pressure of the sun's light upon the matter of the comet which is thus driven out in the direction of the tail. But the photographic evidence of the phenomenon is a very beautiful one.

It would be difficult to describe the photographs of this comet obtained with the 10-inch lens, for they are very rich in detail. The scale of this lens was sufficiently great to show the structure of the tail and streamers in the best manner, and at the same time to secure enough of the length of the tail to exhibit its peculiarities to the best advantage. A brief effort at description, however, may serve to call attention to some of the features that might otherwise be overlooked. These descriptions refer to the photographs made with the 10-inch lens.

DESCRIPTIONS OF THE PHOTOGRAPHS

September 2. Head large and round. Tail near head very narrow.

September 13. Head large and round with very narrow "neck" from which the tail widens slightly.

September 15. Head large and round. "Neck" a mere thread from which the tail widens slightly.

September 16. Head large and round. "Neck" narrow. Tail widens slowly and consists of a brighter and a fainter portion.

September 18. Head large and round. "Neck" not quite so narrow as formerly. The tail double—a faint and a bright portion. The widest extension of head—very diffused and faint—is 54′; width of "neck" 6′.

September 19. Head large and round. "Neck" very slender. Tail widens slowly. On the north side is a slender ray 6° long.

September 21. Head large and round, extreme nebulosity 54′ in diameter. "Neck" narrow but not as narrow as before. A slender ray runs along the north side of the tail. A few irregularities in the tail.

September 22 (Plate I). In this photograph the tail is straight and slender and diffused, and where it joins the head it is very narrow. The head is very large and round, and from it also issue several incipient streamers.

Bad weather and moonlight interfered greatly after this, so that the views of the comet were few, and the photographs are of little interest until the latter half of October, when they are all that could be desired; but even here the series was badly broken by cloudy weather.

On October 19 (Plate II) the tail appears flat and widens out rapidly in the form of a nearly closed fan, and is bounded by a bright, sharply defined rim on each side, while on the body of the tail are many streaky details, some of which are curiously curved. Though the tail appears flat, it is doubtless a more or less thin shell of conical form. It is a striking and beautiful picture, and very much resembles the photograph of comet Morehouse on November 16, 1908.

October 20 (Plate III). The head is small and the tail rapidly widens out into a great bundle of diffused streamers. The effect is very curious and beautiful.

October 23 (Plate IV). The tail for some degrees from the head is wonderfully rich in fine threadlike streamers, nearly all of which are wavy in form, as if the tail had been combed with a coarse comb with an undulatory motion. At 2°.6 from the head a dark sinuous rift begins and extends beyond the limits of the plate. The head is scarcely any larger than the adjacent part of the tail. In the full extent of that portion of the tail shown, there is a remarkable amount of structural detail in the form of wavy streamers.

October 27 (Plate V). The head is very small. The north side of the tail is gently concave and sharply defined. There are a number of long irregular masses in it, about 6° or 8° from the head and many small streamers. This plate apparently shows the comet's tail fainter near the middle of its length. This peculiarity is verified by the 6-inch plate, but I question whether this is not due in some way to the clouds which covered the comet part of the time.

October 28 (Plate VI). This picture in some respects resembles that of October 20. The head is very small, apparently not any larger than the beginning of the tail. The tail widens out rapidly into a fan-shape with a central bright region. The small streamers near the head are wavy.

PHOTOGRAPHS TAKEN WITH THE LANTERN-LENS

The following are descriptions of the tail as shown on the 1.5-inch lantern lens photographs. They are supplementary to the above.

October 23. At a point 19° from the head the tail suddenly widens from 2° to 3° on the north side. This continues for about 6°, until lost in space. There is another sudden widening of the tail on the north side about 10° from the head; this continues until the second widening occurs. The tail becomes bifurcated at 3°.7 from the head, the southern portion being composed of a slender ray, diffused on the south side. The total length of the tail on this plate is 26°.

LIST OF NEGATIVES OF BROOKS'S COMET

Date	Exposure	Quality			Length of Tail			Remarks
		10	6	3.4	10	6	3.4	
1911 July $26^h\,11^h\,20^m$	$2^h\,50^m$	G	G	—	0°	0°	—	Very difficult to guide on
Aug. 18 10 3	2 40	G	G	—	3	4	—	Mixed up with North America Nebula
21 9 25	1 15	G	G	—	Trace	1	—	
24 10 42	4 0	G	G	—	3-4	1	—	Sky whitish. Mixed up with North America Nebula
28 15 27	0 46	G	G	—	1	0	—	
31 12 52	4 15	G	G	—	4	6	—	Sky not v. transparent
Sept. 2 13 34	3 52	G	G	—	3	8	—	
13 8 9	1 40	G	P	—	6	6	—	Some clouds
14 9 6	1 14	P	F	—	6	8	—	Clouds
15 8 54	3 0	G	G	—	7	8	—	Sky fairly good
16 9 0	2 58	G	G	—	8	9	—	Sky dull
18 9 15	3 3	G	G	G	8	9	11	Clouds
19 8 31	2 10	G	G	G	8	5	11	Clouds
21 9 9	3 40	G	G	G	$10\frac{1}{2}$	12	11	
22 9 9	3 30	G	G	G	$10\frac{1}{2}$	12	12	Sky poor
23 7 46	1 1	F	F	F	6	12	11	Poor sky, and haze in last part
26 9 27	0 25	P	B	P	3	4	9	Clouds
27 8 59	2 4	G	—	F	10	—	12	10-in. neg. broken. Clouds
27 9 23	1 47	—	P	—	—	8	—	
29 9 29	2 10	G	F	P	11	14	12	Moonlight; poor sky
Oct. 7 6 58	0 32	B	B	—	$2\frac{1}{2}$	1-2	—	Full moon; sky poor
11 6 59	0 54	P	P	B	3-4	12	6	Haze
12 7 5	0 35	B	B	—	8-10	12	—	
19 16 38	0 46	G	G	G	9	12	8	Clouds
20 16 44	0 45	G	G	G	10	10	14	Sky poor
23 16 32	1 15	G	G	G	11	14	17	
24 16 58	0 33	P	G	P	3-4	—	6	Through breaks in clouds
25 16 38		—	—	F	—	—	15	Visible only once in a while through breaks in clouds
25 16 49	0 38	G	—	—	11	—	—	
25 16 51	0 42	—	G	—	—	14	—	
27 16 33	1 12	G	F	G	$11\frac{1}{2}$	14	15	In clouds most of time
28 16 38	1 5	G	G	G	11	14	15	Sky good

October 25. The tail gently widens and is better defined and slightly concave on the north side. Its length is 28°.

October 27. The tail is 30° long; it is slightly and irregularly concave and best defined to the north. Two condensations occur in it at 7°5 and 10°5 from the head. The south side is diffused.

October 28. Length of tail 29°5. Concave on north side, which is the best defined but irregular. One marked condensation (perhaps one of those of October 27) occurs at 15° from the head. Near the end on the north side the tail has several double curvatures. Unfortunately the reproduction of these plates (Plate VII) does not show the full extent of the tail as given on the photographs. The length of the tail on other lantern lens plates was as follows:

1911 September	16	11°
	18	17
	19	17
	21	18
	22	18
	23	12
1911 September	27	22°
	29	23
October	11	15
	19	10
	20	26

Of course these different lengths do not mean that the tail varied, but for various reasons they represent the full extent shown on the plates.

On p. 7, not including the lantern lens pictures, is a list of the photographs which I have obtained with the three lenses of the Bruce photographic telescope. The headings 10, 6, 3.4, respectively, represent the 10-inch, 6-inch, and 3.4-inch lenses. In the columns of "Quality" and "Length of Tail" the initials mean B=bad, F=fair, G=good, and P=poor. A dash indicates that no photograph was secured with the particular lens.

NOTES WITH THE 40-INCH TELESCOPE, ITS 4-INCH FINDER, THE 5-INCH BRUCE GUIDING TELESCOPE, AND WITH THE NAKED EYE

July 21, $10^{h}30^{m}$. The central part is small, 0′5 diameter, condensed, and 12.5 magnitude. From this condensed part the comet diffuses very softly beyond the limits of the field (6′). It is faintly visible in the 4-inch finder.

July 23, 14^{h}. It is very diffused and larger than the 6′ field. The central condensation is 10″ in diameter; no nucleus or tail.

July 25, $11^{h}30^{m}$. Very large and very diffused, about 15′ in diameter. The central condensation 10″–15″ diameter and 12.5 magnitude. No nucleus or tail. Quite noticeable in finder and seems brighter than before.

July 30, 11^{h}. Very much larger, and somewhat brighter. With 700 diameters there is a small definite nucleus of 12.5 magnitude in a small inner nebulosity. Very much larger than the field of 460 (6′) and very diffused—except the condensed part. In finder it is 10′–15′ in diameter and quite conspicuous, but not much brighter in the middle.

August 6, 13^{h}. Very large and very diffused. Strong but small condensation to (perhaps) a nucleus of twelfth magnitude.

August 15, 12^{h}. Very large and diffused with small central condensation. Quite conspicuous in finder in spite of moonlight.

August 20, 10^{h}. Very much bigger than the field of view. Small central condensation of twelfth magnitude. In finder it is conspicuous and about 15′ in diameter and about 12.5 magnitude gradually condensed in the middle. It is faintly but distinctly visible to naked eye as a hazy spot of 5.5 or 6 magnitude.

August 21, 10^{h}. Five-inch guiding telescope of the Bruce photographic instrument. It is brighter, but no tail. With naked eye could not be certain of seeing it.

August 24, 12^{h}. Five-inch. Bright but no tail or nucleus.

August 25, 12^{h}. Very easy to naked eye as a small hazy spot 5^{m} or 5$^{m}_{\cdot}$5.

August 28, 15^{h}30^{m}. Very easy to naked eye when looked for, as a small hazy spot of 5 magnitude. Exactly as noticeable as the two stars ω^2 and ω^3 *Cygni* (Proctor's Chart), *B.D.*+48°3142 (4$^{m}_{\cdot}$9) and *B.D.*+48°3145 (5$^{m}_{\cdot}$9), when they appeared as one. It was brighter than either, equaling the combined light of the two.

August 29, 12^{h}. It was conspicuous to the naked eye like a hazy 5^{m} star, but no tail was visible either to the naked eye or in the finder. With the 40-inch telescope the comet was several times larger than the field of view, with a small strong condensation, but no definite nucleus. This view was a very impressive one. I do not know any telescopic view of a comet that has produced such an impression of immensity. It seemed as if one were (as he really was) looking deep down into an enormous sphere of transparent vapor through which the distant stars were shining and in the center of which glowed a brighter condensation that illuminated it.

August 31, 12^{h}. Bright in 5-inch. No tail. To the naked eye it was as noticeable as a 4^{m}–5^{m} star—a small round hazy spot. It was 1^{m} brighter than a small star very close east of it ($+52\overset{\circ}{.}2547$, 5.3 magnitude).

September 1, $9^{h}30^{m}$. It was bigger than the field of view. Perhaps a faint nucleus.

September 2, 15^{h}. To the naked eye about $4\overset{m}{.}5$, like a hazy star. No tail even in 5-inch.

September 3, 13^{h}. Very large with small condensation, but no tail or nucleus. Condensation very small, around it a fainter glow for $0\overset{\prime}{.}5$, which merges into the general soft glow extending away beyond the limits of the field.

Quite noticeable to naked eye as a 4^{m}–5^{m} star. It was equal in brightness to *B.D.*$+49\overset{\circ}{.}3062$ ($4\overset{m}{.}9$). This star, however, is marked "variable."

September 10. Just before the moon rose, the comet was noticeable close north of ξ *Draconis*. It seemed inferior to the star—especially later when it was seen on a moonlit sky.

September 11, $10^{h}30^{m}$. On a moonlit sky it was visible to the eye and appeared to be of the same brightness as ξ *Draconis* (3.5 magnitude).

September 13, 8^{h}. Very conspicuous to the naked eye, but unless looked at closely it would have been taken for a 4^{m} star. No tail. The light was quite intense, but the nebulosity very small. It was of the same brightness as *B.D.*$+46\overset{\circ}{.}2349$ ($4\overset{m}{.}0$). It was as no[illegible]able as β *Draconis* ($3\overset{m}{.}0$) but not so bright, perhaps a magnit[illegible]re less than that star.

September 14, 9^{h}. In 5-inch, very bright and there seemed to be some tail preceding. To the naked eye it was of the same brightness as $+46\overset{\circ}{.}2349$ ($4\overset{m}{.}0$). It was as noticeable as β *Draconis*, but 1^{m} or more less bright. When looked at carefully the nebulosity was very small. If glanced at casually one would readily pass it over as a star.

September 15, $9^{h}30^{m}$. With naked eye it was close to a small star *B.D.*$+57\overset{\circ}{.}1702$ (5.0 magnitude). The comet was very much brighter than this star. It was about the mean brightness of ι *Herculis* and β *Draconis*.

September 16, 10^{h}. Tail quite well seen in 5-inch, for, say, 1°, but only noticeable close to comet. To the naked eye the comet was very noticeable, as much so as a third-magnitude star. It was about the mean brightness of *ι Herculis* and *β Draconis*. Faint tail perhaps seen with naked eye for 1° or more.

September 18, 10^{h}. The comet seemed to fluctuate in its light in the 5-inch while guiding on it.

It was very bright to the naked eye—like a small bright hazy star. It was almost the mean brightness of *ι Herculis* and *β Draconis*. Probably a faint tail for 3° or 4°, but not certain.

September 21, 10^{h}. With naked eye, could not be sure of any tail, but the comet itself was as bright as *ι Draconis* ($3^{\mathrm{m}}.0$).

September 22, $7^{\mathrm{h}}30^{\mathrm{m}}$. When I started to guide upon the comet with the 5-inch, a bright yellow star (+52°.1876) of the 7.0 magnitude was very slightly south from the center—about 15″. There must have been, a few minutes before, almost if not a perfectly central transit of the comet over the star. The star appeared like a bright yellow nucleus; by contrast the nebulosity of the comet was bluish white. The tail was noticeable in the telescope.

To the naked eye the comet was bright and small; could not be sure of any tail. At 8^{h} the comet was somewhat brighter than the star *ι Draconis*, *B.D.*+59°.1654 ($3^{\mathrm{m}}.0$). It was bluish white and the light rather intense. The light of the star was more intense, but at a glance the comet was brighter than the star.

September 23, 8^{h}. To the naked eye it was brighter than +59°.1654. A faint tail 3° or 4° long was visible.

September 26, 9^{h}. To the naked eye the tail was 5° or 6° long and very faint.

September 27, 10^{h}. The tail could be traced about 20°. For one-half this distance it was very straight. Located on the chart it was 15°.5 long.

September 29, 9^{h}. Well seen in moonlight with faint tail. The head was brighter than third magnitude. After moonset the sky was thickish and the comet did not appear as bright as on September 27. The tail could be faintly traced for 15° or 20°.

October 11, 7^{h}. The head was noticeable to the naked eye, but the tail was dim; it (the tail) would pass 3° or 4° to the (right) north of *η Ursae Majoris*. It could be traced 6/10 (14°) the distance to η. The head was much brighter than β *Canum Venaticorum* of 5.0 magnitude. The tail near the head was only fairly noticeable, and very dim the rest of the way.

October 12, 8^h. The sky was better and the comet brighter. The head perhaps as bright as the second magnitude. The tail was long and slender and noticeable. It would pass a couple of degrees to the left (south) of ζ *Ursae Majoris*. I could trace it nearly to that star, or over 25° in length.

October 17, 17^h. In the 5-inch there was a bright nucleus. The tail passed out of field preceding. To the naked eye the head was at least second magnitude and the tail could be traced for a few degrees, but the moon was too bright to make much out of it.

October 19, 15^h (in clouds). The nucleus was very bright in the 5-inch. No structure could be made out in that part of the tail in the field of view. The comet was very beautiful to the naked eye. The head was brighter than the first magnitude. The tail for 6° or 8° was pretty bright, but the rest of it was not striking. It seemed to broaden out, and pointed directly toward γ *Ursae Majoris*. The lower, or east, side of the tail passed 2° to the left of γ *Comae* (*B.D.*$+29^\circ.2288$) $4^m.7$ and could be traced for perhaps 10° beyond that star—about 24°. The sky was poor, being more or less misty.

October 20, $16^h 30^m$. To the naked eye the nucleus was starlike and brighter than the first magnitude. The tail for the first 3° or 4° was bright, then fainter. The left-hand (north) edge passed about a quarter of a degree or less to the right of γ *Comae*, and extended at least 10° farther, or about 25° in length. In the 40-inch telescope, at $17^h 30^m$, the nucleus was round and ill defined and not much brighter than the arc on which it was placed. Micrometric measures of the nucleus made its diameter $= 2''.24 = 1200$ km at $17^h 50^m$.

October 23, 16^h. Nucleus perhaps not so bright to the naked eye as before. It was not far from first magnitude, however. The tail, which was very long, seemed somewhat convex to the west. It could be traced to and slightly beyond *62 Leonis*, or about 25°. It passed over that star. The comet was a very conspicuous object. The tail for 10° or more was quite strong and farther out was rather faint and slender. It was a beautiful object when best seen.

In the 5-inch the head and body were slightly cone-shaped, the outline fairly definite, and very slightly brighter toward the axis. The nucleus was white, but not well defined.

October 24, 16^h30^m. Seen by glimpses through clouds. Nothing of importance.

October 25, 16^h30^m. Clouds, until daylight was killing the comet. The nucleus was much less bright, between second and third magnitude; the tail was long and straight.

To the naked eye the nucleus was just as bright as δ *Leonis* of third magnitude.

October 27, 17^h. In the 40-inch the nucleus was large and ill defined with a strong brushing out south following.

October 28, 16^h30^m. The comet was much less bright to the naked eye. The tail could be traced to β *Leonis*—about 18°—but was noticeable for only half the distance. Nucleus slightly less than δ *Virginis* $(+4\overset{\circ}{.}2669)$, $3\overset{m}{.}0$. Sky good.

FROM MT. WILSON, CALIFORNIA

November 13, 17^h. The comet was still visible to the naked eye with a tail 6° or 8° long. The nucleus was a little less bright than η *Corvi* $(B.D.-15\overset{\circ}{.}3489)$, $5\overset{m}{.}0$.

November 24, 17^h30^m. It could be seen faintly with the naked eye, and would have been brighter earlier in the night. The tail was faint, and the head was about 5.5 or 6 magnitude.

In reference to the pictures that accompany this paper, it may be interesting to add that the beautiful structures shown in the tail on the photographic plates in the latter part of October were not visible in any of the telescopes with which the comet was observed here. On several of the most interesting dates I had examined the head and the tail near it carefully to see if there was any structure that would show on the plates, but apparently there was none.

In several of the large plates accompanying this paper there are irregular defects of brightness on the sky, etc., that are not in the originals.

Micrometric measures of the position of the comet were made on a number of nights. These will be printed elsewhere.

YERKES OBSERVATORY
WILLIAMS BAY, WIS.
June 1, 1912

TERTIARY STANDARDS WITH THE PLANE GRATING[1] THE TESTING AND SELECTION OF STANDARDS

FIRST PAPER

By CHARLES E. ST. JOHN and L. W. WARE

INTRODUCTION

At the meeting, on Mount Wilson August–September 1910, of the International Union for Co-operation in Solar Research, the question arose as to the adaptability of the plane-grating spectrograph of long focus to interpolating between the secondary standards in the determinations of the standards of the third order, and in the measurement of arc, spark, and solar spectra in the international units. In the recommendations of the Committee on Wave-Lengths that were adopted by the Conference occur the following resolutions:[2]

> 4. The laboratories or observatories possessing first-rate concave gratings are invited to determine by interpolation as soon as possible standards of the third order in the spectrum of the iron arc within the above range of spectrum [i.e., λ 4282–λ 6494].
>
> 8. It is very desirable that in different laboratories possessing concave gratings of the best quality, photographs of arc, spark, and solar spectra, and new measurements according to the international system, shall be taken as soon as possible.

This preference of the concave grating over the plane grating was questioned by those who have had experience in the determinations of wave-lengths by means of the plane-grating spectrograph of long focus. The question was raised by Professor Plaskett, and in response the position of the Committee was given by Professor Kayser, who said:[3]

> I think it will be much more difficult to use plane gratings, because the spectrum is not normal and it is necessary to apply corrections. I have no plane gratings of my own and have never tried to measure wave-lengths with

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 61.

[2] *Transactions of the International Union for Co-operation in Solar Research*, **3**, 33.

[3] *Ibid.*, p. 42.

them, so I cannot tell if the order of precision is the same as for concave gratings, but in every case I should prefer concave gratings.

The discussion was later taken up by Newall, Adams, Fabry, and St. John,[1] and the point of view of those who have had experience was expressed by Newall, who said:

> Many of those who are engaged in solar observations are using plane gratings, arranged in the Littrow form, and many of us are of the opinion, rightly or wrongly, that the plane grating can compete quite well with the concave grating, provided the rest of the optical train is accurate enough. So it is hoped that provisionally we may regard the words "concave grating of the first quality" as not to be taken as of the essence of that resolution.

As is well known, considerable use of the plane grating is made at the observatory on Mount Wilson for determining the wave-length of spectral lines; and it is exceedingly important to know what degree of precision can be obtained by the use of the plane grating as compared with the concave grating in interpolating between standards about 50 Ångströms apart, when the former is employed in a spectrograph of great focal length.

The immediate motive for taking up the determination of the wave-lengths of the tertiary standards in the red arose from the need of standards for determining the wave-lengths of the oxygen band in the solar spectrum at about λ 6300. The original intention was to use the wave-lengths of the tertiary standards published by Professor Kayser,[2] but when these were tried it was at once found that they did not form a consistent series on Mount Wilson. If these tertiary standards were omitted, however, the secondary standards did form a consistent series, provided the international values for the wave-lengths were employed, instead of the "adjusted" values of Kayser. This induced us to redetermine the wave-lengths of some of the tertiary standards in this region, with what at first were surprising results. For this purpose we used the international standards of the second order. The results from a preliminary series of determinations are shown in Table I, together with the international standards and the corresponding data from Kayser's table of wave-lengths.

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[1] *Ibid.*, pp. 65–66.

[2] *Astrophysical Journal*, **32**, 217, 1910.

It appears that the wave-lengths for the two lines λ 6297 and λ 6322 agree closely with those given by Kayser and come into almost exact agreement when the differences in the standards are taken into account, while for the three remaining lines the differences from Kayser's values are far outside the limits of error. It was at first exceedingly puzzling that lines in such close proximity to each other and to the standards, as are these five lines, should give values differing so widely from Kayser's. We soon convinced ourselves, however, that these were real differences and could not be attributed to errors of measurement. In seeking for an explanation, we compared our results with the behavior of the lines under pressure as found by Gale and Adams.[1] From their results we give in the sixth column of Table I the group to which the individual lines belong according to their classification, and in the seventh column the displacements to the red under a pressure of 8 atmospheres above normal. In the case of lines that could not be measured on these plates we give the results from inspection. It is in general easy to determine the group or the type of displacement by inspection, even when no measurements are possible. The line at λ 6302 shows on their plates a tremendous displacement to the red, similar to λ 6301, but too diffuse for measurement. It is at once apparent from inspection of the table that our

TABLE I

International Standards	Kayser's Adjusted	Mt. Wilson	Kayser	Kayser — Mt. Wilson	Group	Pressure of 8 Atm. above Normal
6265.145	6265.145	....	....	0.000	*b*	0.070 Å.
....	....	6297.802	6297.804	+0.002	*b*	0.068
....	....	6301.519	6301.528	+0.009	*d*	0.25
....	....	6302.511	6302.522	+0.011	*d*	Similar to λ 6301 but unmeasurable
6318.028	6318.031	....	....	+0.003	*b*	0.080
....	....	6322.697	6322.698	+0.001	*b*	Group *b* by inspection
6335.341	6335.339	....	....	−0.002	*b*	0.074
....	....	6336.843	6336.850	+0.007	*d*	0.26
6393.612	6393.612	....	....	0.000	*b*	0.072

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 58; *Astrophysical Journal*, 35, 18, 1912.

results are related to theirs, and that the lines showing such tremendous displacements to the red under pressure on the laboratory plates are the lines showing a lessened wave-length on our plates. These lines belong to group *d* of their class 5, of which they say:[1]

> Almost all of the lines belonging to it occur in the yellow and red portions of the spectrum, and all show enormous displacements. The widening of these lines and their lack of symmetry are so great, however, that many of them are practically incapable of measurement, and for the others the degree of accuracy is extremely low. The measures when made are upon the maximum, which is always upon the violet side of the center of the line.

The first thought was that in the case of unsymmetrical lines the two types of gratings might conceivably give the lines a different appearance. The plane grating having no astigmatism reproduces the exact form of the lines, while the great astigmatism of the concave grating tends to obliterate the differences between the middle and the extremities of the lines, giving them a more uniform appearance throughout their length. This difference in the behavior of the two types of gratings, if effective, would tend, however, in the case of the plane grating to give a longer wave-length for lines so unsymmetrical on the red side, while our measurements gave shorter wave-lengths for these lines. It then occurred to us that the differences were of the order that the differences in atmospheric pressure between the Pasadena laboratory and the observatory on Mount Wilson would produce. The elevation of Mount Wilson is 5886 feet (1794 m), the mean barometric height being 25.4 inches (62.2 cm). The elevation of the Pasadena laboratory is about 800 feet (244 m), so that the difference of altitude corresponds to a change of approximately one-fifth of an atmosphere. Under this decrease of pressure the lines under consideration would undergo an absolute displacement of 0.006 Ångström to the violet, on the basis of the measurements of Gale and Adams. Considering the difficulty of the measurement of these plates, it seemed probable that the cause of the discrepancies would be found in the extreme sensitiveness of these lines to changes of pressure. As to the appearance of these lines under a pressure of 8 atmospheres above normal, Gale and Adams say:[1]

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 58; *Astrophysical Journal*, **35**, 15, 1912.

> Group *d* is made up of a very few lines in the violet, a fair-sized group in the greenish-yellow, and a small number in the red, all of which show immense displacements. The lines are bright and widened enormously to the red, the wings extending sometimes from 5 to 10 Ångström units. The precision of measurement upon these lines necessarily is very low, in some cases the determination amounting to little more than an estimate of the maximum within the broad band.

The preliminary work seemed to promise interesting and important results, and it was then decided to pursue the investigation further. No one questions the adaptability of the plane grating to differential measurements, but in the plan here proposed the absolute wave-lengths were to be determined both on Mount Wilson and in the Pasadena laboratory. In this way the pressure-shifts would be determined by the differences, while for lines belonging to the same groups as the standards two independent determinations of wave-lengths would be obtained.

APPARATUS

The following investigations were made upon two series of plates:

1. A series taken on Mount Wilson with the 30-foot (9 m) Littrow spectrograph used in connection with the 60-foot tower telescope, in which a plane grating by Michelson was used. This grating has an available ruled surface of 2.25 inches (57.1 mm) × 4.88 inches (124 mm) and gives excellent definition for both bright and dark lines.

2. A series of plates made for us by Mr. Babcock with the 30-foot spectrograph of the Pasadena laboratory, in which another Michelson grating was used.

Thus the two series were made separately by two observers with two different instruments. In general the second order of the spectrum was employed, with a scale of approximately 1 mm = 0.88 Ångström, but for the detailed investigation of particular regions the third order was employed in which the scale of the plates is approximately 1 mm = 0.56 Ångström. The second order was used for the purpose of having a large number of standards upon each plate, and in order that the scale might be comparable with that used by Professor Kayser in obtaining his list of standards of the third order, since at first it was thought that the accuracy obtainable by means of the plane grating might be shown by a comparison of our results with those already published by Professor Kayser. Later it developed that a more convincing test of their precision could be made by a comparison of our results for Pasadena and the summit of Mount Wilson.

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[1] *Op. cit.*, pp. 31–32.

The arc used in both series is of the type suggested by Pfund,[1] in which a bead of iron oxide rests in a small cup-shaped bowl of the lower positive electrode consisting of an iron rod. The upper electrode is also of iron and carries a brass bushing to prevent overheating of the electrode. The arc was fed with a current of about 6 amperes from a 110-volt direct-current circuit. This type of arc proves to be very satisfactory in the steadiness of its burning, even for periods of an hour.

METHOD OF REDUCTION

The objection raised by the users of the concave grating to the employment of the plane grating for interpolating between standards is, of course, that the scale of the spectrum produced by the plane grating varies continuously along the plate. In practice the tilt of the photographic plate in the spectrograph of the Littrow type is such that it is tangent to the focal curve of the lens, and with an achromatic lens of 30 feet focus, plates 17 inches (43 cm) long may be made so nearly coincident with the curve for their whole length that the definition of the lines is excellent over the full extent of the plate. The scale is then so nearly a linear function of the wave-length that for runs of 50 to 60 Ångströms it is practically linear within the limits of measurement. In every case the scale of the plate at any point is given by the ordinates of a smooth curve whose abscissae are the wave-lengths of the spectral region covered by the plate. There can be no irregularities in the course of such a curve, as at each point it is a function of the corresponding point of the smooth color-curve of the lens employed, and of the angular distance from the middle of the plate. It is the practically linear character of the curve representing the scale of the plate that makes it possible to use the plane grating for interpolating between standards, and the inherent smoothness of the curve that makes it feasible to test the consistency of a series consisting of several standards. In the reduction of our plates the curve giving the scale of an individual plate was found by obtaining the factor (interval in Ångströms between two successive standards divided by the measured interval between the standards) for each two successive standards of the second order. These factors were carried to five significant figures, and plotted as ordinates with the mean wave-lengths of the corresponding intervals as abscissae. A smooth curve was drawn through the points, or sometimes straight lines were drawn from point to point, both methods yielding the same final results. Such curves are shown in Plate VIII, and will be discussed later in this paper.

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[1] *Astrophysical Journal*, **27**, 296, 1908.

In interpolating to obtain the wave-lengths of intermediate lines, only lines were considered which were situated between two standards on the plate under reduction; that is, lines near the ends of the plates and beyond the last standards at either end were not measured. To obtain such lines a series of overlapping plates was used. In calculating the wave-length of any line it was referred to the nearest standard of lesser and also to the nearest standard of greater wave-length. The reduction factors were read from the curve at the points whose abscissae were the means of the wave-lengths of the required line and the standards of lesser and greater wave-length, the approximate wave-length of the line required being known or previously obtained. It will be seen that the course of the curve at the points at which the factors are taken is influenced not alone by the ordinate of the curve fixed by the two standards directly involved, but by the standards on either side of those to which the required line is immediately referred. Hence, the wave-length of the intermediate line is based upon at least four standards and the agreement between the results of the forward and back interpolations furnishes a basis for judging of the consistency of the standards involved, and the method binds the determination of the wave-length of the intermediate line to the four standards.

The measurement of the plates was done mainly upon a Toepfer comparator with a screw 300 mm long, and upon a smaller Toepfer comparator with a screw 150 mm long. Some plates, however, were measured upon Gaertner comparators with screws 80 mm long. In all cases the periodic errors were known to be negligible, and the errors of run were carefully investigated and the corrections applied to the measurements. The plates were measured red to right and red to left. At least four settings were made on each line in the two positions, bringing the cross-wire to the center of the line alternately from the right and left. In many instances, particularly in the case of the standards and some lines difficult to measure, the number of settings was doubled and sometimes tripled. Single and double cross-wires were employed, their use depending upon the intensity of the plates. In obtaining the dispersion-curve from the standards, it would be desirable to measure over the whole length of a plate without changing its position on the comparator. This we were not able to do even with the largest comparator, which required one break in the series. The conditions are less favorable as the range of the screws becomes shorter, and there is a great loss of time, as well as greater liability to error, in measuring overlapping sections of the plate in order to piece the sections together. In the latter case special care is needed in lining up the spectra and the screw. The error introduced by lack of parallelism between the spectra and the screw is not so serious in interpolating between two standards as in obtaining the distances between successive standards upon which the course of the dispersion-curve of the plate depends. A comparator is now under construction in the observatory instrument shop especially adapted to work of this kind, with which a plate 50 cm long can be measured without changing its adjustment. With this instrument it will be possible to obtain standards upon plates taken with higher dispersion, and accordingly to reach a higher degree of accuracy in obtaining a dispersion-curve for testing the consistency of a series of standards.

THE TESTING OF THE INTERNATIONAL STANDARDS

REGION λ 5975–λ 6494

For this region international standards of the second order are available which belong to a single group when classified according to pressure-shift, and over the narrow region under investigation have the same pressure-shift per atmosphere. They have, therefore, the same relative wave-lengths under wide variations of pressure, and hence are suitable for comparing the wave-lengths of other lines measured both in Pasadena and on Mount Wilson. For convenience of reference these standards are assembled in Table II together with the data from Gale and Adams as to pressure-shift, Kayser's adjusted values, and their deviations from the international standards.

TABLE II

STANDARDS OF THE SECOND ORDER

International Standards	Group	Δ at 8 Atm.	Kayser's Adjusted	I.—K.
(5975.354)	*b*	0.054	5975.354	0.000
6027.059	*b*	0.062	6027.062	−0.003
6065.492	*b*	0.077	6065.489	+0.003
6137.701	*b*	0.078	6137.703	−0.002
6191.568	*b*	0.086	6191.568	0.000
6230.734	*b*	0.070	6230.734	0.000
6265.145	*b*	0.070	6265.145	0.000
6318.028	*b*	0.080	6318.031	−0.003
6335.341	*b*	0.074	6335.339	+0.002
6393.612	*b*	0.072	6393.612	0.000
6430.859	*b*	0.068	6430.848	+0.011
6494.993	*b*	0.065	6494.994	−0.001
Mean....................		0.071		
Mean shift per atmosphere		0.009		

The line at λ 5975 is not a standard of the second order, but a standard was needed near this wave-length, as it was desirable to include the two lines at λ 6003 and λ 6008 in the investigation. It was known from an inspection of the plates of Gale and Adams that these lines were enormously widened and displaced to the red, under a pressure of 9 atmospheres, and, in fact, Mr. Adams made a tentative measurement of the displacements of these two lines on one plate but did not consider the results of sufficient accuracy to be included in their published list. They are given in our Table VI. The region is poor in lines and λ 5975 is not of good quality. It is, however, the only line available, and it belongs to the same group as the international standards, and therefore retains its relative wave-length at observing-stations at different elevations, an essential property of standards, as will appear in the course of this investigation. The wave-length given by Kayser for this line was adopted provisionally.

The inherent smoothness of the dispersion-curve for a plate makes it possible to test the consistency of a series of standards with a degree of accuracy which depends upon the length of the intervals between the standards and the precision of the measurements of the distances between the standards on the plate. This is of such great importance in the examination of standards that we give in Table III the complete data for such a curve extending over a range of about 300 Ångströms (between λ 6191 and λ 6494).

TABLE III

International Standards	Intervals	Distances	Factors	Mean λ	Kayser's Adjusted Values	Factors
6191.568					6191.568	
	39.166	45.7586 mm	0.85593	6211.151		0.85593
6230.734					6230.734	
	34.411	40.2802	0.85429	6247.940		0.85429
6265.145					6265.145	
	52.883	62.0582	0.85215	6291.586		0.85220
6318.028					6318.031	
	17.313	20.3567	0.85048	6326.684		0.85024
6335.341					6335.339	
	58.271	68.6640	0.84864	6364.476		0.84867
6393.612					6393.612	
	37.247	44.0138	0.84626	6412.236		0.84601
6430.859					6430.848	
	64.134	76.0395	0.84343	6462.926		0.84359
6494.993					6494.994	

The measurements were made on spectra of the second order taken on plates 43 cm long, inclined to the axis of the spectrograph to bring the plate tangent to the focal curve of the spectrographic lens of 30 feet (9 m) focus. In the sixth column are given the adjusted values according to Kayser, and in the seventh column the corresponding factors. Our curves were plotted with the reduction factors as ordinates on such a scale that 1/16 of an inch

(1.6 mm) represented one unit in the fourth place, and in interpolating we read the factor from the curve to the fifth significant figure. The wave-lengths were plotted as abscissae on such a scale that one of the above divisions represented one Ångström. The original curves from which Plate VIII was reduced were prepared in our draughting-room by Mr. Nichols; the data for the curves were given to him and he was requested to locate the points with great precision. Curve *a*, shown on this plate, corresponds to the data in the fourth and fifth columns of the table. A smooth curve was drawn, and, as is seen, the points fall upon the curve in a most satisfactory way. These data are the means of fourteen measurements and are practically free from accidental errors. Each measurement has given a similar curve but not quite so smooth as the composite curve. Upon the same plate are plotted the points given by using Kayser's adjusted values for the standards. The first three standards agree so that points 1 and 2 coincide. Point 3 is very slightly above, point 4 very much below, point 5 above, point 6 below, and point 7 above the smooth curve. That is, the points determined from Kayser's adjusted values fall in such a way that a smooth curve cannot be drawn upon which the points all lie. A glance at the curve shows at once how consistent with each other are the international standards, and how irregular are the adjusted values. The precision of the method is quite remarkable; a change of 0.002 Ångström in an interval distinctly displaces the corresponding point. The first interval changed by Kayser is the third—52.883 Ångströms—and though this long interval is lengthened only by 0.003 Ångström the ordinate of the point is increased five units in the fifth place. The fourth interval —17.313 Ångströms—is decreased by 0.005 Ångström which displaces the point downward twenty-four units in the fifth place. The fifth point is raised by increasing the interval from 58.271 to 58.273 Ångströms. The sixth point is an important one as it involves a line for which Kayser's adjusted value differs much from the international standard. In reference to this he says:[1]

> This [his] method of measurement has also yielded values for the standards themselves, which I regard as somewhat more accurate than the secondary

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[1] *Astrophysical Journal*, **32**, 217, 1910.

international standards, inasmuch as they are adjusted. They are in general in good accord and only in a few cases is the difference large, in the case of λ 6430 reaching the amount of 0.012 Ångström. It remains to be investigated whether my value is more accurate than the other.

Referring to the curve, it will be seen that point 6 involving this line is greatly displaced; in fact, the point is lowered by 25 units in the fifth place; and point 7, also involving this line is raised 16 units in the fifth place. It will be noticed that if the wrong value is assigned to one wave-length of a series, the points in which this wave-length are involved are displaced one up and one down so that in such a case the value at fault can easily be ascertained. Judged by the results of this examination of these eight lines, the values for the international standards form a consistent series, no one of which probably contains an error exceeding 0.001 Ångström. The line λ 6430 is an excellent line for measurement, hard and sharp on all of our plates. The values by the three independent observers employing the interference method agree well, viz.:

Fabry-Buisson	Eversheim	Pfund
6430.859	6430.862	6430.855

It is difficult to understand how all of these measurements could be sufficiently in error for the wave-length to be such as Kayser gives it, namely 6430.848, a value not included within the range of the other three. It would tend to destroy faith in the precision of the interference method if such errors were found in the case of lines so excellent for measurement as λ 6430. The method which we illustrate of testing the adjustment of standards is capable of detecting an error of 0.002 Ångström in the case of standards 50 Ångströms apart, and depends only upon the accuracy of the measurements of their distances apart on the photographic plate, values which can be determined to a very high degree of precision. It is essential that a sufficient number of standards be found upon one plate, and with high dispersion this condition requires a plate of great length. These conditions will be well fulfilled by the new 75-foot spectrograph on Mount Wilson, in which a plate 40 inches (100 cm) long can be used. A series of overlapping plates is necessary in order that the lines under investigation may not be too near the end of the series under consideration. The curve *b* on Plate VIII is drawn from data similar to that given in Table III, and is based on 28 measurements of plates covering the region λ 5975–λ 6265, thus overlapping the first region so that points 5 and 6 of this curve are points 1 and 2 on curve *a*. The international and Kayser's adjusted values are:

International. .	(5975.354)	6027.059	6065.492	6137.701	6191.568	6230.734	6265.145
Kayser.	.354	.062	.489	.703	.568	.734	.145
Int.−Kayser. .	.000	−.003	+.003	−.002	.000	.000	.000

As in the other region, the points determined from the international values fall on the curve in a very satisfactory way, while the four points involving the three adjusted values fall alternately above and below. The line λ 5975, as before mentioned, is not an international standard, but the value assigned to it by Kayser appears to fit into the series of international standards. It belongs to the same group when classified according to pressure-shift. It is, however, a line difficult to measure with accuracy and hence not suitable for a standard of the second order. Judged by this curve the relative wave-lengths of the international standards λ 6027–λ 6137 are free from errors as great as 0.001 Ångström.

REGION λ 5371–λ 5658

This region is considered by itself because of two difficulties that arise. The first is due to the fact that the international standards involved belong to two groups when classified according to pressure-shift, and that in these groups the relative wave-lengths change a measurable amount with a decrease in pressure of one-fifth of an atmosphere, the change between Pasadena and the observatory on Mount Wilson. The second difficulty is that some adjustments are indicated in the international standards. The standards and related data are given in Table IV.

The first section of the table contains lines of group *a* (Gale and Adams), the flame lines, for which the displacements at 8 atmospheres were very accurately determined by them. The lines in the second section of the table were provisionally classified by them in group *d*, the lines showing great widening and displacement to the red under pressure. The precision of measurement of the displacements of these lines at 8 atmospheres is low, as they have said. The relative change in wave-length between the two sections of the table is 0.003 Ångström for a pressure-change of one-fifth of an atmosphere. After our previous experience in the red region we expected to detect this change in wave-length upon the plates taken upon Mount Wilson. It ought to manifest itself by the point determined by λ 5506 and λ 5569, the terminal and initial lines of the two sets of standards, being slightly displaced upward on the dispersion-curve. We show the results in curve *c* in Plate VIII. The data for the curve are the means of 20 measurements and should be practically free from accidental errors. The mean of the two standards λ 5497 and λ 5506 was used in plotting the curve in order to weight the intervals between standards more equally. In the case of an interval so short as that between these two lines, an error of a few ten-thousandths of an Ångström in their wave-length, or a few ten-thousandths of a millimeter in the measured distance between the lines, displaces the point very markedly.

TABLE IV

STANDARDS OF THE SECOND ORDER

International Standards	Group	Δ at 8 Atmospheres	Kayser	I.−K.
5371.495	*a*	0.029	5371.490	+0.005
5405.780	*a*	0.027	5405.778	+0.002
5434.527	*a*	0.027	5434.527	0.000
5455.614	*a*	0.029	5455.616	−0.002
5497.522	*a*	0.030	5497.521	+0.001
5506.784	*a*	0.031	5506.783	+0.001
Mean Δ........................	....	..0.029		
Mean Δ per atmosphere..........	....	..0.0036		
5569.633	sub *d*	0.14	5569.630	+0.003
5586.772	sub *d*	0.12	5586.774	−0.002
5615.661	sub *d*	0.13	5615.667	−0.006
5658.836	sub *d*	0.15	5658.846	−0.010
Mean Δ........................	....	..0.14		
Mean Δ per atmosphere..........	....	..0.017		

Point 5 is the one determined from λ 5506 and λ 5569. It falls, however, very near the curve, while point 6 is distinctly above the curve. This was somewhat surprising, and at first a little disconcerting. The points 2 and 3 are plainly displaced, indicating a small error in the wave-length of λ 5434, the only line concerned with both points. Later when the plates taken in the laboratory in Pasadena were reduced and the dispersion-curves for this region were plotted it at once became evident where the difficulty lay. Curve *f*, Plate VIII, is based upon 20 measurements made on plates taken in Pasadena. Points 2 and 3 are displaced as on the curve for the Mount Wilson plates, confirming the indication of a probable error in λ 5434. It also shows displacements of points 5 and 6 from the mean curve, thus indicating an error in λ 5569. The wave-lengths of the lines λ 5434 and λ 5569, in terms of the other standards, can be obtained by interpolation from these curves. The wave-lengths so determined differ by 0.002 and 0.001 Ångström respectively from the international means for these lines. The second, third, and fourth columns of Table V are adjusted to these values.

TABLE V

PROPOSED ADJUSTMENTS

INTERNATIONAL STANDARDS	PASADENA	MOUNT WILSON	
		A	B
5371.495	5371.495	5371.495	5371.495
5405.780	5405.780	5405.780	5405.780
5434.527	5434.52$\underline{9}$	5434.52$\underline{9}$	5434.52$\underline{9}$
5455.614	5455.614	5455.614	5455.614
5497.522	5497.522	5497.522	5497.522
5506.784	5506.784	5506.784	5506.784
5569.633	5569.63$\underline{4}$	5569.63$\underline{1}$	5569.63$\underline{4}$
5586.772	5586.772	5586.7$\underline{69}$	5586.772
5615.661	5615.661	5615.65$\underline{8}$	5615.661
5658.836	5658.836	5658.83$\underline{3}$	5658.836

Curves are given on Plate VIII corresponding to the four columns of Table V. Those based upon the first column are the curves *c* and *f*, showing the relative wave-lengths of the unadjusted standards for Mount Wilson and Pasadena, respectively. Curve *g* is based upon the adjusted wave-lengths given in the second column as determined from the measurement of the standards on the Pasadena plates. It now has a smooth run and so also has curve *d*, which corresponds to the third column in which the suggested changes have been made on λ 5434 and λ 5569. The wave-lengths of the second group of standards have been corrected for their displacement of 0.003 Ångström, on account of the decreased atmospheric pressure on the mountain, relative to the flame lines of the first section of the table. This change having been allowed for, they ought to form a consistent series; as is seen, the points determined from the mountain measurements now fall upon the curve. Then curve *e* was drawn, using the wave-lengths in the fourth column, in which the suggested changes in the Pasadena values have been made so that they form a consistent series, but no consideration is taken of the relative change in wave-length of the two groups of standards under the pressure-change of one-fifth of an atmosphere that occurs in passing from the altitude of Pasadena to the observatory on Mount Wilson. With these adjustments in the low-altitude values, points 2 and 3 fall on the curve, and point 5, depending upon the terminal and initial lines of the two groups of standards, is displaced slightly upward. It was expected that this would be the case when the mountain plates for this region were first reduced and the point was found to fall on the curve. It appears from the observational data here given that the international values for the wave-lengths of λ 5434 and λ 5569 are slightly in error, and that under the decreased pressure on Mount Wilson the wave-length of λ 5569 was shortened so that its actual wave-length differed only by 0.002 Ångström from that given in the table of the international standards. At the altitude of Mount Wilson it formed a part of a practically consistent series with the flame lines, and therefore no great irregularity was shown in the curve at the point where the two groups joined. The uncertainty regarding point 5 in the first reduction of the mountain plates, using the international values, was caused by the long interval—63 Ångströms—between the terminal and initial lines of the two groups of standards, by the partial compensation by the displacement due to decrease of pressure, and especially by the relatively large displacement of point 6 in the same direction due to the small error in λ 5569 effective over the short interval—17 Ångströms—between it and λ 5586. The manner in which these results confirm one another, however, furnishes strong grounds for believing that they correspond to the facts of the case, and illustrates the precision of the method. The corrections suggested are within the range covered by the international determinations of the individual standards, as the comparison shows.

Fabry and Buisson	Eversheim	Pfund	Suggested	Kayser
5434.530	.524	.528	.529	.527
5569.632	.636	.631	.634	.630

Of the ten standards in this region Kayser suggests adjustments in nine. In case successive lines are given algebraically equal increments, the interval is not changed, but the intervals preceding and following such a series of adjustments are altered and the positions of the corresponding points of the dispersion-curve therefore displaced. The points for the dispersion-curve calculated from Kayser's adjusted wave-lengths in the fourth column of Table IV are shown on curve *h*, Plate VIII. The smooth curve is the same as *g*. Point 2 falls upon the curve. The standards inclosing this point differ by equal increments from those upon which curve *g* is based, so that the interval remains unaltered; the preceding and following points, however, are displaced; 1 and 3 by 8 and 18 units respectively in the fifth place; 4 and 5 in the opposite direction by 5 units in the fifth place. Especially to be considered are points 6, 7, and 8, as these depend upon wave-lengths for which Kayser's suggested adjustments are −0.003, +0.002, +0.006, and +0.010 Ångström. The interval to which point 6, greatly displaced, corresponds is a short one—17 Ångströms—so that decreasing one standard by 0.003 Ångström and increasing the other by 0.002 Ångström produced a change in the factor of 32 units in the fifth place. The position of point 8 depends upon the relative wave-lengths of λ 5615 and λ 5658. Professor Kayser suggests an increase of 0.006 Ångström for the first and an increase of 0.010

Ångström for the last. The point is thereby raised by 9 units in the fifth place, but if the value for λ 5615 had not been increased, point 8 would have been raised 13 units more; that the suggested changes in the preceding wave-lengths were not justified is shown by the positions of points 6 and 7. The evidence, therefore, from this part of the curve is opposed to these changes in the international standards. The adjustments suggested by Professor Kayser indicated that large and systematic errors were involved in the determinations of the international standards by the interference method, as the suggested wave-lengths for these lines are not embraced within the values found by the three independent observers.

Fabry and Buisson	Eversheim	Pfund	Kayser
5615.658	.662	.663	.667
5658.835	.838	.835	.846

From the evidence furnished by the dispersion-curves it appears that none of the international standards in the region λ 5371–λ 5658 is affected with a relative error as great as 0.002 Ångström except λ 5434, and that for other lines the error probably does not amount to 0.001 Ångström except for λ 5569. If any change in the wave-length of λ 5658 is indicated by the dispersion-curve, it is a small decrease rather than a large increase; the only point, however, involving this line is the terminal one, and owing to its position the amount of the error, in case there is a small one, cannot be determined accurately; yet the evidence seems decisive against the large positive correction suggested by Professor Kayser.

The displacements of a few units in the fifth place of the factors are well within the limits of error that occur in reading the factors from the curves. We have usually checked each other when taking the factors from the curves in the interpolations for the standards of the third order, and we rarely differ by 2 units in the fifth place. The scale is greatly reduced in the reproduction shown on Plate VIII, but the larger displacements are easily seen. With the dispersion of the plates taken in the second order an error of 0.002 Ångström in the wave-length of the standard, when referred to the neighboring standards at a distance of 50 Ångströms, would displace the points involving the line in error by 3 units in the fifth place, alternately up and down, and by proportionately greater amounts in the case of more closely spaced standards. The delicacy of the method appeared in adjusting the corrections to λ 5434 and λ 5569. These were found to be slightly in excess of 0.002 and 0.001 Ångström respectively; in using the values 0.003 and 0.002 Ångström it was seen that the lines were overcorrected, while in the case of the values suggested they are slightly undercorrected.

In obtaining the international standards of the second order, it was hoped that the standards would be accurate to 0.001 Ångström in the visible spectrum, and it appears from our examination of the region λ 5371–λ 6494 that practically this hope has been realized, as we have not found errors to exceed that limit except in the case of one line. With secondary standards whose wave-lengths are known to such exactness it will be easily possible to reach a like precision for good lines in the case of the tertiary standards.

WAVE-LENGTH DETERMINATION

REGION λ 5371–λ 6494

In arranging the results of our measurements we found that the lines showing displacements to the red under pressure fall into the groups proposed by Gale and Adams in an important paper on the spectra of iron and titanium under pressure.[1]

Group a.—This involves the well-known flame lines,[2] i.e., the lines relatively strengthened in low-temperature sources, such as the flame of the arc, the low-current arc, and the electric furnace.[3] The lines of this group in the yellow-green show small but definite pressure displacements, the mean being 0.0036 Ångström per atmosphere in the arc.[4]

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[1] *Contributions from Mount Wilson Solar Observatory*, No. 58; *Astrophysical Journal*, **35**, 10, 1912.

[2] De Watteville, *Philosophical Transactions*, A **204**, 139, 1904.

[3] Hale, Adams, and Gale, *Contributions from the Mount Wilson Solar Observatory*, No. 11; *Astrophysical Journal*, **24**, 185, 1906; Adams, *Contributions from the Mount Wilson Solar Observatory*, No. 40; *Astrophysical Journal*, **30**, 86, 1909; King, *Contributions from the Mount Wilson Solar Observatory*, No. 53; *Astrophysical Journal*, **34**, 37, 1911; King, *Contributions from the Mount Wilson Solar Observatory*, No. 60; *Astrophysical Journal*, **35**, 183, 1912.

[4] Gale and Adams, *op. cit.*

Group b.—To this group many lines belong; in fact all lines of moderate displacement under pressure are assigned to it for the present. These are bright and symmetrically widened under pressure, and show a mean pressure displacement of 0.009 Ångström per atmosphere for the lines in the region λ 5975–λ 6678 according to Gale and Adams.

Group c.—Group *c* has no representatives in this region.

Group d.—This contains lines showing immense displacements to the red under pressure. The lines are bright and greatly widened to the red under pressure. We have been led to divide group *d*, calling the separate portion sub-group *d*. The lines are similar in behavior under pressure, but show much smaller displacements.

Group e.—To the groups suggested by Gale and Adams we have added a group consisting of lines that remain bright and are greatly displaced to the violet and unsymmetrically widened to the violet under pressure.

DISCUSSION

LINES GREATLY DISPLACED TO THE RED

In Table VI the Pasadena and Mount Wilson values for the wave-lengths are given in the second and third columns respectively. The sixth column contains the differences Pasadena minus Mountain; the ninth and tenth columns show the differences Kayser minus Pasadena or minus mean, and Rowland minus Pasadena or minus mean. In the last column are given the displacements for 8 atmospheres according to Gale and Adams, in the case of lines for which they have given the data. For the other lines we give the results from inspection of their plates. When the lines are so diffuse under pressure that no measurements of value can be made we have said "unmeasurable," though the displacement of the maximum is easy to see. In some cases it has not been possible to fix the type by inspection, but the differences Pasadena—Mount Wilson, and Rowland—Pasadena have shown the group to which the line belongs.

The mean difference, Pasadena—Mount Wilson, for the lines of group *d* in the yellow-green is 0.0075 Ångström. This corresponds to a change in pressure of one-fifth of an atmosphere, or a

TABLE VI

LINES GREATLY DISPLACED AND UNSYMMETRICALLY WIDENED UNDER PRESSURE

Not Recommended for Standards

GROUP	INTERNATIONAL WAVE-LENGTHS		MEAN DEVIATION		P.−MT.	WIDTHS IN ÅNGSTRÖMS		KAYSER−PASADENA	ROWLAND−PASADENA	UNDER 9 ATMOSPHERES
	Pasadena	Mount Wilson	P.	Mt.		Pasadena	Mt.			
Iron *d*, green	5393.188(20)	.182(21)	5	4	+0.006	0.100	0.105	−0.002	0.187	Δ=0.14 Å
	5476.587(33)	.578(10)	4	3	+0.009	0.108	0.113	−0.006	0.191	Δ=0.11
					+0.0075	0.104	0.109		0.189	
								K.−Mean	R.−Mean	
Iron sub *d*	5569.634(s)							−0.004	0.214	Δ=0.14 Å
	5572.859(9)	.858(10)	4	3	+0.001	0.120	0.128	−0.001	0.216	Δ=0.14
	5576.109(17)	.107(15)	6	5	+0.002	0.086	0.090	−0.006	0.212	Δ=0.16
	5586.772(s)							+0.002	0.219	Δ=0.12
	5602.958(5)	.964(11)	4	4	−0.006	0.100	0.100	+0.001	0.225	Δ=0.15
	5615.661(s)							+0.006	0.216	Δ=0.13
	5624.559(19)	.560(12)	3	2	−0.001	0.125	0.126	+0.003	0.210	Δ=0.16
	5638.275(11)	.280(4)	4	3	−0.005	0.078	0.108	+0.012	0.211	Δ=0.15
	5658.836(s)							+0.010	0.216	Δ=0.15
					−0.002	0.102	0.110		0.215	
								K.−P.	R.−P.	
Iron *d*, red	5976.816(14)	.804(19)	5	4	+0.012	0.093	0.105	−0.016	0.191	Diffuse. Max. to red. Unmeas.
	5983.712(18)	.705(28)	6	2	+0.007	0.119	0.135	−0.003	0.196	5984 displ. to v. overlaps
	6003.051(11)	.038(26)	6	6	+0.013	0.121	0.125	−0.022	0.188	Δ=0.38 Å
	6008.595(11)	.581(25)	6	7	+0.014	0.124	0.125	−0.015	0.190	Δ=0.40
	6147.844(6)	.836(8)	4	3	+0.008	0.064	0.102	0.000	0.196	Indeterm. Normal arc weak
	6151.636(4)	.628(4)	4	4	+0.008	0.097	0.107	−0.003	0.198	Indeterm. Normal arc weak

Iron *d*, red —*Cont'd*	6180.225(7)	.216(10)	5	6	+0.009	0.084	0.120	+0.005	0.195	Indeterm. Normal arc weak
	6232.669(11)	.662(24)	4	3	+0.007	0.089	0.101	0.000	0.187	Diffuse. Max. to red. Unmeas.
	6246.350(12)	.337(36)	4	4	+0.013	0.123	0.135	−0.005	0.185	Δ=0.28 Å
	6301.531(16)	.518(18)	4	4	+0.013	0.093	0.109	−0.003	0.187	Δ=0.25
	6302.520(11)	.511(17)	4	3	+0.009	0.083	0.084	+0.002	0.189	Diffuse. Max. to red. Unmeas.
	6336.851(16)	.843(17)	4	3	+0.008	0.084	0.098	−0.001	0.197	Δ=0.26 Å
	6400.026(6)	.020(14)	4	2	+0.006	0.123	0.137	+0.010	0.191	Δ=0.24
	6408.044(13)	.038(8)	3	3	+0.006	0.072	0.096	+0.001	0.189	Diffuse. Max. to red. Unmeas.
	6411.678(16)	.668(14)	3	3	+0.010	0.108	0.137	−0.005	0.187	Δ=0.23 Å
					+0.0095	0.098	0.114	+0.018 −0.073	0.191	
Mn	6013.528(20)	.516(5)	4	4	+0.012	0.108	0.109	−0.012	0.187	Diffuse. Max. to red. Unmeas.
	6016.668(17)	.663(13)	6	8	+0.005	0.118	0.123	−0.002	0.193	Diffuse. Max. to red. Unmeas.
	6021.830(13)	.824(9)	5	3	+0.006	0.133	0.137	−0.009	0.186	Diffuse. Max. to red. Unmeas.
					+0.008	0.120	0.123	−0.023	0.189	
Iron *e*	5383.353(7)	.371(8)	9	4	−0.018	0.141	0.147	+0.007	0.225	Diffuse. Max. to v. Unmeas.
	5410.890(13)	.911(8)	6	2	−0.021	0.096	0.108	+0.012	0.234	Diffuse. Max. to v. Unmeas.
	5415.175(10)	.195(13)	6	3	−0.020	0.118	0.122	+0.009	0.241	Diffuse. Max. to v. Unmeas.
	5424.038(11)	.060(12)	6	5	−0.022	0.136	0.138	+0.012	0.252	Diffuse. Max. to v. Unmeas.
	5543.173(7)	.185(5)	6	4	−0.012	0.096	0.106	+0.011	0.241	Indeterm.
	5554.872(14)	.887(3)	6	3	−0.015	0.090	0.109	+0.006	0.250	Diffuse. Max. to v. Unmeas.

TABLE VI—*Continued*

GROUP	INTERNATIONAL WAVE-LENGTHS		MEAN DEVIATION		P.—MT.	WIDTHS IN ÅNGSTRÖMS		KAYSER—PASADENA	ROWLAND—PASADENA	UNDER 9 ATMOSPHERES
	Pasadena	Mount Wilson	P.	Mt.		Pasadena	Mt.			
Iron *e* —*Cont'd*	5565.688(17)	.702(4)	7	4	−0.014	0.111	0.112	−0.003	0.243	5563 displ. to red overlaps
	5598.279(10)	.303(4)	8	1	−0.024	0.105	0.125	+0.013	0.245	No material
	5984.796(15)	.807(26)	5	7	−0.011	0.124	0.139	−0.001	0.244	5983 displ. to red overlaps
	5987.050(17)	.058(28)	3	4	−0.008	0.120	0.135	−0.005	0.240	Diffuse. Max. to v. Unmeas.
	6007.961(9)	.969(17)	5	5	−0.008	0.125	0.150	−0.008	0.225	Indeterm. Normal arc weak.
	6024.051(16)	.059(29)	5	4	−0.008	0.188	0.199	−0.004	0.230	Diffuse. Max. to v. Unmeas.
	6042.083(11)	.093(20)	6	4	−0.010	0.088	0.109	+0.009	0.232	Diffuse. Max. to v. Unmeas.
	6055.983(11)	.995(22)	6	6	−0.012	0.107	0.133	+0.009	0.244	Diffuse. Max. to v. Unmeas.
	6078.470(14)	.482(21)	8	5	−0.012	0.135	0.136	+0.006	0.240	Diffuse. Max. to v. Unmeas.
					−0.014	0.119	0.131	+0.094 −0.021	0.239	

displacement per atmosphere of 0.0375 Ångström relative to the standards, on the assumption that the differences, Pasadena—Mount Wilson, are attributable entirely to pressure displacement. The standards are displaced to the red 0.0036 Ångström per atmosphere. The absolute displacement is therefore 0.041 Ångström per atmosphere, an amount in excess of the values found by Gale and Adams for these lines, but they say their measurements were little more in some cases than "an estimate of the maximum within the broad band." The mean difference, Pasadena—Mount Wilson, for the lines of group *d* in the red is 0.0095 Ångström; this becomes 0.048 Ångström per atmosphere relative to the standards of group *b*, which show displacements to the red of 0.009 Ångström per atmosphere. The absolute displacement to the red is then 0.057 Ångström per atmosphere, an amount again in excess of the value found by Gale and Adams. On the whole, however, the degree of agreement is surprising rather than otherwise. Assuming that the displacement varies as the cube of the wave-length, the relation found by Gale and Adams for the lines of iron belonging to a common group, the agreement between the observed and calculated values for group *d*, yellow-green, based upon the value 0.057 Ångström for group *d*, red, is within the limits of error, viz., group *d* (yellow-green), observed 0.041 Ångström, calculated 0.038 Ångström. Neither the small pressure-changes of about one-fifth of an atmosphere taken advantage of in this investigation, nor the high pressures used by Gale and Adams are well adapted to the study of lines of this type, and it is purposed to examine *in vacuo* and under normal pressure the behavior of an extended list of lines belonging to groups *d* and *e*.

Sub-Group d.—The mean displacement per atmosphere for the lines of this group, found by Gale and Adams, is 0.019 Ångström to the red. Our results are in complete agreement with this, as we found that the intervals between the international standards of this group and the standards of group *a*, the flame lines, remained constant when the relative displacements for a decrease of pressure of one-fifth of an atmosphere were taken into account. As a basis, displacements of 0.0036 Ångström per atmosphere for the flame-line standards and of 0.019 Ångström per atmosphere for

TABLE VII

Lines Moderately Displaced and Symmetrically Widened under Pressure Recommended for Standards

Group	International Wave-Lengths		Mean Deviation		P.—Mt.	Mean	Kayser—Mean	Rowland—Mean	Under Pressure of 9 Atm.
	Pasadena	Mount Wilson	P.	Mt.					
a......	5371.495(s)	....	..	..	....	....	−0.005	complex	Δ=0.029 Å
a......	5405.780(s)	....	..	..	....	....	−0.002	0.209	Δ=0.027
a......	5429.702(13)	.702(20)	1	2	0.000	5429.702	−0.001	0.209	Δ=0.029
a......	5434.529(s)	....	..	..	....	....	−0.002	0.211	Δ=0.027
a......	5446.919(33)	.920(20)	3	3	−0.001	5446.919	−0.001	0.211	Δ=0.031
a......	5455.614(s)	....	..	..	....	....	+0.002	0.220	Δ=0.029
a......	5497.522(s)	....	..	..	....	....	−0.001	0.213	Δ=0.030
a......	5501.469(33)	.471(15)	2	2	−0.002	5501.470	−0.001	0.213	Δ=0.030
a......	5506.784(s)	....	..	..	....	....	−0.001	0.216	Δ=0.031
a......	5535.419(12)	.419(12)	3	4	0.000	5535.419	+0.001	0.225	Δ=0.034
?......	5543.944(1)	.944(12)	..	..	0.000	5543.944	0.000	0.213	
?......	5563.613(15)	.612(5)	4	4	+0.001	5563.613	−0.005	0.211	
							+0.003	0.214	
							−0.019		
b......	5975.354	....	..	..	....	....	0.000	0.221	Δ=0.054 Å
b......	6027.059(s)	....	..	..	....	....	+0.003	0.215	Δ=0.062
b......	6065.492(s)	....	..	..	....	....	−0.003	0.217	Δ=0.077
?......	6127.913(15)	.912(16)	5	3	+0.001	6127.912	+0.015	0.212	....
b......	6136.626(22)	.624(21)	1	2	+0.002	6136.625	+0.003	0.204	Δ=0.082
?......	6136.998(14)	7.000(12)	5	3	−0.002	6136.999	+0.013	0.211	....
b......	6137.701(s)	....	..	..	....	....	+0.002	0.214	Δ=0.078
b......	6157.731(18)	.731(13)	5	2	0.000	6157.731	+0.009	0.214	Δ=0.041
?......	6165.367(18)	.366(18)	5	3	+0.001	6165.366	+0.021	0.211	....
b......	6173.341(21)	.343(17)	7	2	−0.002	6173.342	+0.008	0.211	Δ=0.067
b......	6191.568(s)	....	..	..	....	....	0.000	0.211	Δ=0.086

b......	6200.322(9)	.322(21)	3	3	0.000	6200.322	+0.008	0.205	Δ=0.079 Å
b......	6213.437(12)	.439(31)	4	2	−0.002	6213.438	0.000	0.206	Δ=0.072
?......	6215.150(9)	.152(19)	5	4	−0.002	6215.151	+0.002	0.209	....
b......	6219.287(17)	.290(34)	5	2	−0.003	6219.289	0.000	0.205	Δ=0.073
b......	6230.734(s)	....	..	..	....	....	0.000	0.209	Δ=0.070
b......	6252.569(17)	.567(37)	3	2	+0.002	6252.568	−0.001	0.205	Δ=0.077
b......	6254.267(17)	.268(37)	2	2	−0.001	6254.268	+0.001	complex	Δ=0.064
b......	6265.145(s)	....	..	..	....	....	0.000	0.203	Δ=0.070
b......	6297.800(24)	.802(40)	2	2	−0.002	6297.801	+0.003	0.206	Δ=0.068
?......	6315.310(14)	.313(22)	3	3	−0.003	6315.312	+0.011	0.205	....
b......	6318.028(s)	....	..	..	....	....	+0.003	0.211	Δ=0.080
?......	6322.693(21)	.695(34)	2	2	−0.002	6322.694	+0.004	0.213	....
b......	6335.341(s)	....	..	..	....	....	−0.002	0.213	Δ=0.074
?......	6344.159(2)	.159(6)	0	4	0.000	6344.159	+0.005	0.212	....
?......	6355.040(7)	.039(9)	2	2	+0.001	6355.039	+0.006	0.207	....
b......	6393.612(s)	....	..	..	....	....	0.000	0.208	Δ=0.072
b......	6421.363(16)	.362(14)	2	2	+0.001	6421.363	−0.009	0.207	Δ=0.068
b......	6430.859(s)	....	..	..	....	....	−0.011	0.207	Δ=0.068
b......	6494.993(s)	....	..	..	....	....	+0.001	0.220	Δ=0.065
							+0.118	0.210	
							−0.026		

the lines of sub-group *d* were employed. In other words, when the international standards of those two groups formed a consistent system in Pasadena they remained consistent on Mount Wilson when the displacements of the lines of sub-group *d* relative to the lines of group *a* were allowed for, but otherwise not. That the lines of sub-group *d* do not belong to group *d* is again clearly evidenced from the values of the differences, Rowland—Pasadena, of which more will be said later.

Sub-group *d*, as far as now known, consists of 9 lines in the yellow-green, 4 of which are among the international standards of the second order. The other 5 were measured on the Pasadena and Mount Wilson plates with a mean difference between the two stations of 0.002 Ångström. If the lines belong to the same group as the standards used, in this case those of sub-group *d*, the two measurements should agree. Considering the extremely poor quality of the standards and of the measured lines, the agreement may be regarded as satisfactory, their displacements under pressure agreeing within the limits of error with the displacements of the standards, a result already found by Gale and Adams.

In dealing with standards of this quality, the question of their availability for such an exacting purpose at once arises. These lines are greatly displaced and unsymmetrically widened under pressure. If it were a case of displacement only, the lines still remaining symmetrical, adjustments could be made for such changes of pressure as occur between sea-level and the mountain observatories, and the lines would still be usable for standards; but they are in practice usually unsymmetrical, the amount of dissymmetry depending upon the pressure and the quantity of vapor in the source, and, in general, two observers will not agree upon individual lines. This has been the case with the authors of this paper, but the means derived from the measures made upon a large number of lines show a good degree of agreement. To be of service in a series of standards in which errors are expected not to exceed 0.002 Ångström, the quality of a line should be such that a reasonable number of measurements will give that degree of precision. We show in Table IX the results of our individual measures of the distances in millimeters between the lines on 10 exposures

TABLE VIII

LINES OF QUESTIONABLE UTILITY AS STANDARDS

GROUP	INTERNATIONAL WAVE-LENGTHS		MEAN DEVIATION		P.−MT.	MEAN	K.−MEAN	REMARKS
	Pasadena	Mount Wilson	P.	Mt.				
	5379.588(4)	.587(3)	1	2	+0.001	5379.588	−0.005	Material insuff. Quality poor
	5400.507(6)	.504(4)	3	4	+0.003	5400.506	+0.004	Material insuff. Quality poor
	5445.039(8)	.037(12)	5	6	+0.002	5445.038	−0.001	From best plates. Very faint
?.....	5462.958(4)	.956(5)	3	2	+0.002	5462.957	+0.007	From plates showing best separation. Lines diffuse. Merged under pressure. Like λ 6102–6103
?.....	5463.268(9)	.268(4)	2	2	0.000	5463.268	+0.004	
	5473.917(28)	.916(6)	3	5	+0.001	5473.917	−0.007	Measures difficult and uncertain
a.....	5476.295(33)	.298(3)	3	4	−0.003	5476.296	−0.002	Near diffuse line λ 5476.587. Sep. difficult
b?....	6020.178(5)	.176(21)	7	2	+0.002	6020.177	−0.010	Fe line distant 0.169 Å
	6089.571(14)	.568(11)	5	3	+0.003	6089.570	+0.011	Too faint on our plates for good measures
?.....	6102.179(13)	.181(21)	3	3	−0.002	6102.180	+0.002	Diffuse. Merges with 6103 under pressure. Like λ 5462–5463
?.....	6103.183(2)	.181(2)	8	2	+0.002	6103.182	+0.010	Fe line distant 0.114 Å
b.....	6256.	.375(35)	..	2	....	6256.375	+0.002	Blend with nickel on valley plates
	6280.623(3)	.628(28)	4	4	−0.005	6280.626	+0.005	Insuff. valley material. Very faint
	6462.740(8)	.737(10)	8	2	+0.003	6462.738	−0.004	Insuff. material for difficult line

upon a plate in the third order, covering this region, with a scale 1 mm = 0.58 Ångström. We include two lines of group *a* for a comparison between the measures upon good and poor lines. The mean deviations are given in the last line. The abbreviations S. and W. refer to St. John and Ware.

TABLE IX

QUALITY OF SECONDARY STANDARDS λ 5569–λ 5658

GROUP *a*		SUB-GROUP *d*							
5497–5506		5506–5569		5569–5586		5586–5615		5615–5658	
S.	W.	S.	W.	S.	W.	S.	W.	S.	W.
15.918	.920	108.428	.427	29.715	.710	50.175	.174	75.283	.277
.917	.918	.430	.430	.696	.698	.178	.181	.287	.286
.916	.920	.419	.421	.708	.708	.177	.176	.281	.286
.918	.923	.432	.431	.692	.698	.173	.176	.289	.284
.921	.917	.423	.425	.692	.695	.178	.180	.302	.301
.920	.918	.431	.432	.699	.690	.180	.186	.286	.287
.917	.917	.431	.428	.682	.675	.188	.184	.294	.298
.920	.916	.424	.420	.700	.706	.178	.177	.286	.290
.920	.917	.439	.430	.688	.703	.175	.178	.288	.286
.919	.920	.440	.424	.694	.697	.175	.177	.299	.295
15.9186	.9186	108.430	.427	29.697	.698	50.178	.179	75.289	.289
0.0014	.0017	.0049	.0034	.0072	.0070	.0027	.0031	.0051	.0056

The agreement between our measures for the group *a* interval is complete; and if we had been obliged to rely upon one or two exposures, the divergence from the general mean would not have made a serious error. In the case of intervals involving the lines of sub-group *d*, however, our measures for the same exposures vary greatly and capriciously, but the general means are in fair agreement. It is evident that very divergent results would be obtained by using single exposures, and even a fair degree of accuracy can be had only by the use of an inconveniently large number of plates, a sufficient quantity of which in many cases would not be available.

Lines of the character of sub-group *d* will present different appearances upon spectra taken with concave and plane gratings. The astigmatism of the concave grating greatly lengthens the lines and tends to obscure the dissymmetry by distributing it along the length of the line. This would be a source of error, particularly if the ends of the lines were cut off as in the case of a close-lying comparison spectrum. When the image of the middle of the arc is held stationary upon the slit of a plane-grating spectrograph, the dissymmetry of such a line becomes apparent; but when the image plays over the slit as in the case of an unsteady arc, the same line upon the spectrum plate may be long and apparently symmetrical. Since the inherent difficulties of the measurement of such lines are great, and the sources of error so many and insidious, we have not included these lines among those which we recommend for standards. The rejection of these lines from the series of standards would enlarge the region near λ 5800 for which the Committee has recommended that barium lines be used. In a still higher degree, the same reasoning holds in the case of the lines of group *d* and the still greater difficulties of measurement render them in our judgment unsuitable for standards of the third order.

LINES GREATLY DISPLACED TO THE VIOLET UNDER PRESSURE

Group e.—When the data from the Mount Wilson and Pasadena plates were finally assembled, some lines were noticed for which the mountain plates gave larger values than the valley plates. The quality of these lines is poor, and it was at first thought that the errors of measurement were proving greater than was expected; but it soon appeared probable that the differences between the mountain and the valley wave-lengths were real, and that the lines constitute a group the members of which are widened and greatly displaced to the violet under pressure, and therefore to the red under the lessened pressure at the elevation of Mount Wilson. In continuation of the grouping scheme of Gale and Adams, we have formed the group *e*, whose definition has already been given. On inspection of the plates of Gale and Adams taken under a pressure of 9 atmospheres with a comparison spectrum at atmospheric pressure, which they kindly placed at our disposal, these lines are seen to be enormously widened with the maximum displaced to the violet. They appear as though blown to the violet under this pressure without a maximum sufficiently pronounced for measurement, but the direction and character of the displacement can be determined with certainty by inspection.

These lines occur in the yellow-green and the red. The mean of the differences, Pasadena—Mount Wilson, is −0.014 Ångström. On the assumption that the lessened wave-length in Pasadena is due to the difference in atmospheric pressure, the displacement relative to the standards is −0.070 Ångström per atmosphere. The mean displacement of the standards per atmosphere is +0.006 Ångström; hence the absolute displacement is −0.070 Ångström +0.006 Ångström=−0.064 Ångström. This would mean an enormous displacement for these lines, and there has been as much hesitancy in accepting displacements to the violet under pressure even to a small amount, as there has been in accepting displacements to the red of the order of those which the measurements given in this paper appear to show.

Since the lines showing these great displacements are unsymmetrically widened under pressure, in the case of group *d* to the red and of group *e* to the violet, it has been thought by others, notably Fabry and Buisson,[1] that such displacements were only apparent and arose from the difficulties inherent in the comparative measurement of such lines. Widening and displacement under pressure are intimately connected, and it is exceedingly difficult to determine whether there is actual displacement of a line, when the widths of the line are different under the two conditions of pressure. To render our data so far as possible free from such errors, and to guard against their entering into the differences, Pasadena—Mount Wilson, in such a way as to produce or increase these differences we measured the widths of the lines and based the comparison only upon plates for which the widths of the lines on the mountain plates equaled or slightly exceeded the widths of the corresponding lines on the Pasadena plates. The measurements of the widths are given in the seventh and eighth columns of Table VI.

As to the lines showing the enormous displacements to the red, the results of this paper are in accord with the measurements of Gale and Adams, and no doubt of their reality could remain in the mind of anyone who has had opportunity to examine the Gale and Adams plates, even if careful measurements had not been made. The mean width on the mountain plates of the lines showing displacement to the violet under pressure is 0.131 Ångström and on the Pasadena plates, 0.119 Ångström. The increase of width in the case of such lines is unsymmetrical, whether the increase is due to increased pressure or to greater density of the radiating vapor, and is largely on the violet side of the line. The mean width of the lines of this group on the mountain plates exceeds the mean width on the Pasadena plates by 0.012 Ångström, so that in the absence of pressure-shift, the wave-lengths should have been less on the mountain than in the valley by approximately 0.006 Ångström. As a matter of fact we found them longer by 0.014 Ångström. On the assumption of no pressure displacement to the violet this would imply an average error of 0.020 Ångström in the measurement of lines whose mean width is 0.120 Ångström. The mean deviations averaged 0.006 Ångström for the Pasadena plates and 0.004 Ångström for the mountain plates, with mean probable errors of 0.0017 Ångström and 0.0011 Ångström respectively. The mountain and valley plates were measured by the same observers, reduced by the same methods, the lines referred to the same standards, and the plates intermingled in the course of the measurement, so that the effects due to systematic error and personal equations were eliminated as completely as possible. In view of the above considerations, and the degree of precision obtained in the case of normal lines, there seems sufficient ground for adopting the direct interpretation of the data, viz., that the lines of groups *d* and *e* are greatly displaced as well as unsymmetrically widened by pressure, and to the red and violet respectively.

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[1] *Comptes rendus*, **148**, 688, 1240, 1909.

RELATION BETWEEN WAVE-LENGTHS IN ARC AND IN ROWLAND'S TABLE

Reference has been made at various times in this paper to the differences, Rowland—Pasadena. The subject deserves a somewhat fuller discussion. The relation has been of service in the present investigation in classifying the lines into groups which agree with those formed upon the basis of pressure-shift. In the comparison we have used the wave-lengths in Pasadena in the case of the lines in groups *d*, *e*, and Mn; for the lines of groups *a*, *b*, and sub *d*, we have used the means of the mountain and valley plates. The differences, Rowland—Kayser, show the same classification, as appears from a consideration of Table X.

That there is a physical basis for the classification cannot be doubted. This is particularly evident in the case of groups *d*, *e*, and Mn. It is not so evident from the Table in the case of groups *a*, *b*, and sub *d*, but it becomes evident when their respective pressure-shifts 0.0036 Ångström, 0.009 Ångström, 0.019 Ångström, are taken into account. It would be interesting to discuss these results somewhat fully from the point of view of some solar problems, but such a discussion would lead far afield, and investigations now in progress are furnishing so much additional material bearing upon the subject that any discussion would be premature.

TABLE X

ROWLAND DIFFERENCES

	a	*b*	*d*	Sub *d*	*e*	Mn
S. and W.	0.214	0.210	0.191	0.215	0.239	0.189
Kayser.	0.215	0.207	0.195	0.213	0.233	0.196

Much has been said about the irregular and apparently capricious differences that are obtained in comparing the wave-lengths of Rowland's table with wave-lengths in the arc, and efforts have been made to determine some factor or operator by which the Rowland wave-lengths can be converted into international units. It is clear from Table X that the apparently large irregularities in the Rowland differences are not inaccuracies, but represent real differences. This is clearer still when one considers that the lines in groups *d*, *e*, and Mn are nearly all sharp lines in the solar spectrum, and hence the accidental errors in Rowland's table in the case of these lines are a minimum. It is equally clear that no one factor can be found by which the wave-lengths can be converted from one system into the other with satisfactory accuracy, even for a limited region of the spectrum. This late justification of Rowland's values for individual lines has of course no bearing upon the systematic errors pointed out by Fabry and Perot.[1] In fact the curve plotted from the data in Table VII shows the same course as theirs. The method proposed by them for passing from one system to the other is alone capable of yielding satisfactory results.

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[1] *Astrophysical Journal*, **15**, 272, 1902.

LINES RECOMMENDED FOR STANDARDS

The criterion applied in selecting the lines suitable for use as standards of the third order has been the close agreement of the wave-lengths as determined from the mountain and from the Pasadena plates. The lines yielding the same values from the two series of plates either belong to the same group as the standards, or undergo practically the same displacement for small changes in pressure. The lines examined are separated into three sections, namely: λ 5371–λ 5563 referred to the flame-line standards, λ 5569–λ 5658 referred to the standards of sub-group *d*, and λ 5975–λ 6494 referred to standards belonging to group *b*. The second section has already been considered and for the reasons given omitted from the list of recommended lines. Two groups of lines, *d* and *e*, occurring in the first and third sections, have also been dropped from the list of available lines because of their extreme sensitiveness to changes in the pressure and density of the emitting vapor, manifesting itself in displacement and great dissymmetry. There remain only 25 lines which we are able to recommend as tertiary standards. The distribution of these lines in reference to the 17 international standards of the second order may be seen in Table VII. The results of our examination of 100 lines are as follows:

Recommended	Standards of the second order	17
	Standards of the third order	25
Rejected	Lines of group *d*	17
	Lines of sub-group *d*	9
	Lines of group *e*	15
	Lines of manganese	3
Questionable		14
		100

We have not been able to measure the many lines of intensity 2 of Kayser's list, as they either did not appear on our plates, or were too faint for the purpose of measurement. Of the 14 lines in the questionable list, a large proportion will probably prove available and be extremely useful as faint-line standards when their wave-lengths are definitively determined; but our material was so insufficient and the quality of the lines so poor that we do not include them in the list of recommended standards on the basis of our measurements even though the precision is apparently high (mean p.e. 0.0016 Ångström for the Pasadena plates, p.e. 0.0011 Ångström for the mountain plates). It would appear from the agreement of the mountain and valley measurements that the lines belong to the same groups as the standards to which they were referred but in four instances this is not the case. The pair of lines λ 5462–5463 were found by Adams[1] to be much stronger in the flame than in the core of the arc, and therefore would appear to belong to the group of flame lines, but under a pressure of 9 atmospheres their appearance differs widely from the appearance of the flame lines proper. They seem related to the pair λ 6102–6103. In the normal arc they are all hazy, and under a pressure of 9 atmospheres they appear enormously diffused with no indication of displacement.

THE PRECISION OF THE MEASUREMENTS

The precision that may be obtained by the use of a plane grating is best shown by the agreement between the wave-lengths obtained from the two series of plates in the case of lines belonging to the same group as the standards used in their measurement. In Table VII the measurements of such lines are given. The mean of the differences, Pasadena—Mount Wilson, taken without regard to sign, is 0.0010 Ångström. This seems to us a more satisfactory measure of the precision obtained than the mean probable errors of 0.0007 Ångström and 0.0006 Ångström in the case of the valley and mountain plates respectively. As these measurements are absolute and not differential, the precision may be taken as representing the degree of accuracy given by the plane grating in interpolating between the international standards of the second order in the case of lines of good quality. This is the same order of precision obtained by one of us in a previous investigation[2] when the conclusion was drawn that with homogeneous standards an accurracy of 0.001 Ångström could be obtained in the case of the best lines.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 40; *Astrophysical Journal*, **30**, 126, 1909.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 44; *Astrophysical Journal*, **31**, 143, 1910.

SOME RELATED CONSIDERATIONS

In comparing our results with those published by Kayser, systematic differences appear, for some of which we can find no explanation. In the case of the lines of groups *b*, sub *d*, and the three lines of manganese, our values from the Pasadena plates are larger than his in the mean by 0.0037 Ångström; in the case of the lines of group *e*, our values from the Pasadena plates are smaller in the mean by 0.0056 Ångström. As these lines show more or less dissymmetry, the first group widening more on the red edge, the latter on the violet edge when the pressure or density of the emitting vapor is increased, such systematic differences may represent the personal equations of the observers in setting upon lines of this character, or may depend upon the different appearances that the two types of gratings would give to such lines.

There appear systematic differences between our results and those of Kayser in the case of lines belonging to groups *a* and *b*, for which our two series of plates show a high degree of agreement. These differences persist after allowance has been made for the different values of the secondary standards to which the tertiaries are referred in the two instances. For the lines referred to the flame-line standards of group *a*, λ 5371–λ 5463, Kayser−S. and W.= +0.0003 Ångström; for the lines of group *b*, λ 5975–λ 6355, Kayser−S. and W.= +0.005 Ångström; for the two lines, λ 6421 and λ 6462, the differences are accounted for by the large difference in the wave-length of the standard λ 6430. Thus at the beginning and end of the list the values agree, but in the intermediate region the difference is larger than it seems possible to assign to accidental errors. In a paper published during the preparation of this article for the press, Goos[1] reports similar systematic differences between his values and those of Kayser for the blue-green region which he has investigated by means of a plane grating, and thinks that these deviations are to be explained by the fact that Kayser used a simple linear interpolation between each two of the secondary standards and then averaged the results.

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[1] *Astrophysical Journal*, **35**, 229–230, 1912.

The result of his previous investigation of the conditions which influence the dispersion in the case of the concave grating, and which should be taken into account is stated as follows:

> Es ist bedenklich, bei Konkavgitterspektrogrammen, selbst innerhalb enger Intervalle, linear ohne Anbringung von Korrektionen zu interpolieren, da man nicht im voraus weiss, in welchem Bereich der Platte die Konstanz des Maasstabes ausreichend ist.[1]

In practice the interpolations in the case of the concave grating appear fully as troublesome as in the case of the plane grating, when a high degree of precision is required. With the plane-grating spectrograph of long focus, the dispersion is apparently more nearly linear than it is normal with the concave grating, and between linear and normal dispersion there is not much to choose on the ground of convenience in interpolation. There seems to be no good reason for assuming a priori that the precision is in one case higher or lower than in the other. It appears that in either case the dispersion for each plate must be considered by itself, and its departure from the normal or linear character must be determined in order to fix the interval through which the dispersion is normal or linear to the required degree.

The idea that changes in atmospheric pressure, such as occur in passing from sea-level to the altitudes of modern observatories, may alter the relative wave-lengths of spectrum lines by measurable amounts introduces a new point of view, and suggests other conditions that a series of standards should fulfil when the highest precision is to be obtained. The red region of the spectrum which is especially rich in lines showing great displacements had not been systematically studied before Gale and Adams undertook its examination. In view now of the large amount of observational data accumulated, recognition should be given to the possibility and the probability of introducing errors by neglecting the effects of relative pressure displacement. Even in the case of some standards of the second order the change in relative wave-lengths amounts to 0.003 Ångström on Mount Wilson, and for observatories of greater elevation it may reach 0.005 Ångström. For other iron lines the change is 0.010 Ångström on Mount Wilson, and may reach 0.015 Ångström at the higher stations. There are a number of important observatories at which such changes in wave-length would prove disturbing.

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[1] *Zeitschrift für wissenschaftliche Photographie*, **10**, 206, 1911.

OBSERVATORIES AT HIGH ALTITUDES

Heidelberg, Germany	570 m	Astronomical and Astrophysical Institute, Königstuhl
Berne, Switzerland	573	Observatory of the Cantonal University
Madrid, Spain	655	Astronomical and Meteorological Observatory
San Luis, Argentina	800	Southern Observatory of the Carnegie Institution
Mount Hamilton, California	1283	Lick Observatory
Denver, Colorado	1650	Chamberlain Observatory
Mount Wilson, California	1794	Solar Observatory of the Carnegie Institution
Flagstaff, Arizona	2210	Lowell Observatory
Tacubaya, Mexico	2280	National Observatory of Mexico
Palani Hills, India	2347	Kodaikânal Observatory
Arequipa, Peru	2452	Station of Harvard College Observatory
Bogota, Republic of Colombia	2634	National Observatory of Republic of Colombia
Quito, Ecuador	2908	National Observatory of Ecuador

GENERAL SUMMARY

1. The precision obtainable with the plane-grating spectrograph in determining the absolute wave-lengths of standards of the third order by interpolation between the international standards is 0.001 Ångström in the case of good lines. Two entirely different instruments were used and the mean of the differences between the wave-lengths in the two series is 0.0010 Ångström, a quantity less than the sum of the probable errors. This represents the performance of two similar instruments; whether the actual wave-lengths were determined with this degree of precision or whether there is some obscure systematic error in the methods will appear when more determinations are made by other observers with other instruments.

2. The secondary standards between λ 5371 and λ 6494, except λ 5763, were examined in reference to their relative wave-lengths. The conclusion is that there are no errors in their relative wave-lengths exceeding 0.001 Ångström, except in the case of the line λ 5434, where it reaches 0.002 Ångström.

3. Of the 100 lines studied in the region λ 5371–λ 6494, we find 17 thoroughly suitable for secondary standards, and 25 for tertiary standards; 44 are discarded because of abnormal pressure displacements accompanied with dissymmetry; and 14 are provisionally classified as of questionable utility.

4. In comparing the wave-lengths from the two series of plates, Mount Wilson and Pasadena, it was found that in the case of 20 lines the wave-lengths determined from the valley plates were in the mean 0.009 Ångström longer than the wave-lengths determined from the mountain plates when lines of the same width were compared. All of these lines, that are of sufficient intensity to appear upon the Gale and Adams plates at a pressure of 9 atmospheres, show strongly unsymmetrical widening and large displacements of the maxima to the red. They seem to us inherently unsuitable for standards and therefore have been rejected.

5. It likewise appeared from the comparison of Mount Wilson and Pasadena determinations of wave-lengths that 15 lines gave values on the valley plates shorter than on the mountain plates by 0.014 Ångström, when the comparison was made between lines of equal widths. Upon the plates of Gale and Adams taken at 9 atmospheres, these lines are very unsymmetrically widened and the diffuse maxima displaced to the violet. There seems sufficient reason for considering them a distinct group, the components of which are really displaced to the violet and unsymmetrically widened under pressure. The reasons for rejecting the lines showing great displacement to the red hold in even greater degree for these lines.

6. From the large differences that appear in the amounts by which different lines are displaced under pressure, it would be desirable to know the pressure displacements of all lines that are finally accepted as standards of the second or third order. The secondary standards λ 5569–λ 5658 are a case in point, the differences between the displacements of these lines and the neighboring standards being 0.015 Ångström per atmosphere. The difficulties of exact measurement due to their lack of sharpness and to their dissymmetry are so great that we are forced to omit them from the list of recommended lines. In the selection of barium lines to

supplement the iron lines, a preliminary examination of the barium lines with regard to displacements under pressure is to be recommended; and in extending the series of standards to the red, caution should be especially exercised in the selection of lines, in view of the occurrence in the red of lines showing great pressure displacements and the rapid increase of displacement with wave-length.

Mount Wilson Solar Observatory
May 1912

ON THE DETERMINATION OF THE ORBITAL ELEMENTS OF ECLIPSING VARIABLE STARS. II[1]

By HENRY NORRIS RUSSELL

PART III. ECCENTRICITY OF ORBIT

§ 5. *Determination of the elements.*—The problem is now completely solved for a circular orbit. But analogy with the orbits of spectroscopic binaries of similar period leads us to expect that the actual orbits of eclipsing variables will often show a small eccentricity (rarely exceeding 0.1); and this is confirmed in several cases by displacement of the secondary minima from the position midway between the primary minima, or by spectroscopic observations of the variables.

In such a case the elements computed from the light-curve as above on the assumption of a circular orbit will differ from the true values. If, however, the eccentricity is small enough to justify the neglect of its square, simple equations can be found which express very approximately the relation between the true and computed elements.

Let us consider the brighter body as moving about the fainter (for convenience in comparison with spectrographic results). Let e be the eccentricity of the orbit, and ω the longitude of periastron (counted as usual from the ascending node in the direction of motion), r the radius vector, v the true, and M the mean anomaly. Then the projected distance of centers of the stars is given by the equation

$$\delta^2=r^2\{\cos^2 i+\sin^2 i\ \cos^2 (v+\omega)\}\ . \tag{23}$$

We have also, neglecting e^2 and taking the major axis of the orbit as unity,

$$r=1-e\cos M\ ,\qquad v=M+2e\sin M\ .$$

If now we set $\theta=M+\omega-\frac{\pi}{2}$, θ will be an angle increasing uniformly with the time, which when $e=0$ becomes identical with the angle called θ in Part I. It may be defined as the mean longitude, measured from a point 90° in advance of the node. Introducing this into the above expressions, and for brevity setting $e \sin \omega = g$, $e \cos \omega = h$, we find

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[1] Continued from *Astrophysical Journal*, **35**, 340, 1912.

$$r = 1 - g \cos \theta + h \sin \theta\,, \qquad v + \omega = \frac{\pi}{2} + \theta + 2h \cos \theta + 2g \sin \theta\,.$$

Introducing these into (23) and neglecting terms of the second and higher orders in the small quantities g and h, we find after some reductions

$$\delta^2 = (1 - 2g \cos \theta) \cos^2 i + (1 + 2g \cos \theta) \sin^2 i \sin^2 \theta + 2h \sin \theta (1 + \sin^2 i \cos^2 \theta)\,.$$

If now we set

$$\theta' = \theta + h\,(\operatorname{cosec}^2 i + 1)\,, \tag{24}$$

and again neglect terms of the second order, this becomes

$$\delta^2 = (1 - 2g \cos \theta') \cos^2 i + (1 + 2g \cos \theta') \sin^2 i \sin^2 \theta' + 2h \sin \theta' (1 - \cos \theta')(1 - \sin^2 i \cos \theta')\,.$$

Setting

$$A \sin \gamma = (1 + g \cos \theta') \sin i\,,$$
$$A \cos \gamma = (1 - g \cos \theta') \cos i\,,$$

whence

$$\left.\begin{aligned} A &= 1 + g(1 - 2 \cos^2 i) \cos \theta'\,, \\ \cot \gamma &= (1 - 2g \cos \theta') \cot i\,, \end{aligned}\right\} \tag{25}$$

we find

$$\delta^2 = A^2 (\cos^2 \gamma + \sin^2 \gamma \sin^2 \theta') + 2h \sin \theta' (1 - \cos \theta')(1 - \sin^2 i \cos \theta')\,. \tag{26}$$

This expression is a minimum when $\theta' = 0$; that is, $\theta' = 0$ at the middle of eclipse. If the epoch of this phase is t_1, we have at any time t

$$\theta' = \frac{2\pi}{P}(t - t_1)\,, \tag{27}$$

which is identical with equation (8) defining the angle θ employed in the computation of elements for a circular orbit. For the secondary minimum we may set

$$\theta'' = \theta + \pi - h(\operatorname{cosec}^2 i + 1)\,, \tag{28}$$

and find in the same fashion

$$\left.\begin{aligned} \delta^2 &= B^2\,(\cos^2\beta+\sin^2\beta\,\sin^2\theta'') \\ &\qquad -2h\sin\theta''(1-\cos\theta'')\,(1-\sin^2 i\cos\theta'')\,, \\ &\text{where} \\ B &= 1-g(1-2\cos^2 i)\cos\theta''\,, \\ \cot\beta &= (1+2g\cos\theta'')\cot i\,, \\ \theta'' &= \frac{2\pi}{P}(t-t_2)\,, \end{aligned}\right\}\qquad(29)$$

where t_2 is the epoch of middle of secondary minimum. In these equations nothing has been neglected except the square of the orbital eccentricity and terms depending upon it.

The last term in equations (26) and (29) is very small unless θ attains large values. Neglecting it for the moment, we see that the light-curve during either eclipse may with very considerable approximation be regarded as symmetrical, and identical with that resulting from the mutual eclipse of two stars of the same radii r_1 and r_2 as the actual stars, moving in a circular orbit which for the principal eclipse must be assumed to be of radius A and inclination γ, and for the secondary eclipse of radius B and inclination β.

These quantities contain variable terms of the form $g\cos\theta'$. Since θ' vanishes at the middle of the eclipse, the changes in these terms will be small, and we may with considerable approximation replace $\cos\theta'$ by its mean value η during the eclipse, and A and γ by their corresponding mean values A_m and γ_m. If ζ is the value of θ' at beginning or end of eclipse, we have with sufficient approximation $3\eta=2+\cos\zeta$.

The observed light-curve during principal minimum will therefore differ very little from one computed for a circular orbit with inclination γ_m, radius A_m, and stars of radii r_1 and r_2, or, since the linear dimensions of the system may be changed in any ratio without affecting the light-curve, by an orbit of radius unity, inclination γ_m, and stars of radii $\frac{r_1}{A_m}$ and $\frac{r_2}{A_m}$. It follows at once that, if we determine the circular elements which best represent the observed light-curve by the methods already developed (assuming the radius of the orbit to be unity), the ratio $k=\frac{r_2}{r_1}$ will be given by this calculation with approximate accuracy, and the other elements will closely approach the values:

$$\left.\begin{array}{ll} \text{Principal Minimum:} & \text{Secondary Minimum:} \\ r'_1 = r_1(1-g\eta)\,, & r''_1 = r_1(1+g\eta)\,, \\ \cot i' = \cot i\,(1-2\,g\eta), & \cot i'' = \cot i(1+2g\eta), \\ \text{where} & \\ g = e \sin \omega\,, & 3\eta = 2+\cos \zeta\,. \end{array}\right\} \qquad (30)$$

These equations involve only $e \sin \omega$. To this degree of approximation the other component of the eccentricity, $h = e \cos \omega$, affects only the interval between the primary and secondary minima. From the equations (24)–(29) we find at once

$$h\ (\operatorname{cosec}^2 i + 1) = \frac{\pi}{P}(t_2 - t_1 - \tfrac{1}{2}P)\,, \qquad (31)$$

a very well known relation, which suffices to determine $e \cos \omega$ with high accuracy.[1]

The determination of $e \sin \omega$ is less easy and in practice somewhat less exact. The equations (30) cannot often be used as they stand, for the secondary minimum is not usually deep enough to permit an independent determination of the elements from its light-curve. We must distinguish two cases, according as the principal minimum does or does not show a constant phase.

In the first case k, r'_1, and i' are determined for the primary minimum in the usual way, assuming a circular orbit. We have then from (30)

$$r''_1 = r'_1(1+2g\eta)\,, \qquad \cot i'' = \cot i'(1+4g\eta)\,. \qquad (32)$$

Assuming different values of g, the light-curve of the secondary minimum may be computed and the value of g found which best represents the observations. Only the times of the four contacts need be computed in the first approximation, and the work involved is small. If the principal eclipse occurs near periastron, and is nearly grazing, the secondary eclipse may be partial.

If the principal minimum shows no constant phase the problem is more complicated, for the method previously used for finding the elements is inapplicable, as it involved the assumption that the eclipsed area is the same at the two minima. In the present case we have to regard the percentages of area eclipsed at each minimum as unknown, α_1 and α_2. Between these quantities and the light-intensities λ_1 and λ_2 at the two minima, there exists the relation

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[1] C. André, *Traité d'astronomie stellaire* (Paris, 1900), p. 241.

$$\frac{1-\lambda_1}{\alpha_1}+\frac{1-\lambda^2}{k^2\alpha^2}=1\,. \tag{33}$$

The subscript 1 here refers to the eclipse of the smaller star by the larger. Whether this is the principal or the secondary minimum must be determined by trial.

From the light-curve for the principal minimum we can derive a relation between k and α_1 (or α_2, as the case may be) of the form

$$\chi(k,\ \alpha_1,\ \tfrac{1}{4})=\text{const.} \tag{34}$$

Assuming any value of k we may find α_1 from Table III and then α_2 from (33). If the light-curve of the secondary minimum is sufficiently well defined to determine $\chi(k,\ \alpha_2,\ \frac{1}{4})$ with precision, the comparison of the computed and observed values of this function will suffice to determine the correct value of k.

Otherwise we may compute the actual light-curve for the secondary minimum for assumed values of k, proceeding as follows. Having found α_1 and α_2 as above, we take out from Table I the values p_1 and p_2, corresponding to k, α_1 and k, α_2. The projected distances between the centers of the stars at the middle phases of the two minima are then

$$\delta_1=r_1(1+p_1k)=(1-g)\cos i\,,$$
$$\delta_2=r_1(1+p_2k)=(1+g)\cos i\,,$$

whence

$$g=\frac{(p_2-p_1)k}{2+(p_1+p_2)k}\,. \tag{35}$$

We may now derive the circular elements r'_1 and i' which best represent the principal minimum by the equations (20) (Part I), and then, by equations (32), find the elements r''_1 and i'' which will best represent the secondary minimum on the assumption of a circular orbit. From these the light-curve (or in the first trials merely the times of beginning and end of eclipse) may be computed.

The whole sequence of these operations can be completed, with the aid of the tables, in a few minutes, so that the process of finding the value of k which gives the best light-curve for the secondary minimum is not so long as it may appear. It is of course well to start with the elements obtained on the assumption of a circular orbit. A few trials will suffice to determine within what limits an acceptable value of k must lie. It may happen that these limits are wide; in which case the problem is nearly or quite indeterminate.

§ 6. *Asymmetry of the light-curve.*—We have still to consider the final term of equation (26), which, changing sign with θ', evidently gives rise to a lack of symmetry between the ascending and descending branches of the light-curve. Suppose that the same brightness (that is, the same value of δ) is reached for $\theta'=\theta_0+\epsilon$ and $\theta'=-\theta_0+\epsilon$. We then find, easily, neglecting terms of the second order,

$$\epsilon=-h(\sec\theta_0-1)(\operatorname{cosec}^2 i-\cos\theta_0)\,, \tag{36}$$

which gives the displacement of the mean epoch of two corresponding phases of ascending and descending light whose interval is $2\theta_0$. The displacement of the secondary minimum, relative to the epoch following the primary by half a period, is in the same units, by (31),

$$\Delta=\frac{2\pi}{P}(t_2-t_1-\tfrac{1}{2}P)=2h(\operatorname{cosec}^2 i+1)\,,$$

whence we have

$$\frac{\epsilon}{\Delta}=-\frac{(\sec\theta_0-1)(\operatorname{cosec}^2 i-\cos\theta_0)}{2(\operatorname{cosec}^2 i+1)}\,. \tag{37}$$

This is always a small fraction. If we set $i=70°$, $\theta_0=45°$ (values well outside those found ordinarily in practice), this ratio becomes 0.04. If $\theta_0=30°$, it is 0.01. The asymmetry of the light-curve is therefore always very small compared with the displacement of the secondary minimum, and in fact is usually too small to be detected by observation—another well-known result (André, *Traité d'astronomie stellaire*, **2**, 248). The negative sign in (37) denotes that the duration of the light change is longer on that side of the minimum which lies nearest to the adjacent secondary minimum.

On account of this, and also because of other neglected terms, the light-curve computed rigorously from elliptical elements obtained as above will not agree exactly with that derived from the preliminary circular elements, and may not represent the observations as well as the latter, though the deviations will usually be small. In such a case we may obtain a satisfactory approximation by starting with a fictitious light-curve which deviates at every point as far, and in the same sense, from that computed from the circular elements as the empirical light-curve which best represents the observations deviates from that computed from the elliptic elements. Representing this as well as may be by new circular elements, we shall be led to new elliptic elements, which will obviously give a light-curve agreeing very closely with the observed curve. Such refinements of calculation will obviously be very rarely necessary; but their introduction will take much less time than an attempt to correct the elements by differential methods, which will seldom if ever be advisable.

The problem of determining the elements, when the stars are supposed to present uniformly illuminated circular disks, is now completely solved.

PART IV. ELLIPTICITY OF THE STARS

§ 7. *General theory.*—In certain eclipsing variables—the classic example being *β Lyrae*—the light-changes are continuous, and the brightness is sensibly greater half-way between minima than just before or after eclipse.

This phenomenon is reasonably explained by the assumption that the stars are elongated by their mutual attraction, a hypothesis which is the more probable, since in such systems the component stars usually appear to be separated by a distance not greatly exceeding their own diameters.

The researches of Darwin[1] indicate that the figure of such stars is probably nearly ellipsoidal, with the longest axis in the line joining the components, and the shortest perpendicular to the orbit plane.

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[1] *Philosophical Transactions*, A **206**, 160–248, 1906.

If the components are equal in mass and density, they will be similar in form. Otherwise the smaller mass will be the more elongated unless its density considerably exceeds that of the other. In the absence of data bearing on this question, and to avoid what are practically unsurmountable difficulties of calculation, we will assume (along with those who have previously discussed the problem)[1] that the components are similar and similarly situated ellipsoids whose longest axes coincide with the line joining their centers; and also, as in all our work so far, that they appear to the observer as uniformly illuminated disks.

A. Roberts has pointed out[2] that in such a case we cannot hope to find the degree of flattening of the ellipsoids in the direction of their polar axes. A general proof may be given as follows.

The lines of sight which touch the surfaces of the two ellipsoids form two elliptical cylinders with parallel axes, which may wholly or partly overlap one another. The apparent disks are obtained by cutting these cylinders by a fixed plane perpendicular to the line of sight. If we cut them by any other fixed plane, we obtain ellipses whose areas (including the area of any eclipsed segment) bear a fixed and constant ratio to the areas of the apparent disks, so that these latter areas may be used instead of the apparent disks in computing the amount of light which the distant observer receives.

Let us now choose the plane of the relative orbit as our fixed plane and consider two systems, one of which may be obtained from the other by increasing all the co-ordinates perpendicular to the orbit-plane (including that of the distant observer) in any constant ratio, while leaving all co-ordinates unaltered which are measured parallel to the orbit-plane. The areas in the orbit-plane, which may be used as a measure of the light received from the system, are identical in these two cases. Hence, by a proper choice of the assumed surface-brightness of the stars, we may obtain an absolutely identical light variation in the two cases. The two systems of elements will differ only in that the polar axes of the two ellipsoids, and the tangent of the angle between the orbit-plane and the line of sight (or cot i as ordinarily defined) will all be changed in an arbitrary constant ratio.

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[1] G. W. Myers, *Astrophysical Journal*, **7**, 8, 1898.

[2] A. Roberts, *Monthly Notices*, **63**, 528, 1903.

We may in general choose this constant so as to make one of our ellipsoids a prolate surface of revolution, with its longer axis coincident with the line of centers, and, since we have already assumed the two ellipsoids to be similar, this will be true of the other also.

Such a system of two prolate spheroids will be the basis of our calculations. It is physically an impossible one, for all actual figures of equilibrium which are elongated toward one another by mutual attraction must also be flattened at the poles owing to their axial rotation (whose duration must coincide with the orbital period). When we have found the extent of the elongation in the equatorial plane, the amount of the polar flattening may be at least approximately estimated. If a, b, c are the axes of the ellipsoid, in descending order of magnitude, we should have, for nearly spherical bodies

$$b-c=\tfrac{2}{3}(a-b)\ .$$

For more elongated bodies, Darwin's results for equal masses of homogeneous, incompressible fluid, may be used as a guide. They may be represented, with remarkably close approximation, by the empirical formulae

$$\frac{b^2}{a^2}=\frac{1-4.4a^3}{1+3.1a^3}\ ,\qquad \frac{c^2}{a^2}=\frac{1-3.4a^3}{1+8.6a^3} \tag{38}$$

(when the distance of centers is taken as the unit of length), as is illustrated by the following examples, which are either taken from Darwin's work, or computed by means of his equations.

a	$1-\frac{b^2}{a^2}$		$1-\frac{c^2}{a^2}$	
	From Darwin's Formulae	From (38)	From Darwin's Formulae	From (38)
0.099	0.008	0.007	0.012	0.012
.204	.059	.062	.098	.095
.284	.153	.161	.223	.232
.340	.260	.263	.352	.353
.410	.425	.425	.517	.520
.497	.664	.668	.719	.720

The value $a = 0.340$ marks, according to Darwin, the limit beyond which these figures of equilibrium, if composed of homogeneous, incompressible fluid, become unstable. This assumption differs so widely from the conditions prevailing in the stars that the actual degree of ellipticity, and the incidence of instability, may differ considerably from Darwin's figures; but the latter are nevertheless the best guide in existence in this very intricate matter.

In any event, if we have obtained a satisfactory representation of our light-curve on the assumption of stars of prolate spheroidal form, we have only to diminish the computed polar axis in the ratio which seems most plausible, according to the above, and at the same time diminish cot i in the same ratio, to obtain a system of elements, giving the same computed light-curve, and satisfying also the physical condition under consideration.

Taking up the problem in detail, let a_1, b_1 be the major and minor axes of the larger spheroid, and ϵ the eccentricity of the meridian section, and let $a_2 = ka_1$, $b_2 = kb_1$ be the axes of the smaller ellipsoid. The two stars will appear as elliptical disks, whose major axes coincide with the line of centers. Their minor axes will always be b_1 and b_2, but their major axes will usually be less then a_1 or a_2. If their values at any instant are d_1 and d_2, and ϕ is the angle between the line of centers and the line of sight, we find readily $d_1^2 = a_1^2(1 - \epsilon^2 \cos^2 \phi)$, $d_2 = kd_1$. But, with our previous notation, $\cos \phi = \sin i \cos \theta$, whence

$$d_1^2 = a_1^2(1 - \epsilon^2 \sin^2 i \cos^2 \theta)\,. \tag{39}$$

Suppose now that the apparent distance between the centers of the disks is δ. If $\delta > (1+k)d_1$ there will be no eclipse, and if $\delta < (1-k)d_1$ the eclipse will be total. If δ lies between these two limits, the eclipse will be partial—the two elliptical disks overlapping in the direction of their major axes.

We will not modify the ratio of the eclipsed portion to the areas of the two disks, if we alter the dimensions of the ellipses at right angles to the line of centers, i.e., their minor axes, in any constant ratio. By choosing this ratio equal to $\frac{d_1}{b_1}$ we can make the disks circular[1] and of radii d_1 and kd_1. We can now use the notation of our previous work, the only change being that d_1 takes the place of r_1. We then have

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[1] G. W. Myers, *op. cit.*, p. 10.

$$\delta = d_1 \phi(k, \alpha) = d_1 \{ 1 + kp(k, \alpha) \} , \tag{40}$$

where α is the percentage of area of the smaller disk which is eclipsed. Let l_1 and l_2 be the amounts of light which would reach us from each star if there were no eclipse. The light actually received will be $l_1 + (1 - \alpha) l_2$. The quantities l_1 and l_2 are not constant, but vary with the varying area of the apparent disks. If L_1 and L_2 represent their maximum values, which are obviously reached when $\theta = 90°$, we have

$$\frac{l_1}{L_1} = \frac{l_2}{L_2} = \frac{d_1}{a_1} = (1 - \epsilon^2 \sin^2 i \cos^2 \theta)^{\frac{1}{2}} ,$$

whence, if l is the actual amount of light received by us at any time,

$$l = l_1 + (1 - \alpha) l_2 = (1 - \epsilon^2 \sin^2 i \cos^2 \theta)^{\frac{1}{2}} [L_1 + (1 - \alpha) L_2] . \tag{41}$$

When $\epsilon = 0$ this reduces to the familiar formula for spherical stars. For brevity, let us now set

$$\epsilon^2 \sin^2 i = z . \tag{42}$$

§ 8. *Determination of the ellipticity and other elements.*—We have now before us two problems, the determination of this constant z, defining the effective ellipticity of the stars, and that of the other elements when this is known. Various methods have been proposed for the solution of the first problem in special cases. Roberts[1] has published tables which give both ϵ and i, on the assumption of equal stars revolving in contact, and Von Hepperger[2] has developed a very ingenious method, which, however, is available only when the two minima are of unequal depth.

It does not appear to have been previously observed that a satisfactory determination of z can be very simply made by plotting the observed values of l^2 against $\cos^2 \theta$. Since the second factor of (41) is constant when there is no eclipse, the resulting points for values of θ near 90° will be on a straight line whose slope gives the desired value of z. When eclipse begins the observed points fall below this straight line and lie on a descending curve.

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[1] *Monthly Notices*, **63**, 535, 1903.

[2] *Sitzungsberichte der Akademie der Wissenschaften in Wien*, Math.-Naturwiss. Kl., **118**, 933, 1909.

This method might seem to fail when the stars are in actual contact, and more or less of their area is eclipsed, except when $\theta=90^\circ$, so that the curve above described has no rectilinear portion; but in this case the eclipsed area is very small when θ is near 90° (varying, as may be easily shown, approximately as $\cos^3\theta$). The tangent to the observed curve at the point for which $\cos^2\theta=0$ can be drawn, and this gives as before the value of z.

Having found z, the light-curve may be "rectified," removing all apparent influence of the ellipticity of the stars. From equation (41), and the logarithmic nature of stellar magnitude, it follows that the actual change in the visual magnitude of the system at any time, as compared with the maximum, may be found by adding the changes in magnitude due to the two factors, one of which arises wholly from eclipse, and the other from ellipticity. By subtracting from the observed magnitudes the computed variation due to the latter cause, we may obtain a light-curve of the ordinary "*Algol*" form, with constant light between eclipses, which represents the variations in brightness due to eclipse alone.

From this curve it may be determined whether the eclipses are total or partial, and the magnitudes and the values of θ corresponding to assigned values of α_1 (for a total eclipse) or n (for a partial eclipse) may be found just as in the previous discussion. Only slight changes are necessary in the method of solution previously developed. In the case of a circular orbit equation (40) gives

$$\delta^2=\cos^2 i+\sin^2 i\sin^2\theta=d_1^2\{\phi(k,\alpha)\}=a_1^2(1-z\cos^2\theta)\{\phi(k,\alpha)\}^2.$$

Selecting three values α_1, α_2, α_3 we find, as in § 3,

$$a_1^2[\{\phi(k,\alpha_1)\}^2-\{\phi(k,\alpha_2)\}^2]=\frac{(\sin^2\theta_1-\sin^2\theta_2)(\sin^2 i-z)}{(1-z\cos^2\theta_1)(1-z\cos^2\theta_2)},$$

and then

$$\psi(k,\alpha_1)=\frac{(\sin^2\theta_1-\sin^2\theta_2)(1-z\cos\theta_3)}{(\sin^2\theta_2-\sin^2\theta_3)(1-z\cos\theta_1)}. \tag{43}$$

When $z=0$ this reduces to the equation (12) previously obtained.

In the case of total eclipse, therefore, we have only to divide each value of $\sin^2\theta_1-\sin^2\theta_2$ by the corresponding value of $1-z\cos^2\theta_1$, and then proceed to find k from the ratios of these numbers exactly as we did previously. The computation of the light-curve when the mean value of k has been obtained involves no difficulty. For the calculation of the elements, if θ' and θ'' correspond to the beginning of eclipse and of totality, we have the equations

$$\left.\begin{aligned} a_1^2(1-z\cos^2\theta')(1+k)^2&=\cos^2 i+\sin^2 i\sin^2\theta',\\ a_1^2(1-z\cos^2\theta'')(1-k)^2&=\cos^2 i+\sin^2 i\sin^2\theta'',\end{aligned}\right\} \quad (44)$$

which give a_1 and i. We then compute

$$\epsilon^2=z\operatorname{cosec}^2 i \qquad b_1^2=a_1^2(1-\epsilon^2) \qquad a_2=ka_1 \qquad b_2=kb_1, \qquad (45)$$

and the elements are determined.

In the case of a partial eclipse we may proceed in the same way. The fundamental equation (16) may be written

$$\chi(k,\alpha_0,n)=\frac{\sin^2\theta(n)-\sin^2\theta(1)}{\sin^2\theta(\frac{1}{2})-\sin^2\theta(1)},$$

since $\theta(1)=0$. χ is therefore a function of the same nature as the ψ-function, and we have only to divide each value of $\sin^2\theta(n)$ by the corresponding $1-z\cos^2\theta(n)$; so that our equation becomes

$$\chi(k,\alpha_0,n)=\frac{\sin^2\theta(n)\{1-z\cos^2\theta(\frac{1}{2})\}}{\sin^2\theta(\frac{1}{2})\{1-z\cos^2\theta(n)\}}. \qquad (46)$$

We may then proceed as in the case of spherical stars.

When the stars are prolate, and the orbit eccentric, complications ensue; for we must suppose the rotation of the stars to be uniform, while their orbital motion is not so, and the assumption that their major axes are always in the line joining their centers is no longer true. The two will be inclined at a small angle equal to the equation of the center in the elliptic motion. The effect of this inclination on the eclipsed area will obviously be small, and may therefore be neglected. More serious is the probability of tidal effects which may be revealed by a brightening of the light near periastron, or perhaps in other ways. A good approximation to the elements of the system may still be obtained by neglecting these irregularities, rectifying the light-curve as above, and determining the eccentricity and other elements as outlined for spherical stars—being careful to use the equations (43) or (46) instead of (12) or (16).

PART V. REFLECTION, DARKENING AT THE LIMB, ETC.

§ 9. *Reflection* (*or radiation-effect*).—The preceding discussion affords a complete solution of the problem of eclipsing variables, so long as we assume that the individual stars appear as uniformly illuminated disks of constant surface-brightness. This assumption is, however, improbable. It is more than likely that the actual stars of a close binary pair appear, like the sun, to be brighter at the center than the limb, and that each star is brighter on the side which is heated up by its companion's radiation than on the other. Direct evidence of the last phenomenon has been found by Dugan[1] in the cases of *RT Persei* and *Z Draconis*, and by Stebbins[2] in that of *Algol*. There are few if any other stars whose light-curves between minima have been studied with anything like the care bestowed upon these three, and it may well be that observations of comparable accuracy will show the phenomenon to be general.

It is probable, as Dugan points out, that in this case the excess of brightness is by no means uniformly distributed over the hemisphere of each star which faces its companion, but is greatest at the center of this region. As the whole amount of the increase in brightness due to this "reflection" effect is but a very small percentage, this variation in its amount is probably less than the general (and so far disregarded) darkening at the limb. Unless we wish to take the latter into consideration, we may follow Stebbins and treat the stars as exhibiting two hemispheres of unequal brightness—the fainter facing away from the companion. To all practical intents and purposes only the brighter hemisphere of each star undergoes eclipse.

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[1] *Publications of the Astronomical and Astrophysical Society of America*, **1**. 311 (date of actual publication, 1908), and **1**, 320, 1909.

[2] *Astrophysical Journal*, **32**, 200, 1910.

If the light we would receive from the fainter side of the larger star is $l_1 - c_1$ and from its brighter side $l_1 + c_1$, the rotation of this star uneclipsed would give us a variation expressed by the equation

$$l = l_1 - c_1 \sin i \cos \theta ,$$

the minimum occurring when $\theta = 0$ and the companion is behind this star. For the smaller star we have in the same fashion

$$l = l_2 + c_2 \sin i \cos \theta .$$

Hence, for the light of the whole system, we have

$$l = l_1 + l_2 + (c_2 - c_1) \cos \theta . \text{ Si.} \qquad (47)$$

If we assume that the quantities c, which are proportional to the excess of light radiated by each star on the side toward its neighbor, are also proportional to the energy received by the surface of each star from the radiation of its companion, it follows that these quantities (being proportional to the product of the superficial area of the first star by the total light-emission of the second) are in the inverse ratio of the surface-brightness of the two stars. If the orbit is circular, the last ratio is that of the loss of light at the two minima, and we have

$$c_1(1 - \lambda_1) = c_2(1 - \lambda_2) , \qquad (48)$$

where λ_1 and λ_2 are the light intensities at the two minima. The brightness of the system is therefore greater just outside the secondary minimum than just outside the primary—which is in accord with observation. When the two minima are of equal depth no such effect is to be expected, and none has been observed. Observations accurate enough to detect an effect of this sort will usually indicate also a small ellipticity of the stars. As the influence of the latter is symmetrical about the point half-way between the minima, and the former changes sign there, there is no difficulty in separating them. In fact, as the influence of "reflection" is so small, we may write at once, for its effect on the observed stellar magnitude

$$\Delta m = -1.08(c_2 - c_1) \sin i \cos \theta , \qquad (49)$$

and determine it by comparison of points on the light-curve, outside eclipse, for which $\cos \theta$ has values numerically equal but opposite in sign. Having determined $(c_2-c_1)\sin i$ (in practice we can always set $\sin i=1$), we may find c_2 and c_1 by (48).

If the fainter sides of both stars were increased in brightness till they equaled the brighter hemispheres, we should have disks of uniform brightness. As the fainter sides are never eclipsed, we may make this correction very simply by transforming the observed magnitudes into light-intensity and adding the quantity $c_1+c_2+(c_1-c_2)\cos\theta$ to the observed intensity. The resulting light-curve may now be discussed as usual.

When the stars are decidedly prolate, the assumption that the surface of each is divided into two regions of different but constant intensity would lead to more complicated expressions. But as we are dealing with an approximation in any case, it is most convenient, and accurate enough, to assume that the simple formulae given above shall hold good—"rectifying" the curve first, to remove the effects of ellipticity, and then proceeding as above.

In certain cases, where the orbit is eccentric, the observed brightness of the system may be different at periastron and at apastron, owing to tidal action. The effect, being small, may be represented approximately by a simple harmonic term[1] of the form

$$\Delta m=a\cos\theta+b\sin\theta\ .$$

The first part of this fuses with the "reflection" effect already discussed. The term $b\sin\theta$ gives rise to inequality in the magnitude at the two maxima, and may be determined from such a difference, if it exists. For further computation it is sufficient to remove this term from the observed magnitudes (which practically amounts to taking the mean of the observed light-curves at the two maxima as a normal curve for both).

§ 10. *Darkening at the limb.*—We must finally consider the influence upon our methods and results of the assumption that the stars do not appear as uniform disks, but are darker at the limb than at the center. To determine in detail the actual law according to which the brightness of the disk varies from point to point is obviously too much to expect. We may, however, assume a law (involving as few constants as possible) and require the determination of these constants as part of our problem. Schwarzschild[1] has shown that on certain plausible assumptions the apparent brightness J of the disk, at a point whose normal makes an angle i with the line of sight, should vary as $1+2 \cos i$, and that the total energy radiation from the sun's disk nearly follows this law. If we assume the more general law, J varies as $1-x+x \cos i$, where x lies between zero and unity, we shall have an expression capable of representing, approximately at least, the conditions likely to be met with among the stars, and shall have but one more constant to determine from our observations.

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[1] A. Roberts, *Monthly Notices*, 66, 127, 1906.

The solution, in the case of total eclipse, at least, can be carried out with the aid of new tables, exactly similar in nature to those already discussed, if we let α denote now, not the fraction of area eclipsed, but the fraction of the light of the eclipsed star which is obscured. In the general case α will be a function of the three quantities $\frac{\delta}{r_1}$, k, and x. If the first two of these have fixed values α will obviously be a linear function of x. When $x=0$, it reduces to the function $f\left(\frac{\delta}{r_1}, k\right)$ already calculated. If for $x=1$ we have $\alpha=f_1\left(\frac{\delta}{r_1}, k\right)$ we shall have in general

$$\alpha=(1-x)f\left(\frac{\delta}{r_1}, k\right)+xf_1\left(\frac{\delta}{r_1}, k\right). \tag{50}$$

The new function f_1 may be computed and tabulated, the work being much lightened by simple methods of mechanical quadrature.[2] For selected values of x (say $\frac{1}{3}$, $\frac{2}{3}$, and unity) we may invert the equation (50) and write

$$\frac{\delta}{r_1}=\phi_x(k, \alpha)=1+kp_x(k, \alpha) .$$

Constructing new functions $\psi_x(k, \alpha)$ exactly in the manner already employed, we arrive at a method of solution, for any previously assumed value of x, which is formally identical with that already developed, differing only in that different tables of numerical values of ψ are used for the different values of x.

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[1] *Göttingen Nachrichten*, Math.-Phys. Kl., 1906, p. 41.

[2] An example of such a method is given by Blazko, *Annales de l'Observatoire astronomique de Moscou*, II Série, **5**, 91, 1911.

The new tables will, however, have to be twice as extensive as the old, for in this case the light-curves caused by the eclipse of a large star by a smaller one will be different from those arising from that of a small star by a larger, and the ψ-table will be different in the two cases. Moreover, during the annular phase of an eclipse the light will not be constant.

Whether the remarkable empirical relations, which serve as a basis for the method already developed for determining the elements in the case of partial eclipse, will hold good in the case of disks darkened at the limb, can be determined only by trial.

When such tables have been constructed, the most probable amount of the darkening at the limb, for any given star, can be determined by comparison of the discordances between the observations and the best light-curves which can be obtained on various assumptions as to the value of x. The most favorable cases will clearly be those where the eclipse is total, as the number of other constants which must be found from the light-curve is then a minimum.

Whatever our assumed value of x, we can make our computed light-curve cross the ascending or descending branch of the observed curve (which we must in all cases assume to be symmetrical), at any three points we choose. The outstanding differences between observation and calculation will therefore always be small, and very accurate observations will be necessary if we are to discriminate between the different values of x.

§ 11. *Influence of darkening at the limb on the computed elements.*—The influence which darkening at the limb will exert on the form of the light-curve, and the nature of the errors which it will introduce into elements computed on the assumption of uniformly illuminated disks, may be easily followed quantitatively with the aid of the function $\psi(k, \alpha_1)$. If the values of $\sin^2 \theta$ for two assumed values α_2 and α_3 are A and $A-B$, we have in general for any value of θ, $\sin^2 \theta = A + B\psi(k, \alpha)$ and the elements are given by equations of the form

$$\left.\begin{aligned} r_1^2 \operatorname{cosec}^2 i &= \frac{B}{\phi_1(k)}, \quad r_2^2 \operatorname{cosec}^2 i = \frac{k^2 B}{\phi_1(k)}, \\ \cot^2 i &= \frac{B}{\phi_2(k)} - A\,. \end{aligned}\right\} \quad (51)$$

In the present discussion it is convenient to take for a_2 and a_3 not the values of 0.6 and 0.9, used in computing Table II, but other values, say 0.25 and 0.75, symmetrical about the point where half the light is lost. The necessary transformation is very readily effected, and the results are as follows:

TABLE OF $\psi(k,\alpha_1)$ FOR $\alpha_2=0.25$ $\alpha_3=0.75$

	$k=1.0$	0.8	0.6	0.4	0.2
$\alpha_1=0$	+2.65	+2.41	+2.22	+2.02	+1.87
$\alpha_1=0.25$	0.00	0.00	0.00	0.00	0.00
$\alpha_1=0.75$	−1.00	−1.00	−1.00	−1.00	−1.00
$\alpha_1=1.00$	−1.11	−1.23	−1.35	−1.47	−1.61
$\frac{1}{\phi_1(k)}$	+0.69	+0.82	+1.08	+1.56	+3.12
$\frac{k^2}{\phi_1(k)}$	+0.69	+0.53	+0.39	+0.25	+0.12
$\frac{1}{\phi_2(k)}$	+0.11	+0.20	+0.54	+1.03	+2.56

If the eclipsed star is darkened at the limb, it is clear (assuming a total eclipse) that the middle part of the loss of light will take place more rapidly, and the beginning and end more slowly, than if it was uniformly bright.[1] The same will be true of the variations of $\sin^2\theta$. Since we have taken the middle part of the eclipse as our standard, so to speak, in determining ψ, the values for this function for the beginning and end of eclipse will both be numerically greater than for uniform disks of the same radii. But a glance at the above table shows that, on the assumption of uniformly illuminated disks, larger values of ψ for the beginning of eclipse demand a larger value of k, and numerically larger values near totality, a smaller value of k. It is impossible to satisfy both these conditions. In practice the latter portion of the light-curve must have the preference, because when the star is faint, and θ is small, given changes in a or $\sin^2\theta$ result in much greater changes in the computed stellar magnitude and time than near the beginning of eclipse. We shall therefore find too small a value of k. From the equations (51) and the last three lines of the table, it follows that the computed value of r_2 will be too small, and those of r_1 and cot i too great. Near the beginning or end of eclipse our computations will give too rapid a change of light, so that the observed duration of the change will exceed the computed. Such an effect has been detected by Dugan[1] in the case of *RT Persei.*

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[1] S. Blazko, *op. cit.*, p. 79.

The errors in the computed dimensions of the two stars are of course incapable of detection by any known method of control. That in the inclination might perhaps be proved to exist; for, if the eclipsing variables as a class present disks considerably darkened at the limb, there will be more cases in which the computed elements indicate an almost grazing total eclipse (i.e., one in which one star is barely hidden behind the other) than can reasonably be explained on the theory of probability.

In the case of ellipsoidal stars, darkening at the limb presents a complicated problem. As it presumably arises in the atmosphere of the star, and increases with increasing obliquity of the line of sight to the star's surface, the mean effective darkening should be greater when we see the prolate spheroid end on than when it is side on. The variation in light due to the rotation of the star alone will therefore be increased, and the ellipticity computed on the assumption of uniform surface brightness will be too great.

§ 12. *Density of the components.*—It may be appropriate in closing to give the equations which determine the density of the components in terms of the orbital elements.[2] Let the total mass of the system be m, that of the larger star my and that of the smaller $m(1-y)$. If a is the semi-axis major of the orbit, we shall have $a=Km^{\frac{1}{3}}P^{\frac{2}{3}}$, where K is a constant depending on the units of measurement. If we choose the sun's mass, the sun's radius, and the day as units, then for the earth's orbital motion $a=214.9$, $P=365.24$, whence $K=4.206$.

In determining the elements of the system, we have taken a as our unit of length. The actual radius of the larger star is therefore ar_1, and its volume in terms of that of the sun is $K^3mP^2r_1^3$, or $74.4\ mp^2r_1^3$. Its density ρ_1 is therefore $\rho_1=0.01344\frac{y}{P^2r_1^3}$, and similarly, of the smaller star $\rho_2=0.01344\frac{1-y}{P^2r_2^3}$.

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[1] *Contributions from the Princeton University Observatory*, No. 1, p. 46, 1911.

[2] A. Roberts, *Astrophysical Journal*, **10**, 308, 1899; H. N. Russell, *Astrophysical Journal*, **10**, 315, 1899.

The actual densities cannot be computed unless the ratio of the masses of the two stars is known. We may, however, make use of the fact that among all the binaries, visual and spectroscopic, in which this ratio has been determined, the brighter star is found to be the more massive (with the doubtful exception of 85 *Pegasi*) for which Boss finds $y=0.36\pm0.13$[1] while the greatest mass-ratio is 3.6 : 1,[2] in the case of the spectroscopic binary *b Persei*. That is, in all known cases y lies between the limits 0.36 and 0.78 for the brighter star, and 0.64 and 0.22 for the fainter.

If we assume $y=0.5$ for both stars, our formula becomes for both stars

$$\rho=\frac{0.00672}{P^2r^3}\,. \tag{52}$$

This is likely to give too high a density for the faint star, and too low for the bright one, but in neither case is the error at all likely to exceed 50 per cent of the computed values, or to be in the opposite sense from that stated. We may therefore, in spite of this uncertainty, draw fairly accurate conclusions regarding the density of these stars.

When the stars are ellipsoidal, our formula obviously becomes $\rho=\frac{0.00672}{P^2abc}$ where a, b, and c are the three axes of the ellipsoid. Darwin has shown that in this case the ellipsoidal form of the stars increases their mutual attraction, and shortens the period of the system. This would tend to make the density computed without consideration of this influence, a little too great; but the effect is at most but a small percentage and may be neglected in practice.

A discussion of the light-curves of those eclipsing variables which have been sufficiently observed is now in progress at this observatory. If sufficient evidence of darkening at the limb is discovered to warrant the labor of computing the tables described above, they will be prepared. The computations are in the very efficient hands of Mr. Harlow Shapley, fellow in astronomy, to whom the writer is indebted for valuable aid in preparing the present communication for the press.

PRINCETON UNIVERSITY OBSERVATORY
March 19, 1912

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[1] *Preliminary General Catalogue*, p. 278, 1910.

[2] J. D. Cannon, *Journal of the Royal Astronomical Society of Canada*, 5, 270, 1911.

SELECTIVE ABSORPTION OF LIGHT ON THE MOON'S SURFACE AND LUNAR PETROGRAPHY

By R. W. WOOD

In a preliminary communication entitled "The Moon in Ultraviolet Light" (*Monthly Notices R.A.S.*, **70**, 226, 1910, and *Popular Astronomy*, **18**, 67, 1910,) I showed that by photographing the moon with light of the shortest wave-length which is capable of traversing the earth's atmosphere, certain features are brought out which cannot be seen or photographed with visible light. The most conspicuous feature was the black area in the vicinity of the crater *Aristarchus*.

In the present paper I shall show that by combining photographs taken by light from three or more regions of the spectrum it may be possible to commence a study of the petrography of the moon's surface. The first experiments were made at my laboratory at East Hampton, Long Island, in the summer of 1909, with a home-made telescope with an objective of quartz, three inches in diameter, coated with a film of silver opaque to visible light, but transparent to the ultra-violet in the wave-length range $\lambda 3160$ to $\lambda 3260$. The telescope tube was mounted on an equatorial mounting improvised from the frame of an old bicycle and provided with slow motion for following in right ascension. Photographs made with this very clumsy outfit showed the dark spot bordering *Aristarchus*, and in the following autumn some better ones were made with a six-inch speculum of about ten feet focus.

In the following year two papers were published by Miethe and Seegert (*Astron. Nachrichten*, No. 4489, 4502, Band 188), who, however, worked with a screen of nitroso-dimethyl-aniline and a silvered glass reflector, and consequently did not utilize the region of the ultra-violet which is not transmitted by glass. They secured pairs of pictures made through an orange ray-filter and the nitroso screen, and by projecting them superposed and in register on a screen through a blue and a red-orange ray-filter obtained a two-color picture which brought out the local differences in the reflecting power of the moon's surface for the two spectrum ranges in question.

Last summer I took up the work again at East Hampton, setting up a 12-inch equatorial of speculum metal, and ordering a 26-foot glass reflector of 16 inches aperture from John E. Mellish of Cottage Grove, Wis. In the *Astrophysical Journal* for December 1911, I have described the method by which a deposit of nickel was formed on this mirror, for it must be remembered that silver reflects only 4 per cent of the ultra-violet light employed in the work, and is consequently no better than glass. This mirror I mounted in combination with a 20-inch coelostat mirror, also nickeled. The arrangements for following were inadequate, however, and the photographs did not show much more detail than the smaller ones made with the six-inch mirror.

In the late autumn, through the courtesy of the Astronomical Department of Princeton University, I was given an opportunity of mounting the mirror on the 23-inch equatorial. I am under great obligation to Professor Russell for the interest which he took in the work and to Mr. Shapley, fellow in astronomy, who assisted me in handling the telescope and making the exposures. The 16-inch mirror was mounted at the eye end of the telescope and the plate-holder back of the objective, at the edge of the incident beam of rays. The use of the mirror in the Herschellian form of course impairs the definition somewhat with the ratio of focus to aperture which was used. Having had little experience with telescope mirrors, I was of the opinion, when putting the apparatus together, that the definition would be sufficiently good with this arrangement for the mirror in question, and as I was anxious to utilize as much of the light as possible, I adopted it.

The plate-holder was mounted on an old microscope stand turned into a horizontal position, the rack and pinion serving for focusing. Three different regions of the spectrum were utilized. A deep-orange screen used in connection with a Cramer Iso plate furnished the yellow image, which represents pretty closely what we see in visual work. A very short exposure made without any screen gives us what I have designated as the violet image, and a screen made by silvering a sheet of uviol glass 1 mm in thickness, made by Zeiss, gives us the ultra-violet image. This glass, in thin sheets, exercises no appreciable absorption for the range of wave-lengths transmitted by silver, and answers the purpose quite as well as quartz, which was used in the preliminary work. Careful experiments were made at East Hampton during the summer on the best thickness of silver film. If the film is too thin, much blue and violet light is transmitted, and the picture is made chiefly by these rays. If, on the other hand, it is too thick, nothing at all is transmitted, or the time of exposure is unduly prolonged. Photography of the spark through the screen is no criterion at all, for I found that a film which, with a short exposure, yielded only the group of iron lines in the region λ 3160 to λ 3260, when used for the ultra-violet photography of terrestrial objects illuminated by sunlight, gave pictures which could not be distinguished from ordinary photographs, e.g., a glass jar containing cigars appeared transparent, showing the cigars distinctly. This circumstance is due to the fact that, owing to absorption by the earth's atmosphere, the intensity-curve is falling very rapidly in the ultra-violet. The best way to test a screen is to photograph the solar spectrum with a quartz spectrograph, and increase the thickness of the silver film until no trace of the blue, violet, and upper ultra-violet appears, the narrow band in the region specified alone appearing on the plate. Such a film barely shows the filament of a tungsten lamp, when eye observations are made in a dark room. If the filament appears at all bright the film is too thin. One film which I used failed to show the tungsten lamp at all, though this one was a little thicker than necessary. It is better to have them too thick than too thin, for obvious reasons. The jar of cigars photographed through one of these films appears quite opaque, as if made of black glass.

The ray-filters were mounted immediately in front of the photographic plates. In some cases Iso plates were used for the violet and ultra-violet impressions, and in others Hammer Special and Lumière Sigma. It was found very difficult to guide satisfactorily with the large Princeton refractor, in spite of the fact that the large telescope was used for following. The motion in right ascension was especially troublesome, as it was accomplished by pulling a rope. Considering the large amount of motion in declination during the three-minute exposures, I am surprised that the pictures turned out as well as they did. An inspection of the three photographs, reproduced in Plate IX, shows that the reflecting power of different areas is quite different for the three spectral ranges utilized. The violet picture is the sharpest, as it was taken with an exposure of a second or less, and the lack of perfect definition is probably the result of the manner in which the plate was mounted, i.e., off the axis. I have lettered certain regions on the ultra-violet picture which deserve special mention. The most conspicuous object is the large dark patch just above the crater *Aristarchus* marked *A*. Practically no trace of this appears in the yellow picture, while it is faintly visible in the violet one. Two enlargements of this region are reproduced, one from the yellow negative, the other from the ultra-violet. Other regions which are relatively dark in the ultra-violet picture will be found near the moon's limb above and to the right of *Aristarchus*, notably the maria *B* and *C*. On the other hand the maria *D*, *H*, and *G* come out relatively darker in the picture made with violet light. The mare *F* is lighter than *G* in the yellow and violet pictures, whereas in the ultra-violet picture they are of equal intensity.

The small crater indicated by the arrow at *E* and the two small craters to the right of it are equally bright in the yellow and violet pictures, while in the ultra-violet, the crater *E* is much darker, almost disappears in fact.

It is very important in work of this kind to be sure that imperfections on the ray-filters or plates are not responsible for minute differences. A very slight variation in the thickness of the silver film will cause spurious effects, of course. I have verified the points mentioned on a number of negatives taken with the screens in different positions, which is the only way to distinguish between the real and the spurious.

It is obvious that the greater the number of widely isolated regions of the spectrum which we make use of in photographing the lunar surface the greater are the possibilities of drawing conclusions as to the nature of the materials of which the surface is composed. At the present time we can photograph only to wave-length λ 8000 or thereabouts, but if we could extend the range say to 8 μ, where anomalies in the reflecting power are shown by the silicates, we should probably find ourselves in the position to take up the subject of lunar petrography. Even with the range capable of investigation at the present time we can probably make a beginning, as I shall illustrate by a study of the *Aristarchus* dark spot, which, as I have said, is the most conspicuous object brought out by the ultra-violet photography. This spot is invisible in yellow light, begins to appear faintly in violet, and is very dark in ultra-violet.

I made a series of photographs with the three spectrum ranges used in the lunar work of a number of volcanic rocks. In this way I selected two specimens of volcanic tuff, of about the same color, one of which photographed dark in ultra-violet light. Superposing a small chip of this specimen on the other reproduced the conditions of the *Aristarchus* spot almost exactly. In yellow light the small chip was almost invisible against the larger, in violet it could be seen to be a trifle darker, and in ultra-violet it was very much darker than the background. I then analyzed the small chip and found that it contained iron and traces of sulphur. Photographs were now made of rocks with surface stains due to iron, but the stains were equally dark in violet and ultra-violet light. I then formed a very thin deposit of sulphur in a spot at the center of a fragment of that specimen of the tuff which photographed light in ultra-violet. The deposit was absolutely invisible to the eye, and was formed by blowing a small jet of sulphur vapor against the surface for a second or two. I photographed the specimen with the three types of radiation, and found that the spot was invisible in the yellow picture, gray in the violet one, and quite black in the ultra-violet. The three pictures are reproduced. The large area and the general form of the *Aristarchus* spot suggested to me, when it was first discovered, that it was made of some material thrown out by a volcanic blast from the crater. It may be an ash containing sulphur or it may be a deposit of sulphur formed by condensation of ejected vapor. It seems probable, however, that it is due to sulphur. Zinc oxide I have found to be quite black in ultra-violet but it is quite as light in violet light as in yellow; therefore the spot cannot be zinc oxide, which would be less probable than sulphur, aside from the evidence cited. It is apparent that the introduction of the violet pictures has improved the method of analysis, and a further increase is desirable. By means of the infra-red screens, with which I have obtained most remarkable photographs of sunlit foliage showing snow-white against a sky as black as midnight, we can make photographs of the moon in the spectrum range λ 7000 to λ 7400, while the nitroso screen will give us pictures in the upper ultra-violet. With a series of five pictures made with five ranges of the spectrum between wave-lengths λ 7400 and λ 3160, it is probable that new facts would come to light, especially if the lunar pictures were made on a larger scale with an instrument capable of accurate following.

Light within the spectrum range λ 7000 to λ 7400 is but very slightly scattered by the earth's atmosphere, which accounts for the intense blackness of the sky in photographs made by light of this wave-length, and it does not seem impossible that photographs of the brighter planets made through an infra-red screen might prove interesting if the planets are surrounded by a light-scattering atmosphere, for we must bear in mind that the surface of the earth, as seen from a neighboring planet, would be seen through a luminous haze, equal in brilliance to the blue sky on a clear day, that is, it would present much the same appearance as is presented by the moon when seen at noonday.

A solution of cyanine and a yellow dye mixed together in such proportion that only the range λ 7000 to λ 7400 is transmitted makes an even better screen than the one which I used originally, as has been observed by Pfund.

The plates obtained through the ray-filters can be studied to advantage by the methods employed in the three-color process of color-photography. Mr. F. E. Ives, the pioneer in work of this kind, was good enough to make at my request a three-color lantern slide from the three negatives. The negative taken through the ultra-violet screen was printed on a gelatin film and stained blue, the violet and orange pictures being rendered in red and yellow respectively. The three films when superposed resulted in a very pretty color photograph which brought out the differences in the reflecting power of the different maria in a very striking manner.

The prevailing tone of the darker portions of the lunar surface was olive green but certain spots came out with an orange tone and others with a decided purple color. The dark spot near *Aristarchus* came out deep blue, as was to be expected.

I have applied this same process to the two photographs of the *Orion* nebula taken by Professor Hartmann and published in this journal some years ago. One of these was taken through the nitroso-dimethyl-aniline screen which I described in the *Astrophysical Journal* for 1903, and the other through a screen which excluded the ultra-violet light. The marked localization of the two gases which make up the nebula is brought out most beautifully in the resulting color-picture. It is my plan next summer to begin work on photographing stars through the silver ray-filter, for it seems probable that high-temperature stars will come out relatively very bright when photographed with the shortest wave-lengths.

Great care is necessary in the preparation of the silver ray-filter. I find that the uviol glass must be handled very carefully in the acid and alkali cleaning solutions. One of my plates, after having been cleaned a number of times, refused to take a uniform deposit, the silver coming down in streaks and cloudy patches, some parts of the surface refusing to take any deposit at all. More vigorous cleaning only made matters worse, and I found on washing and drying the plate that the surface had become badly corroded, resembling very old window glass which has been exposed to the weather. Repolishing with rouge is the only thing to do when this trouble appears. The silver film must be of uniform thickness, free from even the minutest pin-hole, and appear structureless by transmitted light. This last matter can be best examined into by placing a sheet of velox paper in a plate-holder, covering the holder with the silver filter, and exposing for a few seconds to the light of the blue sky through an *open* window. The window should be at some distance in order that patches of irregular thickness, if they exist, may throw sharp shadows. On developing the paper any bad spots in the film become at once apparent.

As silver films of sufficient uniformity to serve as ray-filters require special precautions in their preparation a few words on this subject may not be out of place. The formaldehyde process gives the best results apparently, but it is very tricky to work with, and extremely sensitive to proportions, dilution, temperature, etc., acting in three or four wholly different manners according to circumstances. Commercial formaldehyde is diluted with ten parts of distilled water to form the reducing solution. Personally I never weigh my nitrate of silver as I enjoy the element of the personal equation, which enters the problem when scales are dispensed with. From one to two grams to 100 ccm of distilled water are about right. Add ammonia diluted with several volumes of water a little at a time until the precipitate is nearly but not quite dissolved. If too much is added and the solution clears add nitrate of silver solution, a drop at a time until a permanent straw tint is produced. The addition of the ammonia should form a good brown precipitate. Sometimes a very thin precipitate forms, the solution appearing pink or blue by transmitted light. I attribute this to too dilute nitrate solution, and find that it does not work as well as when the turbid brown precipitate forms. The plates carefully cleaned with nitric acid and caustic potash, and thoroughly washed with tap water, lifting them several times to get rid of the potash solution which lurks beneath them, are now flowed with a little of the reducing solution, just sufficient to wash over the surface when the dish is shaken. The dish is now tipped to one side and an equal amount of the silver solution is added, taking care not to pour it onto the plate but to one side. If the plate fills the dish it is better to mix the solutions in a beaker and then flow them over the plate. If the proportions and temperature are right the solution will turn pink in half a minute or so, the color deepening rapidly, and a beautifully uniform blue film will deposit. The process can be repeated until the film has the required thickness. I usually mount a tungsten lamp on the floor so that I can watch it through the bottom of the dish. Unfortunately things as described above seldom happen at the first trial. The silver may come down almost immediately, in brilliant streaks and splotches, the solution remaining clear. This means that too much of the silvering solution has been added. Or things may commence all right but the deposit may be brown like lightly smoked glass, the solution changing from pink to gray with a distinct precipitate. These brown deposits I find are non-conducting and are probably made up of very small particles. They have a low reflecting power. The trouble in this case is too little silver solution for the amount of reducing solution used. Sometimes the solution becomes muddy at once forming a brown deposit. I suspect that this results from too dilute formaldehyde solution. It troubled me much when my personal equation contained one more variable than at present, but I have not seen it occur recently. As the production of the uniform blue film depends upon getting the proportions just right I suppose the beginner had best mix measured amounts for each trial, unless he has access to a large jar of silver nitrate which "belongs to the department." It is impossible to form a film of sufficient thickness with one silvering by this process. I usually pour off about half of the solution when the operation appears to be over and add a little more of the silvering solution. This usually doubles the thickness. For a second silvering mix the solutions quickly in a clean dish, using a little more of the silvering solution in proportion to the formaldehyde than for the first coating. Transfer the plate to this without washing. If it is washed, the silver is apt to frill in the second bath. The cause of this I do not know. Probably it is osmotic or perhaps catalytic (!).

I feel quite certain that the method of studying the lunar surface outlined in this paper is worth going on with, but only with the largest instruments, perfectly mounted, and under the best atmospheric conditions. The exposure factors for the screens should be carefully determined in advance by photographing well-lighted white paper through them (strip exposure by drawing slide out by degrees) and developing the plates simultaneously. The work can be done to advantage only by one who has had much experience with photographic plates, for it is no easy matter to get three images of the moon through the three specified screens all of which are properly timed and developed to exactly the same density. Unless this is done, very erroneous conclusions will be drawn, and I am not at all sure that my own pictures are wholly free from such defects.

Though a nickel-plated reflector is necessary if the silver ray-filter is to be used, I believe that results of interest can be obtained with an ordinary silver-on-glass reflector, using a nitroso screen in place of the silver film. This gives us an ultra-violet picture made with slightly longer waves than the ones which traverse silver, but I have found that the spot near *Aristarchus* shows well with one of these screens.

I shall be glad to lend my thin plates of uviol glass to any observatory at which it is planned to take up the work with a nickeled reflector. The plates are 12 cm square and 1 mm in thickness, and I shall be glad to silver them anew with films of the most suitable thickness.

I have been aided in this investigation by a grant from the Elizabeth Thompson fund, made several years ago, and this communication is the final report.

JOHNS HOPKINS UNIVERSITY
May 1912

Reviews

Outlines of Applied Optics. By P. G. NUTTING. Philadelphia: P. Blakiston's Son & Co., 1912. Pp. 234. Figs. 73. $2.00.

This book might be called a miniature encyclopedia of the applications of optics in connection with physical instruments. Primary emphasis is placed on the matter of relative and absolute precision and sensibility, in relation to the design and manipulation of optical instruments and the interpretation of their performance. In an introduction and twelve short chapters it contains a wealth of information and suggestion presented in concise fashion, at times so condensed as to seem like a kind of syllabus.

The introduction, after some initial remarks on sources of light and varieties of spectra, gives a preparatory sketch of the various physical properties of light, quoting as needed a few quantitative laws of emission, reflection, refraction, absorption, and polarization, without proof but with some explanation of their significance.

Geometric optics in the ordinary sense is given three chapters which discuss respectively the theory of formation of images, the designing and testing of optical systems, and the principal forms of complete instruments. The theoretical basis here includes the Gaussian theory, the Seidel classification of aberrations of lowest order, and the listing of the conditions to be imposed in the designing of particular instruments, together with the necessary limitations and compromises occurring in connection with each. This treatment is paralleled by a description of methods of determining how closely any actual instrument meets theoretical expectations. A fourth chapter is devoted to methods of determining indices of refraction, to the influence of physical conditions on index, and to representation by dispersion formulas.

Two chapters, on the eye and vision and on colorimetry, treat of the geometric and physiological conditions of visual perception, of photometric and chromatic sensibility, and of the artificial instruments used in the empirical analysis of color. Three chapters deal with sources and distribution of illumination, with photometry, and with the analysis and measurement of radiation as a flux of energy; then three more with the uses of polarized light, the physical properties of the photographic plate, and the metrologic aspect of interference.

The table of contents is full enough to serve fairly as an index, and each chapter is followed by a short bibliography, partly of general references, partly of papers bearing on items in the text.

There are a number of misprints, most of them fortunately obvious, and some errors in single statements. For example, the ordinary eyepiece should be said to be designed for oblique, not axial, achromatism (p. 86), and the separation of lenses in the Clark objectives to introduce rather than eliminate lateral chromatic errors (p. 88). Gauss points and nodal points are confused on p. 33 but distinguished in the specifications of the eye (p. 116). The enumeration of algebraic conditions on p. 53 is faulty. Moreover, the extreme condensation found necessary has sometimes led to the insertion of statements which are obscure and liable to erroneous interpretation. Thus, it is said that the sine-condition and the condition for elimination of coma are equivalent, which is true only for systems free from spherical aberration (p. 54). The convenient distinction of "pupils" and "windows" seems not always consistently observed. It is hard to see why a magnifier of moderate power is recommended for testing objectives (p. 67), or in what sense the effective pupil of the eye can be said to be in front of the cornea (p. 117); or why known radiation laws are called empirical, when their theory is so prominent in the recent history of physics.

It is apparent, however, that the task of compressing such a range of material into the space chosen can have been no easy one. This may account for the difficulty in securing uniform precision of statement, as well as for the omission of some topics which could rightly claim a place; for instance, a direct study of the spectroscope, or Abbé's discoveries regarding the interpretation of microscopic images. The justice of perspective, amount of really valuable imformation, and enthusiastic spirit of the book show that its composition was in good hands. It is surely to be hoped that it may prepare the way speedily for a more ample treatise of similar scope, such as is hinted at in the preface.

A. C. Lunn

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

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THE SYNTHETIC DEVELOPMENT OF RADIATION LAWS FOR METALS

By EDWARD P. HYDE

INTRODUCTION

A lively interest is being manifested at the present time in the radiating properties of metals. This has been stimulated, no doubt, by the utilization of the metals tantalum, tungsten, and osmium in the manufacture of high-efficiency lamps. An explanation of their high efficiencies has been sought, and this study has led to a consideration of the relative importance of the temperature of operation and the *selectivity*[1] of radiation in determining the efficiency. The primary source of interest to the physicist, however, lies in the development of the physics of radiation, which in turn conduces to a better knowledge of matter in various conditions of thermal equilibrium.

We seek, therefore, to correlate the radiating properties with the other known properties of material substances and the first step in such a correlation is the formulation of the law or laws which express the radiating properties. The laws of radiation for the *ideal radiator* or *black body* are fairly well known, though the exact values of the constants have not yet been determined with entire satisfaction. For ordinary matter, though much progress has been made in the study of the radiating properties of some few substances, our knowledge on the whole is very limited. Platinum and iron oxide with two or three possible exceptions are the only two substances for which any successful attempt[1] has been made to study directly the relation between temperature and radiation at temperatures above 1200° or 1500° abs.[2] The reason for this is to be found in the difficulty in making direct measurements of high temperature, particularly of those substances which cannot be operated in the open air. Consequently in the recent studies of the radiation of such substances as tantalum, tungsten, and carbon, recourse has been had to indirect methods. These indirect methods have yielded certain limited facts. Thus without any assumptions it has been shown[3] by experiment that carbon, tantalum, tungsten, and osmium radiate selectively with respect to a black body. On the basis of a single assumption, which in the light of all our present knowledge is quite probable, these experiments have made possible numerical estimates of the degree of selectivity of the radiation from these various substances, and have indicated certain definite temperature relationships. Other experiments[4] giving the distribution of energy in the infra-red region of the spectrum have led to the evaluation of the so-called *constant* a in the generalized form of the Wien equation

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[1] Throughout this paper the term *selectivity* will be used to indicate a difference in quality of radiation from that of a black body at the same true temperature. It will not signify the extreme variations from black-body radiation to which the term was originally applied, as, for example, in the *bright-line* spectra of luminous gases.

$$J = \frac{C_1}{\lambda^a e^{\frac{C_2}{\lambda T}}} \qquad (1)$$

which is supposed to represent fairly well, at least for short wavelengths and low temperatures, the distribution of energy in the spectrum of a black body when the constant $a = 5$. These experiments have yielded values of a other than 5, and different for different substances. Moreover, they have indicated a variation in a with the temperature. The differences in a for different substances point to differences in the radiating properties of the various metals studied, but the variation in a with temperature would seem to suggest the inapplicability of the generalized Wien law to the radiation of these substances.

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[1] Lummer and Pringsheim, *Verh. d. deutsch. phys. Ges.*, **1**, 226, 1899; Paschen, *Wied. Annalen*, **58**, 455, 1896; *ibid.*, **60**, 662, 1897.

[2] Throughout this paper temperatures will be given in degrees *absolute*, i.e., degrees will be intervals on the Centigrade scale, but the zero will be the absolute zero, or —273° from the melting-point of ice.

[3] Hyde, Cady, and Middlekauff, *Trans. Ill. Eng. Soc.*, **4**, 334, 1909. See also *Ill. Eng.* (London), **2**, 241, 335, 1909.

[4] Coblentz, *Bulletin of the Bureau of Standards*, **5**, 361, 1909.

It has long been a matter of conjecture whether or not the radiation of certain metals can be represented by either the generalized Wien equation or the generalized Planck equation

$$J=\frac{C_1}{\lambda^a\left(e^{\frac{C_2}{\lambda T}}-1\right)} \tag{2}$$

which, when $a=5$, has a better foundation, both theoretical and experimental, as representing black-body radiation. The two equations are practically the same for short waves and low temperatures, but differ somewhat when the product λT becomes relatively large. Platinum, the only metal studied directly, seems to be represented fairly well in its radiation by the above equations, though it is doubtful whether the data on platinum justify discrimination between the Wien and Planck equations. By two different methods a has been determined for platinum as 6.0[1] and 6.42.[2] Moreover, the relation between the total emission and the temperature has been found to be approximately

$$E=\sigma T^\beta \tag{3}$$

where β lies between 5 and 5.5. If the generalized Wien law (equation 1) or the generalized Planck law (equation 2) holds for platinum, then it must follow by integration that the total emission is given by the generalized Stefan-Boltzmann law (equation 3) and that the exponent $\beta=a-1$.

It is thus seen that to a fair degree of approximation the radiation from platinum follows the radiation laws for a black body if the constants of these laws are changed. To what extent the radiation from platinum follows the black-body laws, is, however, not known exactly, and it is a distinct assumption to suppose that the radiation from other metals will follow the same laws by changing the constants in the black-body equations. For each metal this must be tested, and since, as stated before, it is most difficult to make direct temperature measurements, recourse must be had to indirect methods.

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[1] Lummer and Pringsheim, *Verh. d. deutsch. phys. Ges.*, **1**, 218, 1899.

[2] Paschen, *Annalen der Physik*, **60**, 697, 1897.

Two general methods of procedure are available. According to a method which has been used extensively the generalized black-body laws are assumed and then deductions from these laws are made and tested experimentally. If a number of these deductions are verified a high degree of probability is established that the assumption is correct. If, on the other hand, some deductions are verified and others are not it is very difficult to form any exact idea of the degree of approximation to which the assumed laws hold. Thus, if on the assumption of the generalized Wien and Planck equations, a for tungsten is found to be higher than that for tantalum, as determined by Paschen's method, some definite result would seem to have been obtained, but if a for each substance is further found to change with the temperature, or with some other variable, the inapplicability of the generalized Wien or Planck equation is at once evidenced, and doubt is cast on the conclusions from the first observation. This doubt can be dispelled only by a careful analysis of all the data. This method, which may be termed the *analytic*, is, however, a very valuable one, and in some cases is the only one available.

The other general method of procedure is the *synthetic* method. Starting from the observed phenomena, an attempt is made to build up the laws or generalizations which correlate and express the phenomena. After an unsuccessful effort to present in proper perspective by the analytical method a quantity of data which the author has accumulated on the radiation of carbon and of the metals tungsten and tantalum, the synthetic method suggested itself. It has proved itself a much more satisfactory way of correlating these and the other data available at the present time.

In the present paper an attempt will be made, as far as the available data will permit, to build up the equations which express the various relations between T, the absolute temperature, λ, the wave-length, J, the intensity of emission at any wave-length, and

E, the total emission for the various substances—carbon, tantalum, and tungsten. Observations made in the laboratory of the author will form the basis for the synthetic construction of the radiation laws, but the observations of others will also be utilized for confirmation and extension.

SYNTHETIC DEVELOPMENT OF RADIATION LAWS FOR CARBON, TANTALUM, AND TUNGSTEN

Criterion I

The synthetic development of the radiation laws for carbon, tantalum, and tungsten rests primarily on two experimental facts. One of these has already been published[1] and has been confirmed by subsequent experiments. It has been described as a relation between *color-match* and *intensity-match*, but will be referred to hereafter as Criterion I, since the term color-match has led to some confusion.

In order to state Criterion I, which is very closely fulfilled by two of the three substances mentioned above and approximately fulfilled by the third substance, let us assume a black body at some relatively low, known temperature T_0. Let $(J_1)_0$ and $(J_2)_0$ be the intensities of emission in any two wave-lengths λ_1 and λ_2 in the *visible* spectrum. Let us assume a second radiating body, e.g., tantalum, at such an unknown temperature T_1 that

$$\frac{(J_1)_1}{(J_2)_1}=\frac{(J_1)_0}{(J_2)_0} \tag{4}$$

where $(J_1)_1$ and $(J_2)_1$ are the intensities of emission in the wave-lengths λ_1 and λ_2 for tantalum at the temperature T_1.

Now suppose the temperature of the black body is increased to some higher, arbitrary known temperature T'_0, and that the temperature of the tantalum is increased by an unknown amount, such, however, that at the new temperature T'_1 the following relation holds:

$$\frac{(J'_1)_1}{(J'_2)_1}=\frac{(J'_1)_0}{(J'_2)_0} \tag{5}$$

where the primed quantities are the corresponding quantities at the higher temperatures. In previous papers the author has referred to the conditions expressed by equations 4 and 5 as color-matches because, when λ_1 and λ_2 are two wave-lengths of the visible spectrum some distance apart, it has been found, by repeated experiment for the three substances under consideration, that when equations 4 and 5 are fulfilled as determined by spectrophotometric measurements, the integral colors of the two radiating bodies are indistinguishable as seen in the ordinary Lummer-Brodhun contrast photometer. In fact the latter criterion is probably more sensitive than the spectrophotometric determination. In order, however, to avoid any possible criticism, the discussion, at least for the present, will be based on the spectrophotometric observations as given in equations 4 and 5.

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[1] Hyde, *Ill. Eng.* (London), 3, 488, 1910. See also *Physical Review*, **31**, 315, 1910.

Suppose now that when a black body and tantalum are at such temperatures T_0, T'_0, T_1, and T'_1 that equations 4 and 5 are fulfilled, there is also fulfilled this condition:

$$\frac{(J'_1)_1}{(J_1)_1}=\frac{(J'_1)_0}{(J_1)_0}\ . \tag{6}$$

Then Criterion I is fulfilled and certain interesting conclusions can be drawn. Previous experiments[1] confirmed by more recent ones have shown that Criterion I is fulfilled within the limits of accuracy of the experiments for carbon and tantalum and that if there is a deviation in the case of tungsten this discrepancy is small. The experimental data will be presented later. (The approximate method of integral color-match gives one a better idea of the significance of the criterion. If two lamps, one of untreated carbon and one of tantalum, are mounted on opposite sides of a sensitive Lummer-Brodhun photometer and are brought to a color-match first at low voltages, and then at higher voltages, it will be observed that the relative intensities of the two lamps remain the same notwithstanding the different radiating properties of the two substances.)

If Criterion I as expressed by the three equations 4, 5, and 6 is fulfilled for all wave-lengths of a given region of the spectrum, it can readily be shown that for this region of the spectrum

$$\frac{d(J)_1}{dT_1}\cdot\frac{T_1}{(J)_1}=\frac{A}{\lambda} \tag{7}$$

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[1] *Loc. cit.*

in which A is independent of λ but may be a function of the temperature. Equations 4, 5, and 6 give directly

$$\frac{(J'_2)_1}{(J_2)_1}=\frac{(J'_2)_0}{(J_2)_0}. \tag{8}$$

Since the criterion is supposed to hold for all temperatures over a relatively large range, let us take the two temperatures T_0 and T'_0 near together, such that $T'_0=T_0+\Delta T_0$ where Δ is a small fraction. Equations 6 and 8 then become

$$\left(\frac{\Delta J_1}{J_1}\right)_1=\left(\frac{\Delta J_1}{J_1}\right)_0 \tag{9}$$

and

$$\left(\frac{\Delta J_2}{J_2}\right)_1=\left(\frac{\Delta J_2}{J_2}\right)_0. \tag{10}$$

Dividing equation 10 by equation 9 there results

$$\left(\frac{\frac{\Delta J_2}{J_2}}{\frac{\Delta J_1}{J_1}}\right)_1=\left(\frac{\frac{\Delta J_2}{J_2}}{\frac{\Delta J_1}{J_1}}\right)_0. \tag{11}$$

If now ΔT_0 is made to approach zero, it must follow, since the changes in J_2 and in J_1 within each parenthesis, i.e., for each substance, correspond to the same changes in temperature, ΔT_0 for the black body and the corresponding ΔT_1 for tantalum, that

$$\left[\left(\frac{\frac{dJ}{J}}{\frac{dT}{T}}\right)_{\lambda_2}\div\left(\frac{\frac{dJ}{J}}{\frac{dT}{T}}\right)_{\lambda_1}\right]_1=\left[\left(\frac{\frac{dJ}{J}}{\frac{dT}{T}}\right)_{\lambda_2}\div\left(\frac{\frac{dJ}{J}}{\frac{dT}{T}}\right)_{\lambda_1}\right]_0. \tag{12}$$

The right-hand member, since it refers to a black body, can readily be shown to equal $\frac{\lambda_1}{\lambda_2}$. Thus, taking the derivative of the Wien equation (which is identical with the Planck equation in the visible spectrum for the temperature region in which we are interested at present) there results

$$\frac{dJ}{dT}=\frac{d}{dT}\ \frac{C_1}{\lambda^5 e^{\frac{C_2}{\lambda T}}}=\frac{C_1}{\lambda^5 e^{\frac{C_2}{\lambda T}}}\ \frac{C_2}{\lambda T^2}$$

so that

$$\frac{dJ}{dT}\cdot\frac{T}{J}=\frac{\frac{dJ}{J}}{\frac{dT}{T}}=\frac{C_2}{\lambda T}. \tag{13}$$

Hence

$$\left[\left(\frac{\frac{dJ}{J}}{\frac{dT}{T}}\right)_{\lambda_2}\div\left(\frac{\frac{dJ}{J}}{\frac{dT}{T}}\right)_{\lambda_1}\right]_0=\frac{\lambda_1}{\lambda_2} \tag{14}$$

and so (equation 12)

$$\left[\left(\frac{\frac{dJ}{J}}{\frac{dT}{T}}\right)_{\lambda_2}\div\left(\frac{\frac{dJ}{J}}{\frac{dT}{T}}\right)_{\lambda_1}\right]_1=\frac{\lambda_1}{\lambda_2}. \tag{15}$$

It must follow, therefore, that for tantalum

$$\frac{d(J)_1}{dT_1}\,\frac{T_1}{(J)_1}=\frac{A}{\lambda} \tag{16}$$

as given in equation 7, A is independent of λ but may be a function of T. Equation 16 may therefore be written, if at the same time the subscripts are dropped

$$\frac{dJ}{dT}\,\frac{T}{J}=\frac{A(T)}{\lambda}. \tag{17}$$

This equation must hold for any substance, and for any ranges of temperature and wave-length for which Criterion I is fulfilled. The limits of wave-length and temperature within which it applies for carbon, tantalum, and tungsten will be discussed later.

Now by integration of equation 17 a relation between J, λ, and T is obtained, corresponding to the Wien or Planck equation for a black body. Thus

$$\frac{dJ}{J}=\frac{1}{\lambda}A(T)\frac{dT}{T}$$

$$\log J=\frac{1}{\lambda}\int A(T)\frac{dT}{T}+B(\lambda)$$

$$J=e^{B(\lambda)}\cdot e^{\frac{1}{\lambda}\int A(T)\frac{dT}{T}}. \tag{18}$$

If we write

$$\left.\begin{aligned} e^{B(\lambda)} &= C_1 f(\lambda) \\ \int A(T)\frac{dT}{T} &= -\frac{C_2}{\theta(T)} \end{aligned}\right\} \tag{19}$$

equation 18 becomes

$$J = C_1 f(\lambda)\frac{1}{e^{\frac{C_2}{\lambda\theta(T)}}} \tag{20}$$

which is the generalized equation of energy distribution in the spectrum of the substance fulfilling Criterion I within the prescribed limits of wave-length and temperature. If $f(\lambda)=\lambda^{-5}$ and $\theta(T)=T$, equation 20 reduces to the Wien equation. If $f(\lambda)=\lambda^{-a}$ and $\theta(T)=T$, equation 20 reduces to the generalized form of the Wien equation, which has been found to hold approximately for platinum and which has been assumed in the so-called analytical method of studying other metals. It is seen, however, that Criterion I by itself does not demand that $f(\lambda)$ should be a power function. Moreover, it gives no information regarding the nature of the function $\theta(T)$.

It is interesting to consider in connection with Criterion I certain experimental facts established by other observers. It has been shown in various investigations[1] that in the region of the visible spectrum the optical constants of certain metals that have been studied undergo no change with change in temperature. According to Hagen and Rubens[2] the emissive power of metals is independent of temperature, not only in the visible spectrum, but even in the infra-red region up to and beyond 2 μ, notwithstanding the equation derived from the electromagnetic theory of Maxwell according to which the emissivity is proportional to $\sqrt{\frac{\rho}{\lambda}}$ where ρ is the specific resistance of a metal, and hence is, in general, a function of the temperature. For the region of the spectrum and the range of temperature within which the emissivity of a metal does not change with the temperature the equation representing the distribution of energy may at once be written as follows:

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[1] Pflüger, *Wied. Annalen*, 58, 494, 1896.

[2] Hagen and Rubens, *Phys. Zeit.*, 11, 139, 1910.

$$J = C_1 F(\lambda) \cdot \frac{1}{\lambda^5 e^{\frac{C_2}{\lambda T}}} \tag{21}$$

since the distribution of energy is obtainable from that of a black body at the same temperature by multiplying by some function of the wave-length $F(\lambda)$ which is independent of the temperature. Since in equation 21 the specific function $\frac{C_2}{T}$ takes the place of the more general function $\frac{C_2}{\theta(T)}$ in equation 20, being the only difference between the two equations, it is seen that the experimental fact of the constancy of the emissivity of a metal for changes in temperature conduces to more definite knowledge of its law of radiation (within the prescribed limits of wave-length and temperature) than Criterion I. The reason for this, and the physical significance of Criterion I are evident on a little consideration.

Differentiation of equation 21 leads to the expression

$$\frac{dJ}{dT} \cdot \frac{T}{J} = \frac{C_2}{\lambda T} \tag{22}$$

which does not involve the function $F(\lambda)$. According to equation 22 the percentage increase in intensity of emission $\frac{dJ}{J}$ in wave-length λ corresponding to a given small percentage increase in temperature $\frac{dT}{T}$ will be inversely as the wave-length λ. Hence if two substances, radiating in accordance with equation 21 with different functions $F(\lambda)$ and $F'(\lambda)$, are at such temperatures that $\left(\frac{J_{\lambda_1}}{J_{\lambda_2}}\right)_1$ for one substance is the same as $\left(\frac{J_{\lambda_1}}{J_{\lambda_2}}\right)_2$ for the other, and if the temperature of one is increased by a given small amount and the temperature of the other is increased by such an amount that again the condition is fulfilled that $\left(\frac{J'_{\lambda}}{J'_{\lambda_2}}\right)_1 = \left(\frac{J'_{\lambda_1}}{J'_{\lambda_2}}\right)_2$, it must follow that the relative change in intensity of emission in either wave-length λ_1 or λ_2 is the same for the two substances. In other words, Criterion I is fulfilled. Indeed, since equation 21 is only a specialized form of equation 20, it follows at once that for two bodies which radiate according to equation 21, that is, which have emissivities independent of the temperature, Criterion I must be fulfilled.

The converse of this is not necessarily true, viz., that if Criterion I is fulfilled by a radiating substance in comparison with a black body then the emissivity of that substance is independent of the temperature within the limits of wave-length and temperature within which Criterion I holds. If Criterion I is fulfilled one of two conditions must exist: either the emissive power of the substance has remained constant, or else a change in the relative emissive power in the various wave-lengths of the visible spectrum has accompanied a change in the average emissivity in this region. Thus if the average emissive power in the visible spectrum increases, the increase for the shorter wave-lengths must be greater by a definite amount than that for the longer wave-lengths of the visible spectrum—otherwise Criterion I would not be fulfilled.

According to the results of Hagen and Rubens, however, the change would most probably be in the other direction. Thus for metals having a positive temperature coefficient of resistance the increase in emissivity with increasing temperature is found to be greater for the longer infra-red waves than for the shorter ones. If this change in emissivity progresses into the region of the visible spectrum at very high temperatures it would seem most probable that the emissivity in the red would increase more rapidly than that in the blue, rather than that the effect should be in the opposite direction. But if the phenomenon proceeds in the way which seems more probable, the relative change in emissivity in the visible would be in such a direction as to augment the effect of the average change, rather than to compensate it, so that the fulfilment of Criterion I points strongly to the constancy of the emissivity of the substance within the prescribed limits of wave-length and temperature, rather than to the alternate conclusion.

Criterion I lends itself admirably to the study of the constancy of the emissivity of metals at high temperatures in that region of the infra-red spectrum where Hagen and Rubens found a transition from the complete fulfilment of the equation resulting from the electromagnetic theory, to the condition of constant emissivity as found in the visible spectrum.

Criterion II

The second experimental fact, which forms the basis of Criterion II, involves the total radiation from the substance under investigation. The Stefan-Boltzmann law of total radiation for a black body

$$E=\sigma_0 T^4 \tag{23}$$

has both theoretical and experimental foundation. Moreover, if either the Wien or the Planck law of spectral energy distribution is true for the black body the Stefan-Boltzmann law must follow from integration, and the constant σ_0 must be expressible in terms of the constants entering into the Wien or Planck equation. Thus from the generalized Planck equation

$$J=\frac{C_1}{\lambda^a\left(e^{\frac{C_2}{\lambda T}}-1\right)} \tag{24}$$

it must follow that

$$E=\int_0^\infty J d\lambda=\int_0^\infty \frac{C_1}{\lambda^a\left(e^{\frac{C_2}{\lambda T}}-1\right)}$$

or

$$E=\left[C_1\Gamma(a-1)\frac{1}{C_2{}^{a-1}}\sum_{n=1}^{n=\infty}\frac{1}{n^{a-1}}\right]T^{a-1}. \tag{25}$$

The constant σ in the generalized Stefan-Boltzmann law

$$E=\sigma T^{a-1} \tag{26}$$

must then be (equation 25)

$$\sigma=C_1\Gamma(a-1)\frac{1}{C_2{}^{a-1}}\sum_{n=1}^{n=\infty}\frac{1}{n^{a-1}}. \tag{27}$$

If the Wien equation is used instead of the Planck equation, this becomes

$$\sigma'=C_1\Gamma(a-1)\frac{1}{C_2{}^{a-1}}. \tag{28}$$

If a is an integer, as in the case of a black body, $\Gamma(a-1)=(a-2)!$ and so for a black body, for which $a=5$, equations 27 and 28 reduce to

$$\sigma_0=C_1(3!)\frac{1}{C_2^4}\sum\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots, \text{etc.} \tag{29}$$

and

$$\sigma'_0=\frac{6C_1}{C_2^4}. \tag{30}$$

This simple process of arriving at the law of total radiation by integration of the Wien or Planck equation of spectral energy distribution is not applicable to equations 20 or 21 expressing the distribution of energy in the spectrum of certain substances, because these equations have been formed only from data in the visible spectrum. Other experimental data involving the total radiation are therefore necessary. These data are supplied in the second experimental fact forming the basis of Criterion II.

This second experimental fact is based on accurate photometric and electrical measurements on various incandescent electric lamps[1] from which is derived the relation between the supplied watts W and the coefficient k', where k' is defined as the ratio of the relative change in candle-power, I, to the relative change in the supplied watts, W, or

$$k'=\frac{\frac{dI}{I}}{\frac{dW}{W}}=\frac{dI}{dW}\cdot\frac{W}{I}. \tag{31}$$

From the observed values of k' and W it is possible to pass to the relation between the radiated energy E and the coefficient k_λ, where

$$k_\lambda=\frac{\frac{dJ}{J}}{\frac{dE}{E}}=\frac{dJ}{dE}\cdot\frac{E}{J}. \tag{32}$$

It is thus the ratio of the relative change in the emission, J, at some wave-length, λ, to the total emission, E. A previous investigation by the author[1] and others showed how the total radiation, E, for a unit length of a filament can be determined from the total supplied watts, W, and how the candle-power for a unit length of filament can be determined from the average candle-power of the entire filament. The simple method of passing accurately from the relative change in this integral or white-light candle-power to the relative change in the intensity, J, at some wave-length in the visible spectrum will be given subsequently in presenting the experimental data.

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[1] Cady, *Trans. Ill. Eng. Soc.*, **3**, 459, 1908; *Elec. Rev. and West. Elec.*, **59**, 1087, 1911; *Ztschr. f. Beleuch.*, **18**, 109, 1912.

The second experimental fact, which has been observed to hold for carbon, tantalum, and tungsten, is the following relation between k_λ (equation 32) and E:

$$\frac{(k_\lambda)_1}{(k_\lambda)_2}=\left(\frac{E_2}{E_1}\right)^{1/\beta} \qquad (33)$$

where $(k_\lambda)_1$ and $(k_\lambda)_2$, and E_1 and E_2 are values of k_λ and E, respectively, corresponding to the temperatures T_1 and T_2. β is a constant having different values for the different substances. The constancy of β with change of temperature forms the basis of Criterion II.

To determine the significance of Criterion II let equation 33 be rewritten in the following equivalent form

$$k_\lambda=\frac{K}{E^{1/\beta}} \qquad (34)$$

in which K is a constant factor of proportionality. Now k_λ from its definition (equation 32) may be written

$$k_\lambda=\frac{dJ}{dE}\cdot\frac{E}{J}=\frac{dJ}{dT}\cdot\frac{T}{J}\div\frac{dE}{dT}\cdot\frac{T}{E} \qquad (35)$$

and the differential equation from which equation 20 was deduced is (equation 17)

$$\frac{dJ}{dT}\cdot\frac{T}{J}=\frac{A(T)}{\lambda}. \qquad (36)$$

If we assume that Criterion I indicates the constancy of the emissivity with change in temperature, as discussed fully above, then the function $A(T)$ will be replaced by $\frac{C_2}{T}$ (by differentiation of equation 21) and equation 36 becomes

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[1] Hyde, Cady, and Worthing, *Trans. Ill. Eng. Soc.*, **6**, 238, 1911. See also *Ill. Eng.* (London), **4**, 389, 1911.

$$\frac{dJ}{dT}\cdot\frac{T}{J}=\frac{C_2}{\lambda T}. \tag{36'}$$

These two equations (36 and 36′), based on equations 20 and 21 respectively, will be carried through in parallel for completeness.

Substituting in equation 35 the values of $\frac{dJ}{dT}\cdot\frac{T}{J}$ as given in equations 36 and 36′ and the value of k_λ as given in equation 34, there result respectively

$$\frac{dE}{E^{(1/\beta)+1}}=\frac{1}{\lambda K}\cdot A(T)\frac{dT}{T} \tag{37}$$

and

$$\frac{dE}{E^{(1/\beta)+1}}=\frac{C_2}{\lambda K}\frac{dT}{T^2}. \tag{37'}$$

On integration these become

$$E=S'[\theta(T)]^\beta \tag{38}$$

and

$$E=ST^\beta. \tag{39}$$

The first of these two equations (equation 38) follows rigorously from Criteria I and II; the other (equation 39) follows in the special case that the emissivity is independent of the temperature. In the latter case it is seen, from a comparison of equations 34 and 39, that

$$Tk_\lambda=\text{constant} \tag{40}$$

so that T, the absolute temperature, is inversely proportional to the observed quantity k_λ.

EXPERIMENTAL DATA

Criterion I

As stated in previous paragraphs, Criterion I furnishes a means of determining the probable constancy of the emissivity of radiating substances in the visible spectrum over any range of temperature within which the criterion is applicable. The criterion was first used[1] as a means of testing the variation of a with temperature in the generalized Wien equation (equation 1) on the assumption of which Coblentz[1] had found a to decrease generally with temperature. Coblentz had determined a by Paschen's method from infra-red energy measurements. The author, by the application of Criterion I, found that there was little indication of a variation of a with temperature in the visible spectrum. In these early experiments ordinary commercial lamps were used, and no effort was made to avoid the effects produced by the cooling of the filaments at the leading-in and supporting wires.

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[1] *Loc. cit.*

The fulfilment of Criterion I, however, has more significance than was originally appreciated, and it was in the light of this new significance that the recent experiments were conducted. If Criterion I is fulfilled in the comparison of a metal with a black body, it points strongly to the constancy of the emissivity of the metal within the range of temperature and wave-length employed. It should be emphasized, however, as already stated, that Criterion I would also be fulfilled even though the average emissivity in the visible spectrum underwent some change with temperature, provided that this change in the average emissivity was accompanied by a relative change in the emissivity at different wave-lengths, of the right magnitude, and in the proper direction. This alternative significance is well within the range of possibility, but it would seem much more probable that the fulfilment of Criterion I would arise from the constancy of the emissivity in the visible spectrum.

In the recent experiments intercomparisons were made between untreated carbon, treated carbon, tantalum, tungsten, and the electrically heated black body. Various lamps were used, some of them of the ordinary commercial type, and others of special construction so made as to reduce the effect of the end losses to a minimum. In the case of the special lamps the most accurate set of measurements was made using a diaphragm to screen off the cooled ends of the filament. The exact procedure will be outlined below. Moreover, measurements were made in the two ways: (1) using the integral light from the lamps and bringing the two lamps to a color-match by means of an ordinary Lummer-Brodhun contrast photometer; (2) using monochromatic radiation of two wave-lengths and bringing the two lamps to the same relative intensities in these chosen wave-lengths by the use of a Lummer-Brodhun spectrophotometer. In all the experiments the same general results were obtained, but the conclusion reached several years ago in the first experiments,[1] using a color-match method, was corroborated, viz., that two lamps (having energy curves of the black-body type) can be brought to the same relative intensity of energy emission in two wave-lengths of the visible spectrum more accurately by means of the color-match method using the integral light than by means of direct measurements in the two wave-lengths as with a spectrophotometer.

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[1] *Loc. cit.*

The most trustworthy measurements are those which were made using this method and employing specially constructed lamps with long free filaments, the cooled ends of the filaments being screened by diaphragms placed between the lamp and the photometer, and as close to the lamp bulb as possible. The procedure was as follows: An untreated carbon filament lamp was mounted on one side of the photometer and a flashed or treated carbon filament lamp on the other side, each being so screened that only light from that portion of the filament which was of uniform brightness reached the photometer. The untreated carbon lamp was set at some arbitrary low voltage and then the voltage of the flashed carbon lamp was varied until a color-match was obtained as seen in the Lummer-Brodhun contrast photometer. Several determinations of the voltage of the flashed carbon lamp were made and the average value taken. The untreated carbon lamp was then set at some arbitrary high voltage, the range in voltage being made as large as possible consistent with the accuracy of color measurement at the lower limit, and the constancy of the lamp at the upper limit. A second set of measurements was made at this high voltage to determine the voltage of color-match for the flashed carbon lamp, and the average value taken.

Having determined the voltages of the flashed carbon lamp corresponding to color-match with the untreated carbon lamp at the two arbitrary voltages, the two lamps were intercompared as to relative candle-power when they were operated at the color-match voltages at the lower temperatures, and again when they were operated at the color-match voltages at the higher temperatures. If Criterion I were fulfilled the relative candle-powers should be the same in the two cases. If any difference in the relative candle-powers had been found Criterion I would have been shown not to be fulfilled.

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[1] *Loc. cit.*

A set of measurements such as that outlined above was made by at least two different observers for the four radiating substances —untreated carbon, flashed carbon, tantalum, and tungsten. In each case the untreated carbon lamp was placed on one side of the photometer and operated at the same two voltages, the other substances being placed successively on the other side of the photometer. The results of these measurements are given in Table I. In the first column are given the lamps which were compared with the untreated carbon lamp No. 63 operated at the two voltages $V_0 = 35$ volts and $V'_0 = 50$ volts. In the third column are given the individual determinations of $\frac{I'}{I'_0} \div \frac{I}{I_0}$ where I_0 and I'_0 are the candle-powers of the untreated carbon lamp at the two voltages V_0 and V'_0, and I and I' are the respective candle-powers of the other substances at the voltages V and V' corresponding to color-matches with the untreated carbon lamp at its two voltages. In the fourth column are given the average values. A quantity larger than unity indicates that the emissivity of the substance increases more rapidly than that of the untreated carbon and vice versa.

Taking untreated carbon as the standard, it is seen that there is no indication of a relative change in the emissivity in the visible spectrum of treated carbon or tantalum, but that there is marked evidence of a change in the emissivity of tungsten compared with that of the other substances. It is true that the data presented would indicate an increase in emissivity of 0.2 per cent for treated carbon, and a decrease of 0.7 per cent for tantalum, compared with untreated carbon, but it should be emphasized that the accuracy attainable in these measurements is certainly not better than 1 per cent and is more likely of the order of magnitude of 2 or 3 per cent. Indeed, the change of 3 or 4 per cent observed in the case of tungsten is so nearly within the reasonable limits of accuracy attainable that it was at first thought to have no definite significance, but as every one of the number of distinct experiments performed at different times and under different conditions, including one or two made with the spectrophotometer using monochromatic light of two wave-lengths, gave a deviation in the same direction and of the same order of magnitude, the reality of the indicated change seemed to be established.

The degree of uncertainty in the measurements is determined by the accuracy with which the color-match can be made, in the experiments with white light, or with which the filaments can be brought to the same relative intensity of emission in the two chosen wave-lengths, in the experiments with monochromatic light. As has already been stated, experience indicates that the two methods yield the same result within the limits of accuracy that have been reached, and that the color-match method is quite as sensitive, perhaps even a little more sensitive than the spectrophotometric method using monochromatic light, which is theoretically preferable, but which is very tedious. For these reasons the principal set of experiments of which the results are given in Table I was carried out using the color-match method. In a later part of the paper (Fig. 3) spectrophotometric curves corresponding to color-match will be given.

So far nothing has been said regarding absolute measurements of emissivity by comparison with a black body. The experiments cited indicate that untreated carbon, flashed carbon, and tantalum have no relative change in emissivity in the visible spectrum within the limits of temperature employed (about 400° centering around 2000° or 2100° abs.), and that the emissivity of tungsten decreases slightly in going from low to high temperature. Without comparison with a black body, however, it is impossible to say with certainty that the emissivities of untreated carbon, flashed carbon, and tantalum in the visible spectrum are constant, and that the emissivity of tungsten decreases slightly, although this conclusion may appear most probable. It may be that the emissivity of tungsten is constant and that the emissivities of the other filament materials increase slightly with increasing temperature, or even that the emissivities of all four bodies are changing, but that the differential changes are as indicated.

With the idea of settling the above question, Criterion I was applied in a comparison between an electrically heated black body and untreated carbon, using the color-match method. This test was not entirely satisfactory owing to the difficulty of securing a uniformly heated cavity with the furnace at the author's disposal at this time. The result obtained in the experiment indicated that the emissivity of the untreated carbon remained quite constant, but under the conditions an accuracy better than 2 or 3 per cent could not be considered. Hence so far as this experiment is concerned the only conclusion is that the emissivities of the entire group of bodies under investigation remain sensibly constant—the observed changes given in Table I being of the order of magnitude of the absolute changes, and not merely changes of a second order.

This conclusion is further strengthened by computations from earlier experiments made, with a different purpose in mind, on two electrically heated black bodies belonging respectively to Drs. Waidner and Burgess, and to Dr. Coblentz at the Bureau of Standards. Color-match measurements over a range of temperature were made between the two black bodies and a tungsten lamp at various voltages. As the tungsten lamp is still available it has been possible to measure its candle-power and to compare this with the computed change in candle-power of the black body over a given temperature range. The results of this comparison strengthened the conclusion from the other experiment that no large changes in emissivity in the visible spectrum occur in the cases of the filament materials studied.

In the light of all the experiments it would seem probable, therefore, that the emissivities of untreated carbon, flashed carbon, tantalum, and tungsten in the visible spectrum undergo little change over a temperature range of 300° or 400° centering around 1900°–2100°; further, that the observed relative change between tungsten and the other bodies is more probably to be ascribed to a slight change in the emissivity of tungsten, the emissivities of the other substances remaining constant. That the difference between tungsten and the other substances is real was established by a comparison of tungsten directly with tantalum over a much more extended range of temperature than was possible with untreated carbon. This test gave as a mean of four sets of measurements, two sets by each of two observers, a difference in the average of $6\frac{1}{2}$ per cent, the direction being the same as before, viz., that the emissivity of tungsten decreases in going from low to high temperature, as compared with tantalum.

The constancy of the emissivity of metals with change in temperature has been shown by various other observers using entirely different methods. Sissingh,[1] Drude,[2] Pflüger[3] found little if any change in the optical constants of iron, silver, gold, platinum, etc., over a temperature interval of 100° or 200° C. near room temperature. Zeeman[4] studied the reflecting power of platinum up to 800° and failed to find any appreciable change over this range of temperature. Holborn and Henning[5] obtained similar results for gold, silver, and platinum. Finally Henning,[6] using his spectral pyrometer, found no indication of change in emissivity in the visible spectrum over large ranges of temperatures in the neighborhood of 1500°–2000° abs. and for a large variety of substances including carbon, tantalum, tungsten, osmium, platinum, etc. It is true that Henning's test, like that of the author, does not necessarily indicate constancy in emissivity. If the logarithm of the absorptivity (A_0) at $\lambda_0=0.5\,\mu$ increased $\frac{\lambda_1}{\lambda_0}$ times as fast as $\log A_1$, where A_1 is the absorptivity at $\lambda_1=0.7\,\mu$, then (see Henning's equations)

$$\theta_1=\frac{1}{C}[\lambda_0 \log A_0-\lambda_1 \log A_1]$$

would be constant even though A_0 and A_1 should vary. The observed constancy of θ, however, would seem in all probability to point strongly to the constancy of the absorptivity at both wave-lengths. Certainly all the data, taken together, indicate strongly that the emissivities of metals in the visible spectrum do not undergo any appreciable change with change in temperature.

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[1] Sissingh, "Mesures de la polarisation elliptique de la lumiere," *Arch. Neerlandaises*, **20**. See also Dissertation, Leiden, 133, 1885.

[2] Drude, *Wied. Annalen*, **39**, 538, 1890.

[3] Pflüger, *ibid.*, **58**, 494, 1896.

[4] Zeeman, *Communications from the Laboratory of Physics at the University of Leiden*, No. 20, 1895.

[5] Holborn and Henning, *Berl. Berichte*, 311, 1905.

[6] Henning, *Ztschr. f. Instr.*, **30**, 61, 1910.

The emissivities of certain metals in the infra-red regions of the spectrum have, on the contrary, been found to undergo marked change with the temperature. Hagen and Rubens have found that the emissivities in the infra-red regions beyond 6 μ conform in their changes with temperature to the following equation, derived from the electromagnetic theory:

$$\text{Emissivity} = 36.5\sqrt{\frac{\rho}{\lambda}}$$

where ρ is the specific resistance and λ is the wave-length. This equation is derived on the assumption of free electrons and under the condition that neither the electrical conductivity nor the period of the emitted light is large. Hence it is not surprising that Hagen and Rubens found the equation to hold for long waves, but not for short ones. Their experiments on platinum gave no indication of a change in emissivity with temperature at $\lambda = 2\ \mu$ over a temperature interval extending from 400° to 1400°. At $\lambda = 4\ \mu$ a marked increase in emissivity with increasing temperature was observed, though the magnitude of the change was not as large as that demanded by the equation given above. At $\lambda = 6\ \mu$ the equation was quite accurately verified. Observations on other substances gave similar results.

The results which the author found on the change in emissivity in the visible spectrum of tungsten as compared with those of carbon and tantalum stand out uniquely, and merit further investigation, for although Criterion I might be fulfilled and yet the emissivity might undergo change, it is impossible to imagine that Criterion I should fail to be fulfilled and still that there should be no change with temperature of the emissivity of one of the two substances compared.

In the fifth column of Table I are given the ratios of the square roots of the resistances of the various filaments at high and low temperatures as compared with the corresponding ratios for untreated carbon. If the equation cited above were fulfilled for carbon, tantalum, and tungsten for emissivity in the visible spectrum, then the values given in the fourth column for $\frac{I'}{I'_0} \div \frac{I}{I_0}$ should be equal to the corresponding values of $\sqrt{\frac{R'}{R'_0} \div \frac{R}{R_0}}$ given in the fifth column. That is, the emissivity of tungsten should have been found to increase about 20 per cent compared with untreated carbon, rather than decrease $3\frac{1}{2}$ per cent as observed.

TABLE I

CHANGES IN CANDLE-POWER (I) AND RESISTANCE (R) OF VARIOUS RADIATING BODIES, AS COMPARED WITH CANDLE-POWER (I_0) AND RESISTANCE (R_0) OF UNTREATED CARBON, CORRESPONDING TO COLOR-MATCH WITH UNTREATED CARBON AT $V_0=35$ VOLTS AND $V'_0=50$ VOLTS

Lamps	Observers	$\frac{I'}{I'_0} \div \frac{I}{I_0}$ Individual Determinations	Means	$\sqrt{\frac{R'}{R'_0} \div \frac{R}{R_0}}$
Untreated carbon No. 63 (taken as standard)	....	.....	1.000	1.000
Flashed carbon No. 40	F.E.C.	0.999	1.002	1.04
Flashed carbon No. 40	A.G.W.	1.006		
Tantalum No. 6	F.E.C.	1.004	0.993	1.05
Tantalum No. 6	E.P.H.	0.997		
Tantalum No. 15	F.E.C.	0.979		
Tungsten No. 79	A.G.W.	0.965	0.963	1.19
Tungsten No. 79	F.E.C.	0.967		
Tungsten No. 79	E.P.H.	0.954		
Tungsten No. 82	A.G.W.	0.959		
Tungsten No. 82	F.E.C.	0.971		
Black body	....	.....	1.00	

Attention should be called, in passing, to the applicability of Criterion I (using two wave-lengths) to the study of the change in emissivity with temperature in the infra-red spectrum by spectrobolometric measurements. The criterion does not require a knowledge of the temperature, and, moreover, is quite sensitive. It is hoped that such application of the method to various metals will be made in this laboratory.

EQUATION OF ENERGY DISTRIBUTION

In the light of the results of the application of Criterion I, taken together with the evidence of other investigations, it would seem fairly well established that the emissivities in the visible spectrum of untreated carbon, flashed carbon, tantalum, and tungsten undergo little change with change in temperature over a temperature interval of about 400° centering around 2000° or 2100° abs. The observed difference between tungsten and the other three filament materials, which appears to be larger than the experimental error, will be ignored in the following discussion and the conclusions for each material will be correct only to the extent to which the constancy of the emissivity is verified.

Assuming, then, the constancy of the emissivity, the radiation from each of the five materials is distributed throughout the visible spectrum in accordance with equation 21 (with the modification according to Planck)

$$J=C_1F(\lambda)\frac{1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}}-1\right)} \tag{41}$$

in which C_2 is the black-body constant, but $C_1F(\lambda)$ depends on the material and is different for each substance. Two questions arise: (1) Over what range of wave-lengths does the equation hold? and (2) What is the form of the function $F(\lambda)$? In reply to the first question, the equation holds over the range of wave-lengths for which the emissivity is independent of the temperature. At 2 μ Hagen and Rubens found the emissivity constant for platinum and for an alloy of platinum-rhodium. At 4 μ the emissivity changes somewhat with the temperature. At what wave-length the emissivity begins to change is not known, but if it remains constant to 3 μ then the emissivity is constant throughout by far the largest part of the spectrum for platinum at the temperature of 1673° abs., the highest temperature (1400° C.+273°) at which the measurements on platinum were made. If the constancy of the emissivities of the substances with which we are dealing is established to 3 μ at temperatures of about 2000°, then the total energy will vary with the temperature quite approximately according to the integral of equation 41 taken over the entire spectrum. This is on the assumption that the emissivity in the ultra-violet undergoes no marked change with temperature, a condition which probably holds, particularly if there are no marked absorption bands. It has not yet been established, however, for the substances under investigation that the emissivities remain constant out to 2.5 μ or 3 μ.

The answer to the second question is found in the absorption coefficients as computed from Coblentz'[1] measurements of the reflecting power of the various materials in the visible spectrum. These absorptivities (or emissivities) show clearly the selectivity in favor of the visible spectrum for graphite, tantalum, and tungsten at room temperature. And since the constancy of the emissivity in the visible spectrum is assumed, they hold for glowing temperatures also.

It is of interest to know whether $F(\lambda)$ can be represented by a power of λ, since in that case the energy distribution, over the range of wave-length for which the emissivity is constant, would be represented by the generalized form of the Planck equation (equation 2)

$$J=C_1\frac{1}{\lambda^a\left(e^{\frac{C_2}{\lambda T}}-1\right)}$$

where $F(\lambda)$ is replaced by $\frac{1}{\lambda^{a-5}}$. It was concluded, however, from computations using Coblentz' data that no power $a-5$ could be obtained so that $\frac{1}{\lambda^{a-5}}$ would represent $F(\lambda)$. Fig. 1 shows the relative distributions of energy in the visible spectrum according to the generalized Planck equation if a is taken successively as 5 (black body), 6, 7, and 8, the temperatures being so chosen that the relative emission at $\lambda=0.5\ \mu$ and $\lambda=0.7\ \mu$ is always the same. Fig. 2 shows corresponding comparative curves for graphite, tantalum, and tungsten as computed from Coblentz' data, and, finally, Fig. 3 shows the actual observed curves for tantalum and tungsten as compared with untreated carbon corresponding to a color-match and plotted to the same ordinate at $\lambda=0.5\ \mu$. The actual temperatures at which the comparisons in the three figures were made are not exactly the same, but the differences would produce only a negligible effect on the conclusions deduced from a comparison of the three figures.

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[1] Coblentz, *Bulletin of the Bureau of Standards*, 7, 197, 1912.

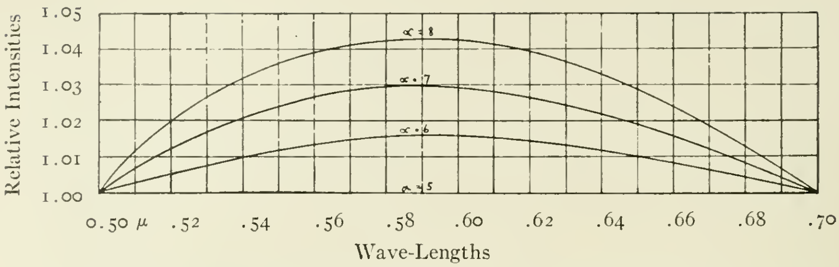

FIG. 1.—Relative distributions of energy in the visible spectrum for various values of a in the generalized Planck equation for temperatures, such that $J_{0.5\mu} \div J_{0.7\mu}$ is constant.

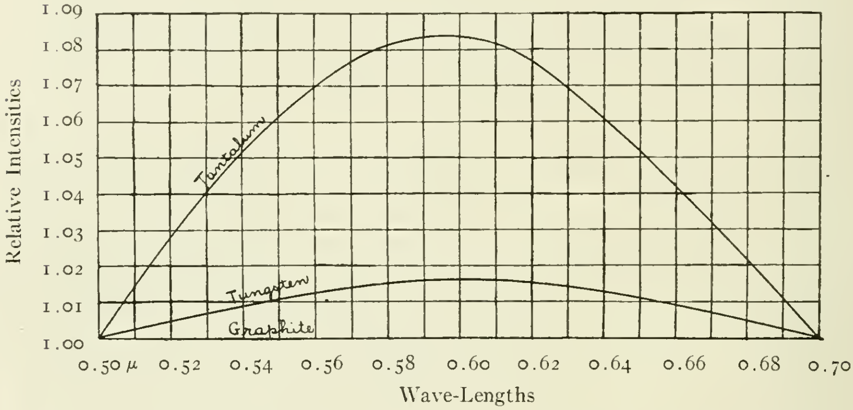

FIG. 2.—Relative distributions of energy in the visible spectrum for graphite, tantalum and tungsten, computed from reflection coefficients, and for temperatures such that $J_{0.5\mu} \div J_{0.7\mu}$ is constant.

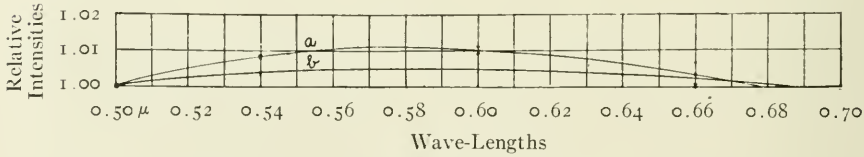

FIG. 3.—Relative observed distributions of energy in the visible spectrum for tantalum and tungsten as compared with untreated carbon for temperatures such that $J_{0.5\mu} \div J_{0.7\mu}$ is approximately constant.

a = Tantalum { · Observations by F. E. C.
o Observations by A. G. W.
b = Tungsten { × Observations by F. E. C.
+ Observations by A. G. W.

In Fig. 3 the observations of two different observers are plotted and show clearly the agreement between the integral light color-matches and the corresponding spectrophotometric relations. A comparison of the curves in this figure with those in Fig. 2 shows no inconsistency in the case of carbon or tungsten between the relative emissivities obtained by Coblentz at room temperatures and those indicated by the relative emission curves obtained at glowing temperatures. In the case of tantalum there is a pronounced discrepancy, but this is not unexpected in view of the difficulties encountered by Coblentz in securing a satisfactory reflecting surface of tantalum.

No attempt has been made to find a mathematical expression for the function $F(\lambda)$.

Before passing on to a discussion of the experimental data regarding Criterion II, it is of interest to consider certain of the data before us in connection with previous observations of the author and others[1] on the relative selectivity of various lamp filaments. The data on the reflectivity of carbon, tantalum, and tungsten showing an increasing emissivity with decreasing wave-length from the deep infra-red to the shortest wave-length in the visible where measurements were made, confirm the assumption made in the previous work regarding the relative temperatures at color-match. It was assumed at that time as highly probable that when tantalum or tungsten was at a color-match with a black body the latter was at the higher temperature. This must be true if the emissivities of tantalum and tungsten increase with decreasing wave-length throughout the visible spectrum as indicated in Coblentz' measurements. Moreover, the measured differences in lumens per watt between untreated carbon and the metals at a color-match must represent, as stated in the previous paper, the minimum effect of selectivity on the relative luminous efficiencies of these substances.

The data in the previous paper which showed that the relative selectivity, as indicated by the relative lumens per watt at color-match, decreased with increasing temperature is of interest in connection with the temperature change of the emissivity in the long wave-lengths. If a black body is compared with a selective body by measuring the relative lumens per watt at color-match at both low and high temperatures, then if the emissivities of the selective body for different wave-lengths are independent of the temperature throughout the entire spectrum, it can readily be shown by analysis that the relative selectivity, as indicated by the relative lumens per watt, would appear to increase with the temperature. The observed differences for tantalum and tungsten as compared with untreated carbon were on the contrary found to decrease by a somewhat larger amount. The explanation of this would seem to depend on the increasing emissivity in the long wave-lengths with increasing temperature.

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[1] Hyde, *Jour. Frank. Inst.*, **169**, 439, 1910; and *ibid.*, **170**, 26, 1910.

Criterion II

Criterion II is fulfilled if it is found experimentally that k_λ, the ratio of the relative change in the emission, J, at some wave-length, λ, to the relative change in the total emission, E, is inversely proportional to some root of the total energy, E (see equations 33 and 34)

$$\frac{(k_\lambda)_1}{(k_\lambda)_2}=\left(\frac{E_2}{E_1}\right)^{1/\beta}.$$

The fulfilment of the criterion rests on the constancy of β.

In most of the experiments measurements were made of k', the ratio of the relative change in the integral light or ordinary candle-power, I, to the relative change in the supplied watts, W, rather than of k_λ as defined above. It is then necessary to correct the relative change in candle-power to the relative change in emission in some wave-length, λ, and to correct the relative change in supplied watts to the relative change in total emission. The latter correction is quite small in all ordinary lamps, and was therefore neglected in most of the present measurements as disproportionately small compared with other sources of error.

The other correction, however, needs further consideration. In the case of a black body obeying Planck's law, the following relation holds in the visible spectrum at glowing temperatures (equations 13 and 23):

$$k_\lambda\equiv\frac{dJ_\lambda}{dE}\ \frac{E}{J_\lambda}=\frac{dJ_\lambda}{dT}\ \frac{T}{J_\lambda}\div\frac{dE}{dT}\ \frac{T}{E}=\frac{1}{4}\ \frac{C_2}{\lambda T}. \qquad (42)$$

The relative change in intensity, J_λ, at any wave-length, λ, corresponding to a given change in total emission, E, is inversely proportional to the wave-length. The relative change in total candle-power, I, will be some mean value of k_λ defined by the following equation:

$$k' = \frac{\int_0^\infty k_\lambda J_\lambda S_\lambda d\lambda}{\int_0^\infty J_\lambda S_\lambda d\lambda} \tag{43}$$

in which S_λ is the sensibility of the eye for the radiation of wave-length λ. Knowing the function S_λ, and also the distribution of energy at the given temperature, i.e., J_λ, it is possible to compute the relation between k' and k_λ for any wave-length and at any temperature. At each temperature there is some wave-length λ_0 for which $k_{\lambda_0} = k'$. As the temperature is raised, λ_0 becomes smaller and smaller, i.e., the relative change in white light increases more rapidly than the relative change in any monochromatic light, which latter increases inversely as the temperature.

Assuming J_λ, the distribution of energy in the spectrum of the black body, to be given by the Planck equation with $C_2 = 14{,}500$, and S_λ, the sensibility function, to be represented by the curve in Fig. 4, k' (equation 43) has been computed for different temperatures. The results of this computation are given in Fig. 5, curve *a*.[1] The data, as given in this curve, are not in the most convenient form for the present use. It is seen (equation 42) that

$$k_\lambda T = \frac{1}{4}\frac{C_2}{\lambda} = \text{constant}. \tag{44}$$

Hence $k'T$ (equation 43) will vary with the temperature by an amount proportional to the change in the ratio of k' to k_λ at any given wave-length. In curve *b* are plotted the values of $k'T$ against the temperature. The percentage difference between the ordinate of this curve at any temperature T, and the ordinate at some given temperature T_0, will be equal to the percentage correction which must be applied to the white-light coefficient k' at T in order to make the ratio of the white-light coefficient k' at T to the white-light coefficient k'_0 at T_0 the same as the ratio of the values of any monochromatic coefficient at the same two temperatures.

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[1] This curve is similar to that computed by Eisler (*E.T.Z.*, **25**, 189 and 443, 1904), but depends on a somewhat different sensibility curve which is probably more nearly correct.

Curve *b* is plotted to give the corrections for a black body at different temperatures, but it is equally applicable to any body which has a constant emissivity in the visible spectrum and which has a distribution of energy in the visible spectrum approximately the same as that of a black body at some temperature. For such a radiating body the relative corrections to two k''s at two different temperatures, in order to make the ratio equal to that of the two values of some monochromatic coefficient k_λ, is the same as that for the black body at such temperatures as correspond to color-matches with the radiating body under investigation. In order then to apply curve *b* to carbon, tantalum, and tungsten it is necessary to know the temperatures of the black body at which it has the same colors as these substances under given conditions of operation.

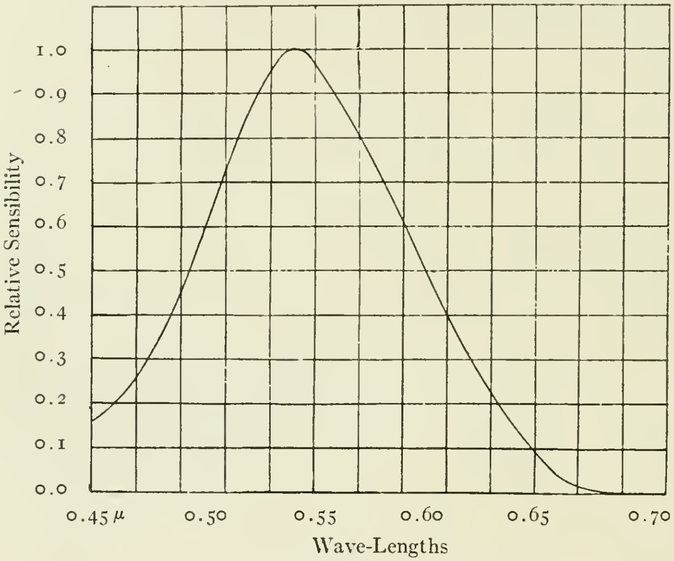

FIG. 4.—Sensibility curve of the eye

In earlier work of the author on the radiation of metals certain colors, i.e., certain distributions of energy in the visible spectrum, were arbitrarily fixed by arbitrarily chosen voltages of a given carbon lamp. Thus a certain 110-volt carbon lamp was operated successively at 75, 100, and 125 volts, and an attempt was made to determine the temperatures of a black body giving the same energy distribution in the visible spectrum. The determination at 75 volts was made by direct comparison with a black body, and gave as a result 1689° abs. (1416° C.+273°). The determinations at 100 and 125 volts were made by use of a Lummer-Brodhun spectrophotometer and gave the values 1886° abs. and 2058° abs. At the time these measurements were made, however, no investigation had been made of the effect of impurity of spectrum owing to the finite width of the spectrophotometer slits, and so these temperatures, as stated when they were published, are subject to correction. New determinations of these temperatures, using the results of the author's investigation[1] of the effect of the slit-width correction, have led to the following values, assuming the lowest

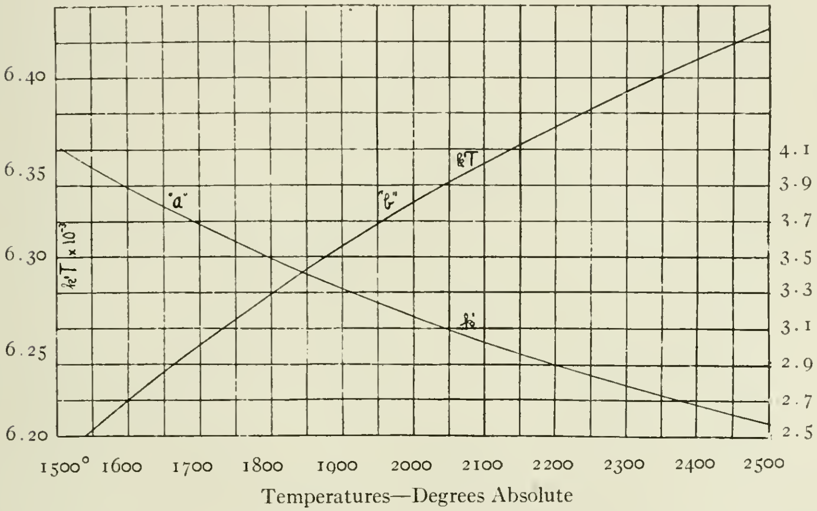

FIG. 5.—Computed values of k' and $k'T$ for a black body at various temperatures

temperature to be 1690° abs., viz., 1935° and 2130° abs. To the original three arbitrarily chosen "colors" corresponding to the colors of a black body at 1690°, 1935°, and 2130° abs. have been added a fourth corresponding to the color of a black body at 2295° abs.

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[1] *Physical Review*, **34**, 233, 1912, and *Astrophysical Journal*, **35**, 237, 1912.

Knowing the watts of the carbon, tantalum, and tungsten lamps corresponding to these colors it is possible, by reference to curve *b*, Fig. 5, to plot the corrections which must be applied to the

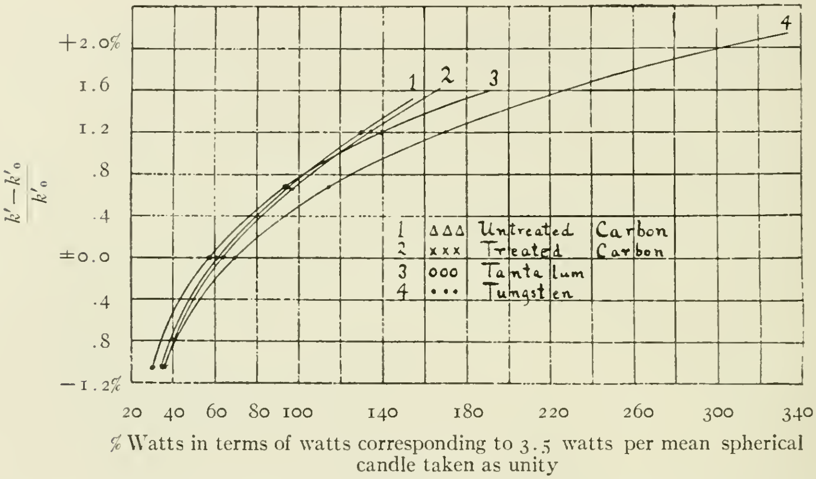

Fig. 6.—Corrections to be applied to observed k' (integral light) in order to get $k'_{\lambda=\lambda_0}$ (monochromatic light, λ_0 being so chosen that $k'=k'_{\lambda_0}$ at color match with a black-body at 1935° absolute).

observed k' at any watts to make it comparable with the k' at some other watts so that the ratio of the corrected white-light coefficients may be the same as that of the monochromatic coefficients, and hence applicable in equation 33:

$$\frac{(k_\lambda)_1}{(k_\lambda)_2}=\left(\frac{E_2}{E_1}\right)^{1/\beta}.$$

In Fig. 6 are plotted such correction curves, the abscissas being in percentage watts, where 100 per cent indicates a specific consumption of 3.5 watts per mean spherical candle for each of the four types of lamps, untreated carbon, flashed carbon, tantalum, and tungsten. The ordinates give percentage corrections which must be applied to the observed values of k' corresponding to given percentage wattages. Since only relative corrections are of interest, the correction at the color corresponding to a black body at 1935° is taken as zero for each substance. In other words, the monochromatic coefficient k_{λ_0} to which the white-light coefficients are corrected is that which is the same as the white-light coefficient at 1935°. At colors corresponding to higher black-body temperatures the white-light coefficients are relatively too large and must hence be reduced. At colors corresponding to lower black-body temperatures the white-light coefficients must be increased.

TABLE II

DATA FOR UNTREATED CARBON

W (Relative Watts)	Observed k' (White Light)	Corrected k' (Monochromatic Light)	β	$W^{1/\beta}$ (in Terms of Value at 90)	$1/k$ (in Terms of Value at 90)
30	3.593	3.636		.757	.758
35	3.467	3.501	4.44	.789	.788
40	3.352	3.376	3.67	.816	.817
50	3.180	3.190	3.94	.863	.864
60	3.050	3.051	4.10	.903	.904
70	2.950	2.944	4.32	.939	.937
80	2.857	2.846	3.94	.971	.969
90	2.774	2.758	3.75	1.000	1.000
100	2.700	2.680	3.68	1.027	1.029
110	2.627	2.603	3.27		
			Mean 3.99		

These corrections, given in Fig. 6, are applied to the observed white-light coefficients as recorded in Tables II to V. The method used in determining the values of k' is fully described in two papers[1] by F. E. Cady.

Table II contains the average data observed on four untreated carbon lamps. In the first column are given the relative watts expressed in terms of the watts corresponding to 3.5 watts per mean spherical candle taken as unity. In the second column are given the observed values of k', the percentage change in candle-power corresponding to 1 per cent change in watts. In the third column are given the corrected values of k' in accordance with the correction curves given in Fig. 6. The fourth column contains the computed values of β for the successive intervals, obtained by application of equation 33 which may be written in the form

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[1] *Loc. cit.*

$$\beta=\frac{\log E_{n+1}-\log E_n}{\log k'_n-\log k'_{n+1}},$$

or, putting for the total emission, E, its equivalent, W, the supplied watts,

$$\beta=\frac{\log W_{n+1}-\log W_n}{\log k'_n-\log k'_{n+1}}. \tag{45}$$

The weighted mean value of β over the entire range is given at the bottom of this column. The comparatively large variations in β are evident, but when it is considered that over the small intervals employed an error of 0.1 per cent in k' is sufficient to make an error of as much as 5 per cent in β, the magnitude of the differences is surprisingly small. It is evident, however, even with the large variations, that there is no consistent change in β. This is more strikingly shown, as is also the conformity of the data to equation 33, when a comparison is made between $E^{1/\beta}$ and $1/k'$. In the fifth column are given values of $W^{1/\beta}$ expressed in such an arbitrary unit that this quantity has the value of unity corresponding to $W=90$, β being the weighted mean value of the different β's for the various intervals. In the sixth column, for comparison with this

TABLE III

DATA FOR FLASHED CARBON

W (Relative Watts)	Observed k' (White Light)	Corrected k' (Monochromatic Light)	β	$W^{1/\beta}$ (in Terms of Value at 80)	$1/k'$ (in Terms of Value at 80)
40	3.330	3.356		.845	.840
50	3.162	3.173	3.98	.892	.889
60	3.033	3.035	4.10	.933	.929
70	2.928	2.923	4.10	.968	.966
80	2.831	2.821	3.76	1.000	1.000
90	2.749	2.734	3.76	1.029	1.032
100	2.684	2.665	4.12	1.056	1.059
110	2.627	2.604	4.11	1.081	1.083
120	2.580	2.554	4.49	1.104	1.104
130	2.541	2.512	4.83	1.126	1.123
140	2.506	2.474	4.86	1.147	1.140
			Mean 4.10		

are given the corrected values of $1/k'$ plotted in terms of a corresponding arbitrary unit. If the data conformed to equation 33, which expresses Criterion II, the figures in these two columns would be identical. The agreement is seen to be most satisfactory.

TABLE IV

DATA FOR TANTALUM

W (Relative Watts)	Observed k' (White Light)	Corrected k' (Monochromatic Light)	β	$W^{1/\beta}$ (in Terms of Value at 100)	$1/k'$ (in Terms of Value at 100)
50	2.793	2.798		.863	.862
60	2.701	2.696	4.91	.897	.895
70	2.618	2.610	4.75	.927	.924
80	2.541	2.529	4.23	.954	.954
100	2.430	2.412	4.70	1.000	1.000
130	2.302	2.276	4.54	1.058	1.060
160	2.205	2.174	4.53	1.105	1.110
190	2.141	2.107	5.49	1.146	1.145
			Mean 4.71		

TABLE V

DATA FOR TUNGSTEN

W (Relative Watts)	Observed k' (White Light)	Corrected k' (Monochromatic Light)	β	$W^{1/\beta}$ (in Terms of Value at 100)	$1/k'$ (in Terms of Value at 100)
37.64	2.85	2.876		.849	.850
53.76	2.71	2.719	6.35	.901	.899
80	2.544	2.539	5.80	.963	.962
90	2.496	2.487	5.69	.982	.983
100	2.456	2.444	6.04	1.000	1.000
110	2.420	2.405	5.93	1.016	1.016
120	2.389	2.371	6.10	1.031	1.031
140	2.329	2.307	5.63	1.058	1.059
170	2.260	2.233	5.96	1.093	1.095
200	2.203	2.172	5.86	1.123	1.125
230	2.158	2.123	6.14	1.150	1.152
260	2.119	2.081	6.14	1.174	1.174
300	2.070	2.029	5.65	1.203	1.204
			Mean 5.95		

Nowhere is there a difference of more than 0.2 per cent, which is within the observational error.

Similar data for flashed carbon, tantalum, and tungsten are given in Tables III, IV, and V. The agreement for these substances is almost as good, though the values of β are seen to be quite different. That the four filament materials quite closely fulfil Criterion II is indicated strongly by the data presented here.

In Table VI are collected the observed average values of β for the different substances given to the nearest figure in the first decimal place. It is significant that the value of β for untreated carbon should come out exactly 4.0, identical with that of a black body. For flashed carbon the value is 4.1, for tantalum 4.7, and for tungsten 6.0, the differences being in the direction one would expect from the observations of Coblentz and the earlier work of the author on selectivity. But it cannot be emphasized too strongly that the actual values of β as given are not insisted upon as being highly accurate, particularly in the case of tungsten, which gives evidence of a change in emissivity with temperature, even in the visible spectrum. A very slight uncertainty in the determination of k' would also be productive of error, so that the values of β actually obtained are surprisingly close to what might have been anticipated from accumulated evidence from other sources.

TABLE VI

AVERAGE VALUES OF β

Filaments	β
Untreated carbon	4.0
Flashed carbon	4.1
Tantalum	4.7
Tungsten	6.0

The present author[1] and others in the preceding work on the selectivity of radiation of filament materials found flashed carbon to be slightly more selective than untreated carbon; tantalum was found to be still more selective, and tungsten quite a little more selective than tantalum. Coblentz' reflection data indicate the same order and approximately the same degree of selectivity and the β's found in the present investigation are not inconsistent with the earlier data, though this would still be true if the value of β for tungsten was 5.5 instead of 6.0, a value more probable from some considerations.

To make sure that the observed value of β for tungsten was not in error owing to errors in observation, the determination was repeated using monochromatic light and making the measurements on a spectrophotometer, two observers making independent sets. A special lamp was employed and the end losses were measured so that only the central portion of the filament was in actual use. The two sets of measurements were in excellent agreement and gave as a mean value of β over a fairly large interval of watts the value 6.2. If, therefore, the change in energy radiated does not proceed as rapidly as the sixth power of the temperature, the discrepancy would seem to the author to lie in the unwarranted assumption of the constancy of the emissivity of tungsten in the visible spectrum.

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[1] *Loc. cit.*

Criterion II, which demands the constancy of β in the relation between k_λ and E (equation 33), is seen to be well fulfilled for untreated carbon, flashed carbon, tantalum, and tungsten, within the limits of experimental accuracy, and over the range of temperature within which the observations were made. If Criterion I, taken to indicate constancy of emissivity in the visible spectrum, were accurately fulfilled, then k_λ (or k' corrected) would be proportional to $1/T$ and so Criterion I in conjunction with Criterion II would lead to the generalized Stefan-Boltzmann law in the form

$$\frac{E_2}{E_1}=\left(\frac{T_2}{T_1}\right)^\beta \tag{46}$$

where β for the various substances has the values given in Table VI over the range of temperature employed. The theoretical necessity that β approach "4" will be discussed later, but before undertaking this discussion it is interesting to consider the connection between selective emission and the relation between total emission and temperature; in other words, to consider the influence of selective radiation on the exponent β in equation 46.

In the case of a black body the rate of change of intensity of emission with respect to a change in temperature varies with the wave-length. Corresponding to a given increase in temperature the intensity of emission at every wave-length is increased, but the rate of increase varies approximately inversely with λ, being therefore relatively large for the visible or ultra-violet region of the spectrum, and relatively small for the long infra-red rays. The average change for the total emission is, however, according to the

Stefan-Boltzmann law, as the fourth power of the temperature. Now if the emissivity in the shorter wave-lengths for another body is relatively larger than that in the longer wave-lengths, relatively more energy at any temperature will be radiated in the shorter wave-lengths as compared with that emitted in the longer wave-lengths than would be the case for a black body. Hence for a small increase in temperature the rate of increase of the intensity of emission in the shorter wave-lengths would have greater weight in determining the rate of increase of total emission than that in the longer wave-lengths. And since the coefficient of increase is greater for the shorter wave-lengths therefore the total emission would increase more rapidly than as the fourth power of the temperature, as in the case of a black body.

Assuming the emissivities of tungsten as given by Coblentz to remain constant, β was computed by the method outlined above, and yielded as a result the value 5.25, being thus somewhat lower than the value 6.0 found by the other method, notwithstanding the fact that since the emissivities in the longer wave-lengths probably increase with temperature, the selectivity at glowing temperatures is less pronounced than at room temperatures at which the reflection coefficients were measured. There is, however, one other element entering, still supposing the emissivity in the visible to be independent of the temperature, and that is the effect of the changing emissivities in the long wave-lengths. Since these latter increase with the temperature, one would expect the total emission to increase more rapidly with the temperature than would be computed on the assumption of their constancy. Hence corresponding to any given selectivity at any temperature, β would be larger than "4" for the two reasons: (1) the predominating effect of the larger coefficients in the shorter wave-lengths; and (2) the increase in the emissivities in the longer wave-lengths. No attempt has been made to compute the relative magnitudes of these two factors.

As the emissivities in the infra-red become larger, however, the selectivity decreases and so the first of the two factors must decrease. The effect of the second factor will depend on the rate of change of the emissivity in the long wave-lengths at increasing temperatures. This rate of increase must ultimately approach zero as the emissivity in this region approaches that of a black body. As this condition is approached the effect of the first factor, i.e., of the selectivity, must also become negligibly small, since the selectivity will have diminished until it is negligible. Hence, if, for a given radiating substance, one assumes that the substance approaches a black body in its radiation, and that if it should undergo no change of state but should continue to radiate in the same general way, it would ultimately radiate exactly like a black body, then it is clear that β would approach the value "4" as its limit. Indeed it is difficult to conceive how the exponent β in equation 46 can fail to decrease as the temperature is increased indefinitely, since otherwise the emission at sufficiently high temperatures would become greater than that of the black body. This argument for the decrease in β must not be confused, however, with the similar but less rigorous argument for the decrease in a in the generalized Planck equation (equation 24). The generalized Stefan-Boltzmann law (equation 26) contains two constants, σ and a, but since σ is itself a function of a (equation 27) the effect of a decrease in a with increasing temperature is not immediately determinable; it is necessary to consider the effect both in the exponent and in the coefficient.[1]

Since β must ultimately decrease, the question immediately arises why such a decrease would not manifest itself over the range of temperature used in the present experiments. The only answer is that it may possibly be changing continuously but that the magnitude of the change within the prescribed limits of temperature may be less than the experimental errors. With the exception of tungsten, which owing to the uncertainty regarding its conformity to Criterion I is somewhat unsatisfactory, the largest range of temperature is only of the order of magnitude of 600° or 800° as in the case of tantalum, and since β for tantalum is approximately 4.7 it is only necessary to assume that the decrease in β should be 0.2 or 0.3 every 1000° in order to have it reduced to 4.0 at 5000° or 6000° provided there was no change in state. But a change in β of 0.2 or 0.3 over the entire range of temperature used in the present experiments is well within the range of experimental error, and, moreover, there is no reason to believe that the radiating body would become a black body at so low a temperature as 5000° or 6000° abs. Of course this question is interlinked with that of the absolute emissivity at a given temperature, and the necessary changes in β are partly determined by this factor.

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[1] *Loc. cit.*

FILAMENT TEMPERATURES

Nothing has been said so far regarding the actual temperatures of operation of the various filaments. Moreover, there is no intention of entering into a general discussion of this question in the present paper. However, as certain deductions regarding temperature follow immediately from the preceding theory, it seems advisable to refer to them briefly. If Criterion I and Criterion II are both fulfilled, assuming the former to indicate constant emissivity in the visible spectrum, then (equations 35 and 39)

$$k_\lambda=\frac{dJ_\lambda}{dE}\cdot\frac{E}{J_\lambda}=\frac{dJ_\lambda}{dT}\cdot\frac{T}{J_\lambda}\div\frac{dE}{dT}\,\frac{T}{E}=\frac{C_2}{\lambda T}\div\beta=\frac{C_2}{\beta\lambda T} \tag{47}$$

and so

$$T=\frac{C_2}{\beta\lambda k_\lambda}. \tag{48}$$

Since $C_2=14{,}500$ as for a black body, and the other quantities are all derivable from the data given in the preceding paragraphs, it is possible to compute the temperature, T, corresponding to any measured k' corrected to k_λ for some given λ. This computation for untreated carbon at 4 watts per mean horizontal candle, flashed carbon at 3.1 wpc, tantalum at 2.0 wpc, and tungsten at 1.25 wpc, gives the values in Table VII.

The computed temperatures for all except tungsten would seem to be of the right order of magnitude, but it must be remembered that β enters as a first-power factor, and since this quantity is certainly not to be depended upon to an accuracy higher than 5 per cent, the computed temperatures may be in error to that extent, even though the radiators fulfil the conditions quite accurately. The temperature found for tungsten would seem to be entirely too low, but since there is direct evidence that tungsten does not conform to the conditions imposed in the fulfilment of the two criteria, equation 48 is not applicable.

The temperatures of tantalum and tungsten were also evaluated in two other ways, both of them, however, depending on the assumed melting-point temperatures of the two substances. For these the values found by Forsythe[1] were taken, viz., 3070° abs. for tantalum, and 3250° abs. for tungsten.

TABLE VII

NORMAL WORKING TEMPERATURES COMPUTED FROM EQUATION 59

Lamps	Watts per Mean Horizontal Candle	Absolute Temperature (°C.+273°)
Untreated carbon	4.0	2250
Flashed carbon	3.1	2290
Tantalum	2.0	2350
Tungsten	1.25	1910

According to one of these methods the wattage, W_0, corresponding to normal operation was measured and then the voltage was increased very rapidly until the filament melted and the maximum wattage, W, reached just before the filament melted was read. The β-root of the ratio of $W_0:W$ was taken as the ratio of the temperature T_0, of normal operation to the melting-point T. The results obtained were: 2175° abs. as the temperature of normal operation of tantalum, and 2375° abs. as the temperature of normal operation of tungsten. For each metal the mean of four or five determinations was taken, and the agreement among the individual determinations was of the order of magnitude of 20°.

Confining our attention to tantalum, since, as before, no great significance is attached to the data on tungsten, it is to be noted that the assumption is made that $\beta=4.7$, the mean value found for tantalum at lower temperatures. Since β may decrease with increasing temperature, the mean value of β between the temperature of normal operation and that of melting may be slightly smaller, the effect of this being to reduce the computed temperature of normal operation. Moreover, since the watts at the melting-point are measured for the entire filament, whereas the melting takes place at but one point, the measured watts must be too small by a greater or less amount, and the effect of this is such as to reduce the computed temperature of operation.

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[1] Forsythe, *Astrophysical Journal*, **34**, 353, 1911.

The second method is somewhat similar to the preceding. An image of a piece of the filament of the test lamp is focused in the plane of the filament of a comparison lamp, and the latter is viewed through a telescope with a monochromatic transmission screen in the eyepiece, the whole[1] constituting a modified form of the Holborn-Kurlbaum optical pyrometer. The test lamp (tantalum or tungsten) is set at normal operation, and the wattage of the foreground comparison lamp (an untreated carbon lamp for which $\beta=4.0$) is determined for a match. The voltage of the test lamp is then increased until the filament melts, and a measurement is made, as near the melting-point as possible, of the watts in the foreground carbon lamp corresponding to a match. From these measurements, and a knowledge of the melting-point of the test filament, it is possible to compute the temperature of normal operation on the basis of the following assumptions: (1) that the test filaments comply with Criterion I; (2) that the comparison carbon filament complies with Criterion I and Criterion II, having $\beta=4.0$; (3) that the temperature of normal operation of the carbon lamp is known.

The results obtained by this method, which is open to some of the criticisms of the previous method, are: 2100° abs. as the normal temperature of tantalum, and 2320° as the normal temperature of tungsten if the normal temperature of untreated carbon is taken as 2100°; and 2130° for tantalum, and 2350° for tungsten if the normal temperature of untreated carbon is taken as 2200°.

It is beyond the scope of the present paper to enter into a discussion of the above temperature measurements or to compare the results with those of other observers. The computations are given only for the purpose of anticipating a reasonable query as to the temperature relations deducible from the given data.

SUMMARY

The principal purpose of the present investigation has been to construct synthetically, so far as may be possible, the laws governing the radiation from untreated carbon, flashed carbon, tantalum, and tungsten, starting from certain simple photometric and electrical measurements. Two criteria have been established which conduce to certain more or less definite conclusions regarding the laws of radiation of glowing solids, and these criteria have been applied to the filament materials mentioned above.

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[1] For a full description of this method and its applications see Hyde, Cady, and Worthing, *loc. cit.*

Criterion I (which see), if fulfilled, points strongly to the constancy of the emissivity of a substance with change of temperature for the region of the spectrum in which it is tested. If not fulfilled it indicates definitely a change of emissivity with temperature. As applied to the various filament materials it has led to the conclusion that the emissivities in the visible spectrum of untreated carbon, flashed carbon, and tantalum are probably constant within the limits of attainable accuracy over the range of temperature employed, but that the emissivity of tungsten in the visible spectrum probably undergoes change with a change in temperature.

The criterion conduces definitely to the following equation representing the distribution of energy in the spectral region in which it is fulfilled:

$$J=C_1 f(\lambda)\frac{1}{e^{\frac{C_2}{\lambda\theta(T)}}}.$$

If the criterion is taken to indicate constancy of emissivity the equation becomes

$$J=C_1 F(\lambda)\frac{1}{\lambda^5 e^{\frac{C_2}{\lambda T}}}$$

which is the Wien equation for a black body, multiplied by the function $F(\lambda)$.

Criterion II (which see), taken in conjunction with Criterion I, leads to the following relation between the total emission and the temperature:

$$E=S'[\theta(T)]^{\beta}.$$

If Criterion I is taken to indicate constancy of emissivity the above equation becomes:

$$E=ST^{\beta},$$

the generalized form of the Stefan-Boltzmann law for a black body. All four of the investigated filament materials conform to the requirements of Criterion II and hence for all, except tungsten, the latter equation for total emission probably holds within the limits of temperature employed. On this assumption the following values of β have been computed: 4.0 for untreated carbon; 4.1 for flashed carbon; and 4.7 for tantalum. A similar computation for tungsten, neglecting the indicated lack of constancy of its emissivity, gives 6.0. These computed values of β are not insisted upon to a high degree of accuracy.

In addition to these specific results the paper contains a general discussion of the radiating properties of metals.

In conclusion the author desires to express his gratitude to his colleagues, Mr. F. E. Cady and Dr. A. G. Worthing, for valuable assistance in both the observations and computations.

Physical Laboratory
National Electric Lamp Association
Cleveland, Ohio
April 22, 1912

ELEMENTS OF THE ECLIPSING VARIABLES *W DELPHINI, W URSAE MAJORIS,* AND *W CRUCIS*

By HENRY NORRIS RUSSELL

As an illustration of the methods developed in the preceding papers,[1] details are here given of the computation of the elements of three eclipsing variables. Reference will be made throughout to the notation and tables of these papers, to which the reader is referred for explanation.

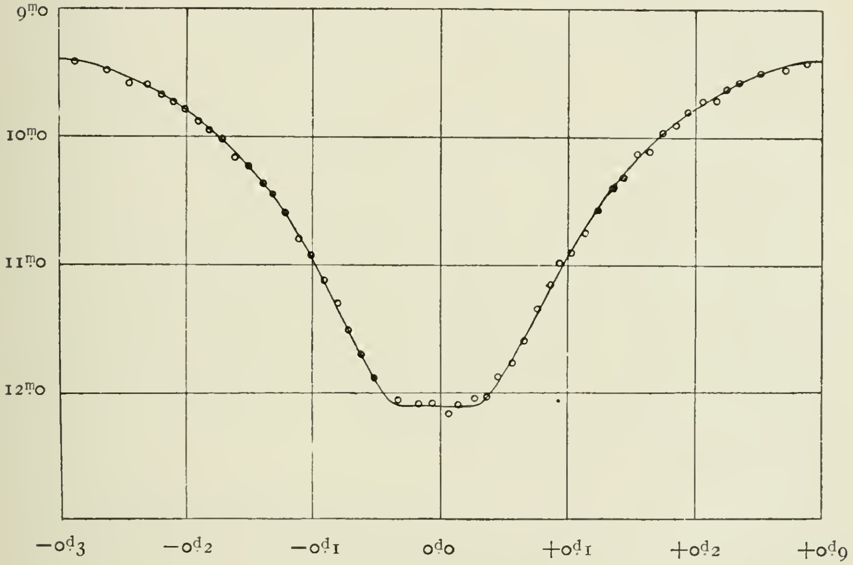

Fig. 1.—Light-Curve of *W Delphini*

I. *W Delphini* ($20^{h}33^{m}.1$, +17°56′ [1900]) is a good example of the typical *Algol* variable, with a deep primary minimum, showing a constant phase, and little or no secondary minimum. Its light-curve is very well defined by the 500 observations by Professor Wendell, with a polarizing photometer, which are published in the

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[1] *Astrophysical Journal*, **35**, 315, 1912, and **36**, 54, 1912.

Harvard *Annals*, 69, Part 1. These have been combined into normal places as follows, the assumed period of 4.8061 days requiring no alteration:

TABLE I

OBSERVED MAGNITUDES

Phase	Mag.	No. Obs.	O.−C.	Phase	Mag.	No. Obs.	O.−C.
$-0^d.2894$	9.41	6	$+0^m.01$	$+0^d.0560$	11.76	7	$+0^m.01$
.2637	9.49	5	+ .02	.0659	11.58	8	+ .01
.2458	9.58	5	+ .04	.0753	11.33	7	− .04
.2306	9.59	4	− .01	.0859	11.14	5	− .02
.2200	9.67	5	.00	.0937	10.97	5	− .05
.2106	9.73	8	+ .01	.1036	10.88	8	+ .02
.2007	9.79	10	.00	.1147	10.73	8	+ .05
.1911	9.88	12	+ .02	.1246	10.56	12	+ .03
.1817	9.95	10	+ .01	.1351	10.39	14	.00
.1718	10.02	8	.00	.1445	10.31	11	+ .04
.1615	10.16	17	+ .04	.1546	10.13	10	− .02
.1506	10.23	14	.00	.1641	10.10	11	+ .04
.1396	10.37	14	+ .01	.1744	9.97	10	.00
.1311	10.44	16	− .03	.1847	9.90	9	+ .02
.1212	10.59	17	− .03	.1941	9.79	9	− .02
.1121	10.78	14	+ .01	.2050	9.71	8	− .02
.1013	10.91	17	− .04	.2157	9.71	6	+ .04
.0906	11.12	14	− .01	.2242	9.63	8	+ .01
.0809	11.30	10	− .02	.2345	9.57	7	.00
.0715	11.51	12	.00	.2507	9.50	7	.00
.0617	11.69	10	.00	.2708	9.48	7	+ .03
.0509	11.88	7	.00	.2811	9.43	4	+ .02
.0313	12.05	5	− .04	.94	9.42	5	+ .02
.0169	12.08	4	− .02	1.90	9.35	5	− .05
− .0082	12.07	7	− .03	2.04	9.41	7	+ .01
+ .0060	12.16	5	+ .06	2.67	9.38	5	− .02
.0139	12.09	4	− .01	3.04	9.42	3	+ .02
.0261	12.03	5	− .07	4.04	9.44	6	+ .04
.0356	12.02	6	− .03	4.48	9.36	7	− .04
+ .0460	11.87	6	− .04				

From the 38 observations outside minimum we find the magnitude during constant light to be $9^m.395 \pm 0.009$. There is no evidence of any change during this period. With a circular orbit, the secondary minimum should occur at phase $2^d.40$. As none of the observations fall within $0^d.27$ of this, they give us no information whether such a minimum exists. The light-curve of the principal minimum is very well determined. The eclipse lasts from about $-0^d.28$ to $+0^d.28$, and there is a short constant period at the middle, of apparently a little less than one-tenth the total duration of the eclipse. The mean of the 20 observations lying within $0^d.02$ of the middle of eclipse gives for the magnitude at this phase $12^{\mathrm{m}}.10 \pm 0.014$. The range of variation is therefore $2^{\mathrm{m}}.70$, and the light-intensity at minimum 0.0832 times that at maximum. This shows at once that the eclipse is total, for if it was annular, the companion (even if perfectly dark) must cut off at least 0.917 of the light of the primary, and hence its radius cannot be less than 0.956 times that of the latter. In such a system the duration of the annular phase could not exceed $\frac{0.044}{1.956}$, or 0.022 of the whole duration of eclipse. The observed constant phase is almost five times as long as this.

The brighter star therefore gives 0.9168 of the whole light of the system, and if isolated would appear of magnitude 9.49; while the fainter but larger star which eclipses it gives out only one-eleventh as much light, and when seen alone at minimum is of magnitude 12.10.

We have now to determine the values of $\sin^2\theta$ corresponding to given values of α_1 (the percentage of obscuration). The loss of light $(1-l)$ will in any given case be $0.9168\,\alpha_1$. The corresponding change in magnitude can be taken at once from Table A. We now draw a free-hand light-curve to represent the observed points, and read from this the epochs t_1 and t_2 at which the magnitude just computed is reached. Taking half the difference of these as the interval t from the middle of eclipse to the given phase, we have $\theta = \frac{2\pi t}{P} = 1.3065\,t$, where θ is expressed in radians, and t in days. Taking the values of $\theta - \sin\theta$ from Table B, we find $\sin\theta$ and $\sin^2\theta$. The results are given in Table II.

From the values of t_1 and t_2 it appears that the observed curve is remarkably symmetrical, and that the actual epoch of mid-eclipse is $0^{\mathrm{d}}.0015$ earlier than that assumed by Wendell. The times of beginning and end of the light-changes cannot be read accurately from the curve, and are marked with colons to denote uncertainty.

We have now to find k from the values of $\sin^2\theta$, with the aid of Table II. If A is the value of this quantity when $\alpha_1 = 0.6$, and $A-B$ its value when $\alpha_1 = 0.9$, we have in general $\psi(k, \alpha_1) = \frac{\sin^2\theta - A}{B}$. For every value of α_1, we may now find a value of k, by inverse interpolation in Table II (or better graphically, by plotting $\psi(k, a_1)$ against k for the tabular values of a_1). When the tabular differences of ψ are small, the resulting values of k are less accurate, and are marked with colons.

As a first approximation we take $A=0.0369$, $B=0.0258$ (which values are derived as above from the observed values of $\sin^2 \theta$ corresponding to $a_1=0.6$ and $a_1=0.9$, and partake of any errors which may affect the latter). The resulting values of ψ and k are given in the last columns of Table II. The deduced values of k are fairly accordant, except for those corresponding to values of a_1 near 0.6. A glance at Table II (of the preceding paper) shows that this discrepancy may be almost removed by increasing all the values of ψ by 0.024—which may be done by diminishing A by 0.024 B. Our new value of A is therefore 0.0363. The resulting values of ψ and k are those given under the heading "2d Approximation" in Table III. The values of a_1 near 0.9 now give discordant values of k; but by diminishing B by $2\frac{1}{2}$ per cent, and hence increasing all the computed values of ψ in the corresponding ratio, we obtain a third approximation of a very satisfactory character. Excluding the uncertain data for the beginning of eclipse and of totality, and giving the values of k corresponding to $a_1=0.2$, 0.3, 0.4, 0.95, 0.98, and 0.99 double weight, we find for the mean value of k:

TABLE II

a	$1-l$	Mag.	t_1	t_2	θ	$\sin^2 \theta$	$\sin^2 \theta - A$	$\psi\ (k, a_1)$	k
0.0	0.0000	$9^{\mathrm{m}}400$	$-0^{\mathrm{d}}304$:	$+0^{\mathrm{d}}300$:	0.394:	0.1474:	0.1105:	+4.28:	0.56:
.1	.0917	9.505	.2540	.2515	.3304	.1050	.0681	2.64	.504
.2	.1834	9.620	.2285	.2258	.2968	.0860	.0491	1.908	.505
.3	.2750	9.749	.2075	.2030	.2681	.0702	.0333	1.290	.480
.4	.3667	9.896	.1884	.1830	.2426	.0576	.0207	0.802	.462
.5	.4584	10.066	.1682	.1644	.2173	.0462	.0193	+0.361	.36:
.6	.5500	10.266	.1486	.1470	.1931	.0369	.0000	......	....
.7	.6417	10.514	.1270	.1274	.1661	.0272	− .0097	−0.376	.64:
.8	.7334	10.835	.1070	.1048	.1381	.0190	− .0179	−0.694	.56:
.9	.8250	11.292	.0824	.0788	.1054	.0111	− .0258	−1.000	....
.95	.8709	11.624	.0655	.0624	.0836	.0071	− .0298	−1.155	.58:
.98	.8985	11.884	.0505	.0462	.0632	.0040	− .0329	−1.277	.525
.99	.9076	11.985	.0430	.0390	.0536	.0029	− .0340	−1.318	.528
1.00	.9168	12.100	− .021:	+ .019:	.026:	.0007:	− .0362:	−1.404:	.50:

From the upper part of the light-curve........... 0.530
From the middle part of the light-curve.......... 0.525
From the lower part of the light-curve........... 0.528

The general mean gives $k=0.528$. If we were to compute the probable error of this mean, from the accordance of the individual determinations, remembering that three quantities—A, B, and k—have been determined from the observations, we would find $k=0.528\pm0.005$. This process is by no means rigorous, but suffices to show that k is very accurately determined. A and B must be known with still greater precision, for changes of only 0.0006 in either one led to the large differences between the successive approximations.

We may now compute a theoretical light-curve, using the constants resulting from our final approximation: $A=0.0363$, $B=0.0252$, $k=0.528$. The results are given in the last columns of Table III, and explain themselves.

TABLE III

	2D APPROX.		3D APPROX.		FINAL LIGHT-CURVE				
α_1	ψ	k	ψ	k	$\psi(0.528,\alpha_1)$	$B\psi$	$\sin^2\theta$	$\sin\theta$	t
0.0	+4.30:	0.56:	+4.40:	0.58:	+4.100	0.1032	0.1395	0.373	0.292
.1	2.665	.512	2.73	.534	2.713	.0683	.1046	.324	.252
.2	1.932	.517	1.974	.538	1.949	.0491	.0854	.292	.227
.3	1.314	.500	1.344	.527	1.348	.0338	.0701	.265	.205
.4	0.826	.503	0.845	.532	0.843	.0212	.0575	.240	.185
.5	0.385	.47:	0.394	.51:	+0.400	.0101	.0464	.215	.166
.6	+0.024	.....	+0.025	.....	0.000	.0000	.0363	.191	.146
.7	−0.352	.48:	−0.360	.54:	−0.358	− .0090	.0273	.165	.127
.8	−0.670	.40:	−0.685	.51:	−0.689	− .0173	.0190	.138	.106
.9	−0.976	.....	−1.000	.....	−1.000	− .0252	.0111	.105	.081
.95	−1.131	.714	−1.157	.564	−1.162	− .0293	.0070	.084	.064
.98	−1.253	.610	−1.282	.507	−1.276	− .0322	.0041	.064	.049
.99	−1.294	.597	−1.324	.512	−1.318	− .0332	.0031	.056	.043
1.00	−1.38:	0.55:	−1.412:	0.48:	−1.389	− .0350	.0013	.036	.028

Plotting the magnitudes computed in Table I against the epochs $-0^{\mathrm{d}}0015\pm t$, we obtain the computed light-curve. The residuals (O.−C.) are given in the last column of Table I. Their average value, regardless of sign, is $0\overset{\mathrm{m}}{.}020$.

We may now proceed to the determination of the elements, which we might just as well have carried out before computing the light-curve. From Table II*a* of the previous paper[1] we find for $k=0.528$, $\phi_1(k)=0.382$, $\phi_2(k)=0.507$, whence

$$\cot^2 i=\frac{B}{\phi_2(k)}-A=0.0133,\quad \cot i=0.115,\quad i=83^\circ\ 25'.$$

$$r_1^2 \operatorname{cosec}^2 i=\frac{B}{\phi_1(k)}=0.0660,\quad r_1^2=0.0652,\quad r_1=0.256,$$

and finally

$$r_2=kr_1=0.135.$$

In other words, we have, taking the radius of the orbit as unity,

Radius of larger star	0.256
Radius of smaller star	0.135
Inclination of orbit plane	83°25′
Least apparent distance of centers	0.114
Light of larger star	0.0832
Light of smaller star	0.9168

At the middle of eclipse, the larger star overlaps the other by only 0.007 of the radius of the orbit, or about one-twentieth of the radius of the smaller body, so that the eclipse is very nearly grazing. The smaller star gives off eleven times as much light as the other, and exceeds it forty-fold in surface-brightness.

The loss of light at secondary minimum should be k times the light of the fainter star, or 0.023 of that of the system. The corresponding change in stellar magnitude is 0.027, which could only be detected by refined observations.

Applying our formulae for the density of the components, we find, on the hypothesis of equal masses, 0.118 of the sun's density for the density of the smaller and brighter star, and 0.017 for the density of the larger and fainter one. Since the brighter component of a pair differing in magnitude as much as this is usually decidedly the more massive (at least among the visual binaries which alone are so far available for investigation), it is probable that the actual density of the brighter star is about one-third greater, and that of the fainter star one-third less, than the values just computed for them. It is probable that the bright star is at least ten times as dense as its duller companion. The spectrum of the former is of Class A. It would be very interesting to know that of the latter; but its extreme faintness, and the short interval—only about $1^{h}.3$—during which it may be observed without interference by the light of the primary, probably put it beyond the range of direct observation.

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[1] *Astrophysical Journal*, **35**, 337, 1912.

II. The variable *W Ursae Majoris* ($9^{h}\ 36^{m}.7$, $+56° 25'$) enjoyed for some time after its discovery the distinction of possessing the shortest known period. The discoverers, Müller and Kempf,[1] published an excellent light-curve based upon 140 observations with the photometer of the Potsdam Observatory. They found the variation continuous, with a period of $4^{h}\ 0^{m}\ 13^{s}.267$, and much more rapid near minimum than near maximum, and called attention to a curious flattening of the light-curve about 45^{m} on each side of the maximum. They state that the variation cannot be explained by the rotation of a single ellipsoid, but can be well accounted for by almost central eclipse of two ellipsoids nearly in contact—in which case the orbital period must obviously be 8^{h}. They do not however publish elements, or any comparison of observation with theory.

Five years later, J. M. Baldwin[2] published the results of 260 observations made with the same instrument, which completely confirm the character of the variation, though indicating a somewhat smaller range. The odd and even minima were found to be sensibly equal in depth, and to follow one another at sensibly equal intervals.

The present discussion is based on the results of both observers. The normal places derived from the observations of each, and those finally adopted, are given in Table IV. The differences between the results of the two observers for the same phase vary considerably. On the average, Müller and Kempf appear to have measured the star $0^{m}.027$ fainter than Baldwin. After correction for this, the average difference, regardless of sign, between the normal magnitudes derived by the two observers is $0^{m}.040$. Ten of the individual differences (in the sense M. and K.—0.027—Baldwin) are positive, and ten are negative, and, when arranged in order of phase, they show ten changes and ten permanences of sign. There does not therefore appear to be any evidence of real change in the star's variation, or of other systematic differences between the observers. The means of their results, with weights proportional to the average number of observations combined into a normal place, has been taken as definitive. Baldwin's normals have received weight 3, except for the last three, which are formed by rearranging observations originally grouped by him into two normals, and have weight 2. Müller and Kempf's normals have all received weight 2.

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[1] *Astrophysical Journal*, **17**, 201, 1903; *Sitzungsberichte der Preussischen Akad.*, 1903, p. 173.

[2] *Monthly Notices*, **69**, 78–90, 1908.

TABLE IV

M. and K.		Baldwin		Adopted		Corr. for Ellipticity	Rectified Mag.	O.—C₁	O.—C₂
Phase	Mag.	Phase	Mag.	Phase	Mag.				
$0^h\ 3^m$	$8^m.55$	$0^h\ 3^m.2$	$8^m.48$	$0^h\ 3^m.1$	$8^m.51$	$-0^m.30$	$8^m.21$	$0^m.00$	$0^m.00$
0 11	8.46	0 11 5	8.42	0 11 3	8.44	−0.29	8.15	− .03	− .02
0 20	8.36	0 21 4	8.39	0 20 9	8.38	−0.27	8.11	+ .02	+ .02
0 27	8.30	0 29 8	8.24	0 28 7	8.26	−0.25	8.01	.00	.00
0 35	8.20	0 38 4	8.19	0 37 1	8.20	−0.22	7.98	+ .04	+ .03
0 44	8.20	0 45 5	8.09	0 44 9	8.14	−0.19	7.95	+ .04	+ .03
0 59	8.11	0 53 5 1 0 7	8.06 8.02	0 57 6	8.06	−0.14	7.92	+ .01	+ .01
1 14	8.07	1 12 1 1 21 6	7.99 8.00	1 16 1	8.01	−0.07	7.94	+ .03	+ .03
1 29	7.98	1 32 0	7.97	1 30 8	7.98	−0.04	7.94	+ .03	+ .03
1 44	7.89	1 42 0 1 53 4	7.91 7.95	1 46 8	7.92	−0.01	7.91	.00	.00
2 5	7.88	2 2 3 2 10 9	7.90 7.89	2 6 2	7.89	0.00	7.89	− .02	− .02
2 29	7.96	2 28 9	7.91	2 28 9	7.93	−0.01	7.92	+ .01	+ .01
2 46	7.99	2 41 2 2 55 4	7.96 7.94	2 47 7	7.96	−0.08	7.88	− .03	− .03
3 9	8.05	3 7 6	8.06	3 8 2	8.06	−0.16	7.90	− .01	− .01
3 22	8.08	3 17 7	8.11	3 19 4	8.10	−0.21	7.89	− .02	− .03
3 28	8.16	3 26 7	8.13	3 27 2	8.14	−0.23	7.91	− .03	− .04
3 35	8.23	3 35 2	8.25	3 35 1	8.24	−0.26	7.98	− .04	− .04
3 42	8.35	3 42 4	8.37	3 42 2	8.36	−0.28	8.08	.00	+ .01
3 47	8.47	3 48 4	8.45	3 47 7	8.46	−0.29	8.17	+ .04	+ .04
3 54	8.57	3 54 2	8.43	3 54 1	8.50	−0.30	8.20	+ .02	+ .02

The observations appear to indicate an asymmetrical light-curve, with the rise to maximum slower than the fall after it; but a symmetrical curve, with minimum at phase $2^m.2$, represents them with no residual exceeding $0^{m}.04$, and an average residual, regardless of sign, of $0^{m}.015$. In view of the differences between the results of the two observers, this may be regarded as quite satisfactory.

Applying the principles of the eclipse theory to this system, we have first to find the ellipticity of the stars, and to "rectify" the light-curve. Reading off from the empirical curve the magnitude at intervals of fifteen minutes, and calculating the values of L^2 and $\cos^2 \theta$, we find derive Table V.

TABLE V

t	θ	Mag.	$1-L^2$	$\cos^2 \theta$	$0.420 \cos^2 \theta$	$m-m_0$	M
$0^{h}00^{m}$	0	8.510	0.669	1.000	0.420	0.296	8.206
0 15	$11\frac{1}{4}$	8.420	.609	0.964	.405	.282	8.192
0 30	$22\frac{1}{2}$	8.224	.439	.854	.358	.240	8.150
0 45	$33\frac{3}{4}$	8.106	.303	.691	.290	.186	8.098
1 00	45	8.038	.210	.500	.210	.128	8.038
1 15	$56\frac{1}{4}$	7.985	.129	.309	.130	.075	7.985
1 30	$67\frac{1}{2}$	7.943	.059	.146	.061	.034	7.944
1 45	$78\frac{3}{4}$	7.920	.018	.036	.015	.008	7.918
2 00	90	7.910	0.000	0.000	0.000	0.000	7.910

The values of $1-L^2$ are taken from Table A, with the argument $2(m-m_0)$ where m_0 is the magnitude at maximum. Plotting $1-L^2$ against $\cos^2 \theta$, it is found that the last five points lie very nearly on a straight line, whose equation is $1-L^2=0.420 \cos^2 \theta$. The values of $1-L^2$ computed from this equation, and the corresponding magnitudes, are given in the last columns of Table V, and represent the observed curve for an hour each side of maximum with extraordinary closeness. The ellipticity of the stars therefore accounts for this part of the variation, and explains the flattening of the curve noticed by Müller and Kempf. The eclipse—marked by the observed magnitude being lower than the computed—extends only a little more than 45 minutes on each side of the minimum. The constant $z=\epsilon^2 \sin^2 i$ is now determined as 0.420.

If from each observed magnitude we subtract the corresponding value of $m-m_0$ we obtain the material for a rectified light-curve, from which the influence of ellipticity is practically removed. The values of this correction, and the "rectified" magnitudes, are given in the seventh and eighth columns of Table IV. A plot of the latter shows a light-curve of the ordinary *Algol* type, the magnitude outside eclipse being 7.912, and the eclipse $0^m.30$ in depth, without constant phase.

The eclipses being partial, we must solve our problem with the aid of Tables III and III*a* of the previous paper.[1] We first find the values of $\sin^2\theta$ corresponding to different values of n—that is, to different fractions of the maximum obscuration—which is done, with the aid of a free-hand symmetrical light-curve, just as in the case of the preceding star. The loss of light corresponding to the greatest eclipse is 0.240, and we find:

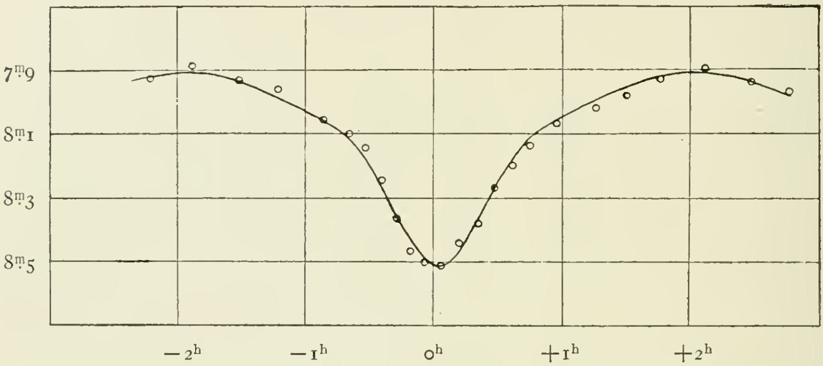

FIG. 2.—Light-Curve of *W Ursae Majoris*

TABLE VI

n	$1-l$	Mag.	t	θ	$\sin^2\theta$	$\frac{\sin^2\theta}{1-z\cos^2\theta}$	k_1	k_2	t_1	t_2
0.0	0.000	7.912	±47ᵐ:	0.616:	0.333:	0.461:	0.435	0.492	±44ᵐ1	±48ᵐ7
.1	.024	7.938	36 8	.483	.216	.322	.329	.360	37 3	39 5
.2	.048	7.965	31 8	.417	.164	.252	.264	.283	32 6	34 0
.3	.072	7.993	28 4	.372	.132	.208	.218	.228	29 1	29 9
.4	.096	8.022	25 4	.333	.107	.172	.177	.181	25 6	25 9
.5	.120	8.051	22 8	.299	.087	.141	.140	.140	22 7	22 7
.6	.144	8.081	20 0	.262	.067	.110	.107	.104	19 7	19 3
.7	.168	8.112	17 4	.228	.051	.085	.080	.076	16 8	16 3
.8	.192	8.144	13 9	.182	.033	.055	.050	.046	13 2	12 6
.9	.216	8.176	9 7	.127	.016	.028	.025	.022	9 2	8 6
1.0	0.240	8.210	0 0	0.000	0.000	0.000	0.000	0.000	0 0	0 0

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[1] *Astrophysical Journal*, **35**, 337–338, 1912.

We have now to determine two constants C and D, so that the equation

$$\frac{\sin^2\theta}{1-z\cos^2\theta}=C\omega_2(n)+D\omega_1(n)$$

shall represent the observed values as closely as possible. The ratio $\frac{C}{D}$ must lie within certain limits, determined by the depth of the minima. From the equations

$$\alpha_0=1-\lambda_1+\frac{1-\lambda_2}{k^2}\,, \quad \frac{C}{D}=\chi(k,\,\alpha_0,\,\tfrac{1}{4})\,,$$

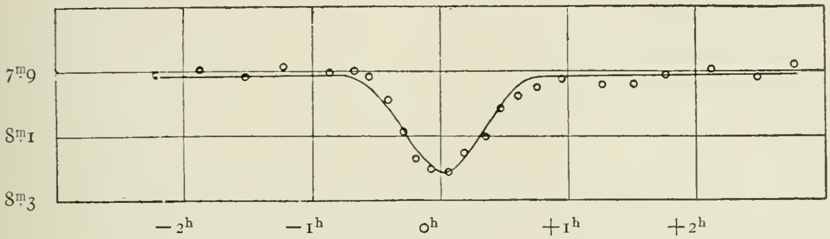

FIG. 3.—Rectified Light-Curve of *W Ursae Majoris*

we find, with the aid of Table III, setting $1-\lambda_1=1-\lambda_2=0.240$, the following values of this ratio, when k and α_0 vary through their whole possible range.

k	1.00	0.90	0.80	0.70	0.65	0.60	0.565
α_0	0.482	0.539	0.618	0.733	0.812	0.911	1.000
$\chi(k, \alpha_0, \frac{1}{4})$	1.817	1.805	1.805	1.802	1.798	1.778	1.720

The ratio $\frac{C}{D}$ must therefore be very nearly 1.80 unless the eclipse is almost total; and, except in this last event, the solution is practically indeterminate.

Setting $C=1.72\,D$, and plotting the values of $\frac{\sin^2\theta}{1-z\cos^2\theta}$ against those of $\omega_1(n)+1.72\omega_2(n)$, we find the best value of D to be 0.140. The corresponding values of $C\omega_2+D\omega_1$ are entered in the column headed k_1 in Table VI. The representation of the observed values is well within the errors of observation. With the other limiting value, $C=1.82\ D$, we find again that the best value of D is 0.140, and obtain the numbers in the column headed k_2, which appear less satisfactory. If however we derive from k_1 and k_2 the corresponding values of $\sin^2\theta$ and of t (which are given in the last two columns of Table VI) by the equations

$$\sin^2\theta=\frac{k(1-z)}{1-k}\ , \quad t=\frac{P}{2\pi}\theta\ ,$$

and plot the light-curves corresponding to the two sets of elements, we find that the differences between the magnitudes computed on the two hypotheses are as follows:

Phase	0^{m}	10^{m}	20^{m}	30^{m}	35^{m}	40^{m}	45^{m}
(1)–(2)...........	$0^{\mathrm{m}}.000$	$+0^{\mathrm{m}}.006$	$+0^{\mathrm{m}}.003$	$-0^{\mathrm{m}}.005$	$-0^{\mathrm{m}}.011$	$-0^{\mathrm{m}}.012$	$-0^{\mathrm{m}}.005$

In the only region where the difference between the two curves exceeds $0^{\mathrm{m}}.01$, the observed magnitudes during decreasing light lie about $0^{\mathrm{m}}.03$ above them, and those during increasing light as far below. Normal points formed by combining observations equidistant before and after minimum lie between the two curves. It therefore appears that we cannot hope to distinguish between the two hypotheses (or any of the intermediate ones) on the basis of the existing observations; nor could we do so, even if their already high accuracy were greatly increased. The elements are practically indeterminate within this range—owing mainly to the small magnitude of the partial eclipses.

The residuals, in the sense O.–C., from the light-curves computed on the limiting hypotheses, are given in the last two columns of Table IV. None of the residuals exceeds $0^{\mathrm{m}}.04$, and their average value, regardless of sign, is $0^{\mathrm{m}}.021$—exactly the same on the two hypotheses. Since five unknowns[1] have been determined from the 20 observations, the probable error of one normal magnitude is $0^{\mathrm{m}}.019$. From the average value ($0^{\mathrm{m}}.040$) of the difference between the results of the two observers, the probable error of one of these same normals comes out $0^{\mathrm{m}}.018$—an excellent agreement. The eclipse theory therefore furnishes a satisfactory explanation of the observed variation of this star.

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[1] The epoch of minimum, magnitude at maximum, and the constants z, C, and D.

We may now compute elements corresponding to the limiting hypotheses, as an indication of the range within which the true values must lie. Our equations are (if θ' is the value of θ at the beginning of eclipse):

$$a_1^2(1-z)[1+kp(k, \alpha_0)]^2=\cos^2 i.$$

$$a(1-z\cos^2\theta')(1+k)^2=\cos^2 i\cos^2\theta'+\sin^2\theta'.$$

$$\epsilon^2=z \operatorname{cosec} i;\quad b_1^2=a_1^2(1-\epsilon^2);\quad a_2=ka_1;\quad b_2-kb_1.$$

The function $p(k, \alpha_0)$ is given in Table I of the preceding paper.

On our two hypotheses we now find:

		I	II
Ratio of radii of stars	k	0.565	1.000
Maximum percentage of eclipse	α_0	1.000	0.482
Function	$p(k, \alpha_0)$	−1.000	−0.161
Value of $\sin^2\theta$ at beginning of eclipse	$\sin^2\theta'$	0.310	0.359
Ellipticity constant	z	0.420	0.420
Major semi-axis of larger star	a_1	0.431	0.366
Minor semi-axis of larger star	b_1	0.326	0.273
Major semi-axis of smaller star	a_2	0.242	0.366
Minor semi-axis of smaller star	b_2	0.184	0.273
Eccentricity of meridian section	ϵ	0.654	0.667
Inclination of orbit plane	i	81°47′	76°30′
Least apparent distance of centers	($\cos i$)	0.143	0.234
Light of larger star	L_1	0.760	0.500
Light of smaller star	L_2	0.240	0.500
Mean density of the system	(the sun being 1)	2.24	2.22

The mean density of this system is much greater than in the case of any other eclipsing variable, but is not intrinsically improbable. The spectrum of the pair is of Class G; that of most of the eclipsing variables is of Class A, and their densities are in all other cases less than the sun's, and usually very much less. In this case, therefore, the solar type of spectrum appears to go with a late stage of evolution.

If the mass of this system is equal to that of the sun, the relative orbital velocity of the components must be 300 km per second. This star is therefore a very tempting object for spectroscopic investigation, though too faint for any but the greatest instruments. Spectrographic observations would not only give the real dimensions and mass of the system, but would remove the uncertainty regarding those elements which are derived from the photometric data; for the ratio of the amounts of light emitted by the two stars could be estimated from the relative intensities of their spectra, and this, since the surface brightness of the two is the same, would give the ratio of their radii, (k), and remove all ambiguity.

III. The third variable here considered, *W Crucis* (*C.P.D.* $-58^\circ 4151$; $12^h\ 6^m.7$, $-58^\circ\ 14'$), was discovered photographically by Miss Leavitt at Harvard in 1906. The present discussion is based on unpublished observations, which were very kindly communicated to the writer by Miss Leavitt, with the generous approval of Professor Pickering, to both of whom the writer's very hearty thanks are due.

The observations were made by estimation on photographs belonging to the Harvard Photographic Library. Concerning the details of observation, Miss Leavitt says:

> The variable was compared with four stars, and the magnitudes employed were 8.40, 9.00, 9.58, and 10.10. Magnitudes have now been determined by means of comparisons with the South Polar Sequence, on three plates having duplicate exposures. . . . . While the magnitudes of the South Polar Sequence are provisional, it is hoped that they are approximately correct, at least in scale. The magnitudes of the comparison stars for *W. Crucis*, as found by this method, are 8.24, 9.08, 9.68, and 9.99. As the variable, practically, was compared only with the second and third of these stars, between which the earlier and later intervals are 0.58 mag. and 0.60 mag., respectively, and estimates were made only to the nearest tenth of a magnitude, there seems to be no reason to suppose that the range of the variable, as indicated by the light-curve already plotted, differs greatly from the true range. The uniformity of the scale in different parts of the curve evidently depends entirely on the judgment of the observer. An examination of recent photographs gives no conclusive evidence that the period of this variable has changed.

The light-curve is clearly of the *β Lyrae* type; the elements for prediction of the principal minimum being, according to Miss Leavitt, Min. = J.D. $2{,}410{,}158 + 198^d.5\ E$.

The observations as communicated to the writer consisted of 68 normal places, each representing the mean of the estimates on five plates, adjacent in order of phase. In Table VII, to save space, adjacent pairs of these have been combined into single normals, except near principal minimum (between phases -20^d and 20^d), when the variation is rapid. The normal places in the lower part of the table are therefore of twice the weight of those in the upper part.

From a free-hand curve drawn to represent these normals, it appears that the middle of the principal minimum occurs at about

TABLE VII

Phase	Mag.	Corr.	Rectified Mag.	O.−C.	Phase	Mag.	Corr.	Rectified Mag.	O.−C.
−17^{d}.7	9^{m}.05	−0^{m}.13	8^{m}.92	−0^{m}.02	55^{d}.5	8^{m}.90	+0^{m}.02	8^{m}.92	+0^{m}.02
−15.5	9.11	−0.14	8.97	−0.03	66.2	8.92	−0.02	8.90	0.00
−12.5	9.27	−0.15	9.12	+0.02	69.7	8.96	−0.04	8.92	+0.02
−10.1	9.33	−0.16	9.17	−0.02	74.9	9.04	−0.07	8.97	+0.05
− 7.0	9.50	−0.17	9.33	+0.01	80.6	9.03	−0.10	8.93	−0.04
− 4.5	9.45	−0.17	9.28	−0.05	86.5	9.13	−0.13	9.00	−0.01
− 2.5	9.45	−0.17	9.28	−0.05	94.8	9.19	−0.16	9.03	+0.02
− 1.1	9.47	−0.17	9.30	−0.03	100.4	9.16	−0.17	8.99	−0.02
− 0.5	9.55	−0.17	9.38	+0.05	106.3	9.15	−0.16	8.99	+0.03
+ 1.0	9.52	−0.17	9.35	+0.02	118.2	9.00	−0.12	8.88	−0.02
+ 6.5	9.52	−0.16	9.36	+0.03	124.8	8.96	−0.09	8.87	−0.03
+11.4	9.40	−0.13	9.27	+0.02	128.2	8.94	−0.07	8.87	−0.03
+13.2	9.28	−0.11	9.17	0.00	133.8	8.98	−0.05	8.93	+0.03
+17.0	8.94	−0.09	8.85	(−0.21)	142.4	8.93	−0.03	8.90	0.00
+23.2	9.01	−0.07	8.94	+0.04	150.2	8.92	−0.02	8.90	0.00
+26.6	8.96	−0.05	8.91	+0.01	153.5	8.92	−0.03	8.89	−0.01
+28.8	8.96	−0.04	8.92	+0.02	156.1	8.94	−0.03	8.91	+0.01
+33.8	8.90	−0.01	8.89	−0.01	162.5	8.94	−0.05	8.89	−0.01
+38.5	8.90	+0.01	8.91	+0.01	168.0	8.97	−0.07	8.90	0.00
+43.8	8.87	+0.02	8.89	−0.01	176.1	9.02	−0.11	8.91	+0.01
+49.2	8.85	+0.03	8.88	−0.02					

phase 1^{d}, and that of the secondary about 97^{d}. The interval from primary to secondary is shorter than that from secondary back to primary; hence the orbit is eccentric, and periastron passage occurs during the first interval. The maximum light during this interval is 8^{m}.87, while near apastron it is unquestionably less—about 8^{m}.92. This difference may arise from a combination of causes, connected with the eccentricity of the orbit, such as increased ellipticity of the stars when nearest one another, mutual heating, or tidal effects. It is small enough to justify summary treatment; and therefore the observed magnitudes have simply been corrected for this effect by subtracting the quantity 0^{m}.025 cos θ. This would in any case be the first term in the Fourier series by which a more complicated correction might be expressed, and is ample for the present purpose.

Having thus eliminated what may be called the "periastron effect," we proceed, just as in the case of the last star, to find the ellipticity-constant z—assuming the maxima to come halfway between the minima, and taking means of the observed values of L^2 at equal intervals before and after both maxima. Plotting these against $\cos^2 \theta$, we find $z=0.27$. (In this computation the angle θ, which here expresses the rotation of the stars, is supposed to increase uniformly with the time, in spite of the eccentricity of the orbit.) We may now compute the correction for ellipticity of figure, and adding it to that for the periastron effect, obtain the corrections given in the third column of Table VII, and the rectified magnitudes given in the fourth column.

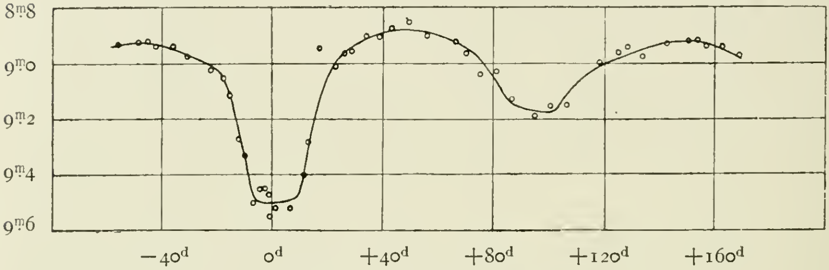

FIG. 4.—Light-Curve of *W Crucis*

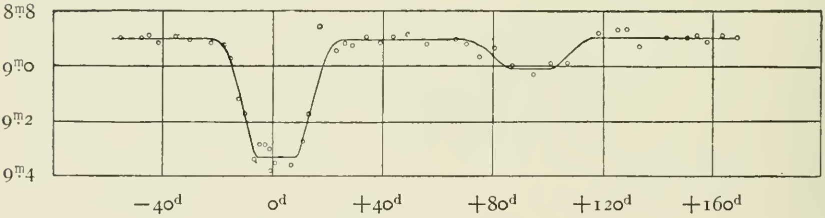

FIG. 5.—Rectified Light-Curve of *W Crucis*

A plot of these shows a light-curve of the normal *Algol* type, with a conspicuous constant phase during the principal minimum, the rectified magnitude being 9.33 for about fourteen days. The secondary minimum is about $0^m.10$ in depth. The mean rectified magnitude between the principal minimum and the secondary is 8.902, and between secondary and primary is 8.895—an excellent agreement. The middle of the principal minimum can be fixed with considerable accuracy at phase $1\overset{\mathrm{d}}{.}3$. That of the secondary minimum is much more uncertain, owing to the small amplitude of the variation, especially as there are very few observations on the ascending branch of the curve. A curve obtained by reducing the depth of the principal minimum (measured of course in light-intensity, not in magnitude) so that the change in magnitude is 0.11, and with the middle of eclipse at 93^{d}, gives a good representation of the observations.

We may now determine the orbital eccentricity. The equation

$$e \cos \omega = \frac{\pi}{2P}(t_2 - t_1 - \tfrac{1}{2}P)$$

gives in this case $e \cos \omega = -0.06$. The corresponding quantity, $e \sin \omega$, can be found only by comparing the lengths of the two minima. The secondary is so shallow that it is impossible to determine its duration with any accuracy. The assumption that it is of the same length as the primary represents the observations as well as there is reason to expect. We may therefore set $e \sin \omega = 0$, with relatively high uncertainty. This gives, for the elements of the orbit of the star which is eclipsed at principal minimum, about the other as a center, $e = 0.06$, $\omega = 180°$. Periastron passage occurs halfway between the principal and the secondary minima.

From the discussion in Part III of the preceding paper, it appears that when $e \sin \omega = 0$ the only sensible effect of the orbital eccentricity upon the form of the light-curve for either minimum is to make them very slightly asymmetrical. The minima will be of equal length, and the radii of the stars and inclination of the orbit can be determined from the light-curve of either one, just as if the orbit was circular. We shall see later that the theoretically predicted asymmetry is also practically insensible.

Proceeding to determine the remaining elements, we note that the ratio k of the radii of the components may be found at once from the depths of the minima. The loss of light at the principal minimum is 0.324 of that of the whole system, and that at the secondary is 0.096. If the principal minimum is assumed to be total, the smaller star must emit 0.324 of the light of the system, and obscure $\frac{0.096}{0.676}$ or 0.142 of the light of the larger star when it eclipses it annularly at secondary minimum; whence $k=\sqrt{0.142}=0.38$. In the same way we find that if the secondary minimum is total, the value of k is 0.60. To distinguish between these values we must make a solution for the principal minimum, along the same lines as in the case of the first star discussed in this paper. Our fundamental equation is

$$\sin^2\theta-A=B(1-z\cos^2\theta)\psi(k,\alpha_1)$$

and the numerical data are given in Table VIII.

TABLE VIII

a_1	MAG.	t	$\sin^2\theta$	$\frac{\sin^2\theta-A}{1-z\cos^2\theta}$	ψ	k	COMPUTED FOR k 0.38			
							ψ	$\sin^2\theta$	t	m^1
0.0	8.900	$23^{d}1$:	0.450:	0.345:	3.92:	0.49:	3.42	0.422	$22^{d}3$	8.90
.2	8.973	17.4	0.275	0.148	1.68	0.36	1.70	0.280	17.7	8.92
.4	9.051	15.0	0.210	0.068	0.77	0.40	0.76	0.212	15.1	8.94
.6	9.135	12.8	0.156	0.000	0.00	....	0.00	0.156	12.8	8.96
.8	9.226	10.7	0.110	−0.060	−0.68	0.46	−0.67	0.109	10.7	8.99
.9	9.276	9.6	0.090	−0.088	−1.00	....	−1.00	0.087	9.4	9.00
1.0	9.335	7.8:	0.059:	−0.130:	−1.48:	0.30:	−1.45	0.056	7.6	9.01

From the data for $a_1=0.6$ and $a_1=0.9$ we find $A=0.156$ and $B=0.088$, and thence the values of ψ and k in the sixth and seventh columns of the table. It is evident that the smaller of the two values of k derived above is the correct one, and therefore that the principal eclipse is total.

We may now improve our determination of the other constants. Plotting $\frac{\sin^2\theta-A}{1-z\cos^2\theta}$ against the new values of ψ, we find the most satisfactory value of B to be 0.092, while A retains its former value. We may then compute the values of $\sin^2\theta$ and t given in the latter part of the table—which are in excellent agreement with those derived from observation. For the secondary minimum, we may use the same values of t, with the magnitudes given in the last column of the table.

The light-curve may now be drawn, and the residuals found for the individual observations. These are given in the last column of Table VII. With the exception of the observation at phase 17.0 (which is evidently affected by some error, and has been neglected in the discussion), the representation is very good. The average value of the residuals, regardless of sign, is $0^m.027$ for the 13 normals of unit weight, and $0^m.018$ for the 27 of double weight. Since nine unknowns in all have been determined from the observations, the probable error of a normal place of unit weight is $\pm 0^m.025$. When it is considered that these normals are means of but five estimates, made only to $0^m.1$, on photographs taken with no special reference to this star, or to photometric use, this accuracy is amazing. The value of the Harvard photographs in the study of all classes of regular variables will evidently be exceedingly great.

The distribution of the residuals, in sign and magnitude, shows no anomalies, and there can be no doubt that the eclipse theory, as here developed, gives a very satisfactory account of the variation of this star.

We may next proceed to determine the elements, using the equations

$$a_1^2(1+k)^2(1-z\cos^2\theta') = \cos^2 i \cos^2\theta' + \sin^2\theta',$$
$$a_1^2(1-k)^2(1-z\cos^2\theta'') = \cos^2 i \cos^2\theta'' + \sin^2\theta'',$$

where θ' and θ'' correspond to the beginning of eclipse and of totality. Introducing the numerical values of our constants, we have

$$1.608a_1^2 = 0.578\cos^2 i + 0.422,$$
$$0.286a_1^2 = 0.944\cos^2 i + 0.056,$$

whence $a_1^2 = 0.270$, $\cos^2 i = 0.0225$.

We may now calculate the asymmetry of the light-curve. The duration of the light-changes is increased on the side toward periastron, and decreased on the opposite side, by an amount ϵ, which, by equation (36) of the preceding paper, is

$$\epsilon = -e\cos\omega\,(\sec\theta - 1)(\operatorname{cosec}^2 i - \cos\theta).$$

At the beginning or end of eclipse $\theta = 40^\circ.8$, and, introducing $e\cos\omega = -0.06$, and changing ϵ from circular measure into time, we find $\epsilon = 0^{d}.16$. The light-curve does not begin to slope steeply until θ has diminished to about 32°, and at this time $\epsilon = 0^{d}.06$. The change of magnitude in this interval is hardly more than $0^{m}.001$, and hence the light-curve is sensibly symmetrical.

Proceeding finally to determine the remaining elements, we have the following summary:

Major semi-axis of the larger star	a_1	0.520
Minor semi-axis of the larger star	b_1	0.442
Major semi-axis of the smaller star	a_2	0.197
Minor semi-axis of the smaller star	b_2	0.168
Eccentricity of meridian section	ϵ	0.526
Inclination of orbit plane	i	81°21′
Least apparent distance of centers	$\cos i$	0.150
Light of larger star	L_1	0.676
Light of smaller star	L_2	0.324
Eccentricity of orbit	e	0.06
Longitude of periastron	ω	180°
Distance of surfaces at periastron		0.22
Distance of surfaces at apastron		0.34
Period	P	$198^{d}.5$

The closeness of the components, and the considerable variation in the distance of the nearest points of their surfaces, make the observed brightening of 5 per cent at periastron, compared with apastron, not at all surprising. Both components are of remarkably small density. On the assumption of equal masses, we find for the smaller star a density of 3.1×10^{-5}, and for the larger star 1.7×10^{-6}, the sun's density being unity. That is, their mean densities are approximately $\frac{1}{30}$ and $\frac{1}{550}$ of that of air under the standard conditions of temperature and pressure. If one of the stars is denser than the assigned value, the other must be less dense; and neither one can be more than twice as dense as the assigned value.

These results may appear at first sight almost incredible; but the density computed for the larger star is that which the sun would have if expanded to a radius of about 60,000,000 km, or the size of the orbit of *Mercury*. The nebulae, which show sensible disks, must be many times larger—probably many hundreds of times—and their mean densities correspondingly less, so that this system is probably far advanced beyond the nebulous condition. It might also be questioned whether bodies of such low density could be opaque; but in traversing 120,000,000 km of gas of the mean density of the larger component, the light of its neighbor would be as much weakened as if it had penetrated 200,000 km of air of standard density. A very much smaller thickness would undoubtedly be practically opaque.

The spectrum of this system was examined by Mrs. Fleming, who made the following notes upon it, dated February 3, 1911:

Plate	Date	Expos.	Spectrum	Remarks
B 37591..	June 6, 1907	60$^{\text{m}}$	G pec.	"Composite, showing many bright lines. May be similar to β *Lyrae*. Near max. br.? Lines fine and clear."
B 38957..	May 20, 1908	60$^{\text{m}}$	G pec.	"Composite. A few bright lines visible. Near min.?"

According to Miss Leavitt's elements, the first of these photographs was taken thirty-two days after the principal minimum, outside eclipse, and the second sixteen days before the principal minimum, when the smaller star was partly concealed behind the larger.

The existence of bright lines in the spectrum of bodies of such low density is not surprising, but the fact that the background is of solar type shows that spectra of this class cannot be confined to stars late in the order of evolution; for this system is clearly in a very early stage.

The opportunity to make certain general remarks concerning this system, even though they may involve a certain speculative element, is too tempting to be passed over.

We may first inquire into what sort of a system this would develop under the influence of tidal evolution. It the mass of the system is M, that of the larger star yM, and the angular velocity of rotation and orbital motion is ω, the orbital moment of momentum is $M\omega y(1-y)$ (neglecting the small orbital eccentricity), while the rotational moments of momentum of the two stars are $c_1 My\omega(a_1^2+b_1^2)$ and $c_2 My\omega(a_2^2+b_2^2)$, where c_1 and c_2 are constants depending on the internal constitution of the stars. For homogeneous ellipsoids, $c=0.2$; but the actual stars are probably strongly condensed toward their centers, so that we will assume $c=0.1$.

Introducing the numerical values of a and b for the two components we find for the ratio of the rotational to the orbital moment of momentum $\frac{0.46c_1}{y}+\frac{0.07c_2}{1-y}$. If $c_1=c_2=0.1$, and the stars are equal in mass, this becomes 0.10; and if the larger star has three times the mass of the smaller, it is 0.19. Tidal action tends to transfer all the rotational momentum into orbital, and the parameter of the orbit increases as the square of the orbital momentum. If this process goes to the limit, the final parameter of the orbit will on our first hypothesis have 1.2 times its present value, and on our second 1.4 times. If during the process the orbital eccentricity should increase to 0.5, the final major axis of the orbit would be 15 per cent greater than the parameter. The final period of the system, on the most favorable hypothesis, would be twice the present period, or a little more than a year.

This system therefore approaches, but does not reach, the hypothetical conditions which must have prevailed in the early stages of evolution of visual binary stars, if the latter originated by the fission of a nebulous mass. A system similar to the present, but of $\frac{1}{30}$ the density, might evolve into a binary pair similar to δ *Equulei*.

In the second place, it is easy to show that any plausible values of the mass and surface-brightness of the system demand that its distance shall be exceedingly great. Assuming, for example, that the mass and surface-brightness of the larger component are equal to those of the sun, we find that its mean diameter must be about 85 times that of the sun, and its light-emission some 7000 times as great, while that of the whole system would exceed 10,000 times the sun's light. Its photographic magnitude at maximum is 8.9, and its visual magnitude probably about 8.2. To appear of this brightness, the sun (whose stellar magnitude is about -26.8) would have to be removed to a distance of 150 light-years, corresponding to a parallax of $0\overset{''}{.}021$. On the above assumptions (neglecting possible absorption of light in interstellar space) *W Crucis* would be 100 times as far away—at a distance of 15,000 light-years, with a parallax of 0."0002. These assumptions may of course be greatly in error; but even if we assume a mass $\frac{1}{8}$ that of the sun (which is less than has been found for any star for which the necessary data are at all reliable), and a surface-brightness $\frac{1}{10}$ that of the sun (which seems very small, considering the general similarity of their spectral types), we still find a distance of 2500 light-years. By modifying our assumptions in the opposite direction, without going outside the range of observed stellar masses, the computed distance might be raised to 50,000 light-years. In any case, there can be no doubt that this star is enormously remote.

Spectroscopic study of the system would be of extreme interest; but it is so faint, and so far south, that it is practically out of reach with existing instruments.

The light-curves shown in the figures are simply graphical representations of parts of Tables I, IV, and VII. The curves are in all cases those corresponding to the finally adopted elements. The agreement between theory and observation is excellent. The "rectified" light-curves show how a curve of the β *Lyrae* type is transformed into one of the ordinary *Algol* type by the application of the correction for the ellipticity of the stars.

Princeton University Observatory
April 10, 1912

ON A NEW METHOD OF DETERMINING THE ANGULAR DIAMETERS OF STARS BY MEANS OF ELLIPTICALLY POLARIZED LIGHT

By S. POKROWSKY

The method which I propose for the determination of the angular diameters of stars by means of the elliptical polarization of light differs essentially from that of Fizeau. It is based on the consideration that, between two rays arriving at any two points from any element of the surface of a distant source of light, there will always be a difference of phase depending on the distance between these points. For the various elements of the surface of the star, this difference will have different values, from zero up to a certain limit determined by the magnitude of its angular diameter. If we polarize all the rays which emanate from the star toward two points not very near together, and if we then superimpose them after turning the plane of polarization of one of the bundles of rays through 90°, we shall obtain, in general, elliptically polarized rays, if the difference of path mentioned above be of the same order of magnitude as the wave-length of light. In the case of an infinitely small angular diameter, the polarization will be rectilinear and will occur in the quadrant through which the aforesaid rotation of the plane of polarization has been effected. Before passing to the theory of my method I shall begin by recalling a case of the composition of two rectilinear vibrations of equal amplitude operating in planes mutually perpendicular. An elliptical vibration results, the axes of which make an angle of 45° with the component vibrations independently of the difference of path which may exist between them. Let us suppose that the two following groups of luminous vibrations proceed together along the axis:

$$\left.\begin{array}{ll}
x_1 = a_1 \sin 2\pi\left(\frac{t}{T} - \frac{z_1}{\lambda}\right); & y_1 = a_1 \sin 2\pi\left(\frac{t}{T} - \frac{z_1+\delta_1}{\lambda}\right); \\
\cdots\cdots & \cdots\cdots \\
x_i = a_i \sin 2\pi\left(\frac{t}{T} - \frac{z_i}{\lambda}\right); & y_i = a_i \sin 2\pi\left(\frac{t}{T} - \frac{z_i+\delta_i}{\lambda}\right); \\
\cdots\cdots & \cdots\cdots \\
x_n = a_n \sin 2\pi\left(\frac{t}{T} - \frac{z_n}{\lambda}\right); & y_n = a_n \sin 2\pi\left(\frac{t}{T} - \frac{z_n+\delta_n}{\lambda}\right).
\end{array}\right\} \quad \text{I}$$

These superposed vibrations will yield a system of elliptically polarized rays with identical axes. The magnitude of the semi-axes is expressed thus:

$$\left.\begin{array}{ll} a'_1 = a_1\sqrt{2}\cos\pi\frac{\delta_1}{\lambda}\,; & b'_1 = a_1\sqrt{2}\sin\pi\frac{\delta_1}{\lambda}\,; \\ \cdots & \cdots \\ a'_i = a_i\sqrt{2}\cos\pi\frac{\delta_i}{\lambda}\,; & b'_i = a_i\sqrt{2}\sin\pi\frac{\delta_i}{\lambda}\,; \\ \cdots & \cdots \\ a'_n = a_n\sqrt{2}\cos\pi\frac{\delta_n}{\lambda}\,; & b'_n = a_n\sqrt{2}\sin\pi\frac{\delta_n}{\lambda}\,. \end{array}\right\} \quad \mathrm{II}$$

This system of elliptical vibrations, having coincident axes, and at the same time being entirely independent, cannot be studied by means of the process ordinarily employed for studying elliptically polarized light, because the difference in path δ_i will be different for the different vibrations. If we receive this system on a doubly refracting prism in such a manner, for example, that the vibrations along the axes a'_i arrive in the plane of its principal section and, as a consequence, the vibrations along the other axes b' arrive in the plane perpendicular to this section, the prism will separate the two groups of vibrations; two bundles of rays will come out rectilinearly polarized and at right angles. The intensity I_a of one bundle will equal the sum of the intensities Ia'_i of the separate rectilinear vibrations having the amplitudes a'_i; further, the intensity of the other I_b will equal the sum of the intensities $I_{b'_i}$, supposing that the intensity of each separate vibration is proportional to the square of its amplitude.

After these preliminary remarks let us consider the theory of the proposed method. Let us assume that the source of the light studied has the form of a sphere. At a distance L (Fig. 1), exceedingly large in comparison to the other distances which we shall have to discuss, a screen is placed with two openings separated by the distance D. To simplify the reasoning, these openings take the place of two mirrors 1 and 2 of the stellar interferometer described below. From a point M of the surface element of the star with the co-ordinates ϕ and θ, which we take as the center of radiation, the luminous vibrations travel to the screen; between the rays passing through the homologous elements $\sigma_1\ (\xi_1, \eta_1, \zeta_1)$ and $\sigma_2\ (\xi_2, \eta_2, \zeta_2)$ of the surface of these openings, there will be a certain difference of path $\delta = M\sigma_2 - M\sigma_1$, which can be determined in the following manner: Take the center of the star for the origin of the system of rectangular co-ordinates shown in Fig. 1. The

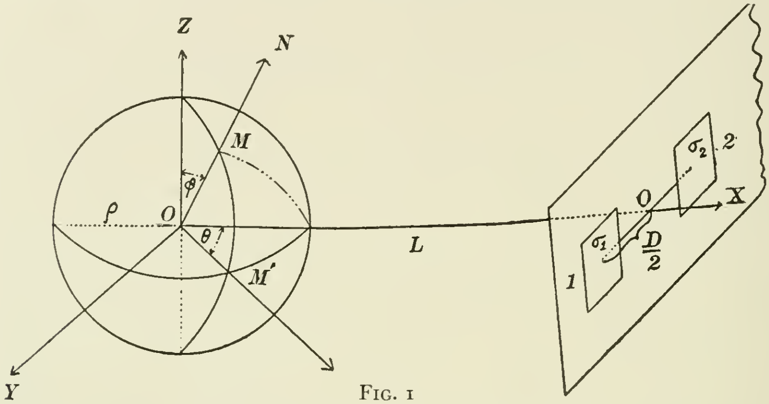

FIG. 1

co-ordinates of the point M and of the homologous elements σ_1, σ_2 of the openings will be

$$\begin{array}{lll} x\ ; & y & ;\ z\ ; \\ \xi_1 = L\ ; & \eta_1 = \frac{D}{2} + \epsilon\ ; & \zeta_1 = \zeta_2\ ; \\ \xi_2 = L\ ; & \eta_2 = -\frac{D}{2} + \epsilon\ ; & \end{array}$$

where ϵ is the distance along the Y axis of the elements σ_1, σ_2 from the centers of the openings of the screen. We now have approximately

$$\delta = \frac{D}{L} y\ .$$

Introducing the spherical co-ordinates ϕ, θ, where $\phi = \angle ZOM$, $\theta = \angle XOM'$, and confining ourselves to the first powers of the ratio $\frac{2\rho}{L} = \omega$, where 2ρ is the true diameter of the star and ω its angular diameter, we get the definite result

$$\delta = D\frac{\omega}{2} \sin\phi \sin\theta\ .$$

For the different elements of the surface of the star this difference of path will have various values between the limits $-D\frac{\omega}{2}$ and $+D\frac{\omega}{2}$. Moreover, the equality of the amplitudes of the luminous vibrations which have reached the elements σ_1, σ_2 from a given point on the surface of the star may be assumed. The difference of path, with the approximation indicated above, does not depend on the co-ordinates of the elements σ_1, σ_2; for all points of the openings it will be the same. Consequently we may consider the luminous vibrations arriving at these openings as plane waves with a certain difference of path

$$\delta_i = D\frac{\omega}{2} \sin \phi_i \sin \theta_i$$

and of equal amplitude.

In passing the interferometer, these plane waves are polarized, as will be indicated farther on. At the same time the plane of polarization of one ray is turned 90° by means of a half-wave plate; thereupon the waves proceed in the same direction, superposed. Thus for each center of radiation on the surface of the star there will be a plane wave elliptically polarized passing out of the interferometer, the axes of these elliptical vibrations will coincide, and will have the values indicated above (see equation II). This group of plane waves elliptically polarized is entirely analogous to the system (I and II) which we have already examined. It falls on a doubly refracting prism which separates the mutually perpendicular vibrations with the amplitudes a'_i and b'_i. In this manner two systems of plane waves rectilinearly polarized will emerge from the prism, inclined toward each other at a conveniently small angle. The objective of the telescope, upon which the waves fall, will reproduce at its focus two images of the star in question, which will be rectilinearly polarized at right angles. It would be more convenient to place the prism inside the telescope between the objective and its focus, as Rochon did in his telescope.

Now let us find the intensity of each image. Since all the elements on the surface of the star may be considered as independent centers of radiation, the intensity of each image will equal the sum of intensities of the images of all the separate surface elements. If there are on the surface element n centers of independent radiation—n being in general a very large number—the quantities of luminous energy furnished by the element to the two images, and, consequently, the intensities themselves, will be

$$dI_b = 2\ \sin^2 [\beta \sin \phi \sin \theta] k \sum^n a_i^2 \,,$$

$$dI_a = 2\ \cos^2 [\beta \sin \phi \sin \theta] k \sum^n a_i^2 \,;$$

where, for brevity, $\beta = \pi \frac{D}{\lambda} \frac{\omega}{2}$, and k is a certain proportionality factor.

If we designate the radiating power of the surface of the star by i, it is evident that

$$k \sum^n a_i^2 = k' \frac{i \cos (NM\sigma_1)}{L^2} \rho^2 \sin \phi d\phi d\theta$$

where k' is another proportionality factor. The equation

$$\cos (NM\sigma_1) = \sin \phi \cos \theta$$

will be approximately correct. We shall then have

$$dI_b = 2\ C \sin^2 [\beta \sin \phi \sin \theta] \cos \theta \sin^2 \phi d\phi d\theta$$

$$dI_a = 2\ C \cos^2 [\beta \sin \phi \sin \theta] \cos \theta \sin^2 \phi d\phi d\theta$$

where $C = \frac{ik'\omega^2}{4}$.

Integrating these two expressions we find after some evident transformations

$$I_b = C \left[\pi - \frac{1}{\beta} \int_0^\pi \sin [2\beta \sin \phi] \sin \phi d\phi \right]$$

$$I_a = C \left[\pi + \frac{1}{\beta} \int_0^\pi \sin [2\beta \sin \phi] \sin \phi d\phi \right].$$

The second integral can be represented in the form of Airy's series:

$$\frac{1}{\beta} \int_0^\pi \sin [2\beta \sin \phi] \sin \phi d\phi = \pi S(\beta)$$

$$S(\beta) = 1 - \tfrac{1}{2}\beta^2 + \tfrac{1}{3}\left(\frac{\beta^2}{2!}\right)^2 - \tfrac{1}{4}\left(\frac{\beta^3}{3!}\right)^2 + \tfrac{1}{5}\left(\frac{\beta^4}{4!}\right)^2 - \ldots.$$

The ratio for the intensities of the two images will be

$$i=\frac{I_b}{I_a}=\frac{1-S(\beta)}{1+S(\beta)}\,.$$

With stars, the value of $\beta=\pi\frac{D}{\lambda}\ \frac{\omega}{2}$ is usually very small. For example, if $D\sim 10^2$ cm, $\lambda=5\times 10^{-5}$ cm, and $\omega\sim 10^{-7}$ (0.″02), we have $\beta\sim 0.3$; consequently one may limit the formula to two members of the series $S(\beta)$. Thus we obtain as an approximation[1]

$$i=\frac{\beta^2}{4}\dots\dots\dots\dots\dots\dots\dots\dots\dots(x)$$

The last expression permits an evaluation in advance of the ratio which will exist between the intensities of the two images of the star, its angular diameter having been given at the outset. We have now seen that if $\omega\sim 10^{-7}$, β will be of the order of 0.3; then

$$i\sim\tfrac{1}{40}\,,$$

that is to say, if an image of the star, for example, appears in the interferometer telescope as a star of the second magnitude, the second image of the same star will be approximately a star of the sixth magnitude. In case the angular diameter is

$$\omega\sim 25\cdot 10^{-9}\ (0.005)$$

$$i\sim\tfrac{1}{640}\,,$$

and the difference of intensity will correspond to a difference of about 7 stellar magnitudes; therefore, if one image in the telescope possesses the intensity of a star of magnitude 0.0, the other will represent approximately a star of magnitude 7.0. This preliminary calculation shows the importance of the intensity of the star under consideration: the more brilliant the star is, the smaller will be the angular diameter that can be detected. Solving the expression (x) for ω, we obtain

$$\omega=\frac{4}{\pi}\ \frac{\lambda}{D}\sqrt{i}\dots\dots\dots\dots\dots\dots\dots\dots(1)$$

---

[1] We shall obtain the same expression if we observe a disk instead of a sphere, because according to Lambert's law the radiation of a sphere is equivalent to the radiation of a disk of the same diameter.

This equation provides the means of calculating the angular diameter of the observed star, if we know D—the distance between the centers of the screen openings—and if, with the aid of an astrophotometer of any kind, we determine i—the ratio of the intensities of the polarized stellar images obtained by the interferometer. Thus we see that the proposed method definitely reduces the determination of angular diameters of stars to a problem of stellar photometry. Its successful application will depend on the lowest feasible value of the ratio i.

But the intensity of the stellar image cannot fall below a certain limit which is determined by the sensitiveness of the eye; we must therefore reckon not only with the ratio of the intensities, i, but also with the minimum intensity, I_o, perceptible to the eye. Suppose it equal to the intensity of a star of magnitude k. Since the intensity I_b of one of the images of the star cannot be less than this limit we put

$$I_b = I_0 \, .$$

We have already seen that with very small angular diameters the two images will show a great difference in intensity; it may therefore be assumed without gross error that the more intense image is formed by the whole luminous energy falling upon the interferometer from the star. Let us assume then that the intensity of the second image will equal the intensity which the star itself had when we examined it with the interferometer telescope. Normally magnified, the intensity of the star increases in the ratio $\frac{S}{\sigma} : 1$, where S is the effective surface of the telescope objective and σ is the area of the pupil. Further, let us assume that of the luminous energy I, when it fell upon the openings of the interferometer, only the part γI reached the eye, γ being a certain fraction usually not very large. It follows that if the star under observation was of the magnitude p to the naked eye, the more intense of the two images, when we observe the star with the interferometer telescope, will appear to be a star of magnitude

$$\left(p - 2.5 \log \frac{S}{\sigma}\gamma\right) .$$

The ratio of the intensities i of the images cannot be smaller in the limiting case than that which exists between I_0 and I_p, the intensities of stars of magnitudes k and $(p-2.5 \log \frac{S}{\sigma}\gamma)$. Thus we can give as a minimum value for i

$$i=\frac{I_0}{I_p}=2.5^{-[k-p+2.5 \log \frac{S}{\sigma}\gamma]},$$

and therefore the corresponding minimum value for ω will be

$$\omega=\frac{4}{\pi}\ \frac{\lambda}{D}\ 2.5^{-\left[\frac{k-p}{2}+1.25 \log \frac{S}{\sigma}\gamma\right]}.$$

The magnitude of the member $1.25 \log \frac{S}{\sigma}\gamma$ depends on the values of σ, S, γ. The last two are determined by the construction and dimensions of the interferometer. This instrument I shall now describe.

The principal part of the apparatus, which determines the optical power by its dimensions, is a rhombohedron of calc spar with polished plane-parallel faces. If two bundles of plane-parallel rays, one of which is polarized in the plane of the principal section and the others in the plane at right angles thereto, fall perpendicularly on the natural face of the spar, the first bundle will traverse it as ordinary rays without refraction, and the second will suffer a parallel displacement as extraordinary rays and will coincide exactly with the first, provided that the size of the beam and thickness of the crystal be properly adjusted to each other. A bundle of elliptically polarized rays can result from the coincidence only when the polarized rays are coherent when they reach the spar: that is to say, in general, capable of interference. It is easy to understand that with natural light one can obtain coherent superimposed ordinary and extraordinary rays in the spar. With such an end in view, it will be more convenient to employ a selenite biplate corresponding to a half-wave. This biplate consists of two plates set side by side (Fig. 2); in one of them the optical axis lies parallel to the line of separation *ab* and in the other it forms an angle of 45°. In the bundle of rays of natural light that have passed through the second half of the biplate, the coherent amplitudes parallel with the line of separation *ab* are turned through an angle of 90°. This biplate is placed before the spar in such a fashion that the line of separation *ab* is perpendicular to the principal section of the rhombohedron.

After all that has been said, the action of the stellar interferometer which I propose will be clear. Two contiguous bundles of parallel rays of natural light emanating from the star under observation will traverse first (Fig. 3) two plates of glass *V*, *V′* to compensate for the difference in path that will result later between the ordinary and extraordinary rays advancing through the spar with unequal velocities. (The mirrors are supposedly removed from the interferometer.) Having passed through the selenite biplate described above, the rays fall upon the spar, which superposes the ordinary rays that come from one bundle and the extraordinary rays that come from another. The diameter *D* of the bundles will represent the distance between the openings of the screen, which we employed before. If we wish to obtain a greater difference of path, we can direct the rays upon the spar by means of mirrors (I→III; II→IV, Fig. 3), or prisms. In this case D will represent the distance between the centers of the mirrors or the prisms. A slit wide enough to permit the passage of only the elliptically polarized rays is placed before the objective of a medium-sized telescope upon which the rays next fall. They traverse the objective and strike the Wollaston prism introduced before the focus. This prism will yield two images of the observed star; revolving the prism in its plane, we can obtain the position which insures the maximum difference of image intensities.[1] This will serve to show that the more intense image is formed by, one may say, the major axes of

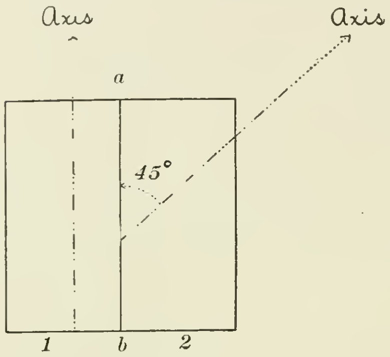

Fig. 2

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[1] In this position the principal section of the prism will make an angle of 45° with the principal section of the rhombohedron.

the elliptical vibrations, and the less intense image by the minor. One may find the ratio i of the intensities of these images by means of a Pritchard astrophotometer.

Passing to the evaluation of the sensitiveness of the proposed method, we remark that the transverse section $S=hD$ of the bundles of rays superposed by the spar determines the effective surface of the objective. The size of S will depend on the dimensions of the given rhombohedron. If, to save material, we employ two crystals of the dimensions of about 45×45×55 mm in place of one large one, and place them in contact, S will have the value $30\cdot 10 = 300 \text{ mm}^2$ ($D=10$ mm; $h=30$ mm). The area σ of the pupil of a normal eye may be supposed equal to $4\pi \text{ mm}^2$. The coefficient γ, indicating what fraction of the luminous energy falling on the interferometer reaches the eye, is determined in the following manner. Assume that upon the average, on each refracting surface, 5 per cent of the energy is lost by reflection during the normal passage of the rays [for glass, for example, $\left(\frac{\mu-1}{\mu+1}\right)^2=0.04$; $\mu=1.5$]; and

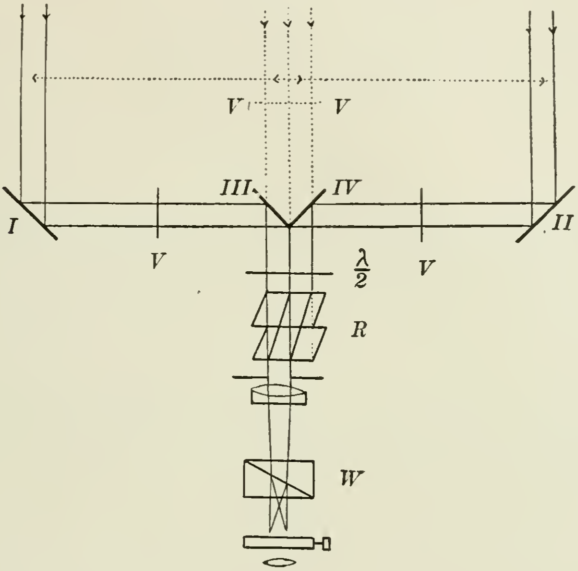

FIG. 3

that as a consequence only 95 per cent passes. After the rays have consecutively traversed the plate of glass (2 surf.); the selenite plate (2 surf.); two rhombohedrons (4 surf.); the objective (3 surf.); the double refracting prism (3 surf.); the astrophotometer (3 surf.); and finally the eyepiece (2 surf.)—$(0.95)^{19}$ of the whole quantity of luminous energy received would reach the eye. Assume, further, that the absorption of light in this apparatus amounts to about 5 per cent, we shall find

$$\gamma=(0.95)^{20}.$$

We thus have for the proposed interferometer

$$S=300 \text{ mm}^2; \quad \sigma=4\pi \text{ mm}^2$$

$$2.5 \log \frac{S}{\sigma}\gamma=2.33 ;$$

that is to say, if the star under observation was of a magnitude p to the naked eye, when we observe it in the interferometer telescope the more intense of its two images will appear to be a star of the magnitude

$$p-2.33 .$$

With these data let us determine the minimum value of the angular diameter to be discovered for stars of the first magnitude ($p=1$). We have

$$\omega=0\overset{''}{.}46$$

if $k=6$; $D=1$ cm; $\lambda=5\times 10^{-5}$ cm.
For the interferometer with its mirrors (allowing for the loss of light by reflection), we shall have

$$\omega=0\overset{''}{.}0048$$

if $D=100$ cm.

For stars of the ninth magnitude ($p=9$) we have

$$\omega=0\overset{''}{.}19.$$

In the case of the brighter stars, as, for example, *Acturus*, *Vega*, *Canopus*, *Capella*, that is to say, on the average of 0.0 magnitude, this limit will fall still farther to

$$\omega=0\overset{''}{.}0031.$$

In the case of *Sirius*

$$\omega = 0''.0017.$$

So far we have supposed that the interferometer itself causes no difference of path between the two rays. If, however, by reason of the lack of symmetry between the two branches, a certain difference of path l is established in the interferometer itself, the ratio i of the intensities will be

$$i = \frac{1-(1-\frac{1}{2}\beta^2)\cos\alpha + \frac{4}{3\pi}\beta\sin\alpha}{1+(1-\frac{1}{2}\beta^2)\cos\alpha - \frac{4}{3\pi}\beta\sin\alpha} \quad \dots\dots\dots\dots\dots (2)$$

where

$$\alpha = 2\pi\frac{l}{\lambda}\,; \quad \beta = \pi\frac{D}{\lambda}\;\frac{\omega}{2}.$$

If $\alpha = k\pi$, $k = 0, 1, 2, 3 \dots$ the ratio of the intensities becomes alternately reciprocal. This circumstance allows us to determine the angular diameters more precisely by several consecutive measurements, obtaining a variable difference of phase by inclining the plates in the interferometer; this difference of phase will alternatively invert the ratio of the intensities i. The careful adjustment of the stellar interferometer will require close attention. but the expected results will largely compensate the trouble expended, for they will considerably enlarge the domain of astrophysics which has hitherto depended almost exclusively upon spectrum analysis. By reason of the extraordinary sensitiveness of this apparatus, one may expect atmospheric conditions to affect the stability of the elliptical polarization strongly. For this reason the second image will sometimes become visible even with stars of very small angular diameter. The apparatus can yield positive results only at mountain observatories.

SUPPLEMENT

Perhaps the simplest form of a stellar interferometer would be the following (Fig. 4). A bundle (1) of parallel rays proceeding from a given star traverses the polarizing prism P, similar to the Glan-Thompson prism, which allows only the extraordinary rays E_1 to pass. The second bundle of parallel rays (2), reflected by the mirror R, traverses the half-wave plate whose principal section is at 45° with the plane of incidence. The rays next enter the aforesaid prism in which the ordinary rays O_2, reflected by the dividing surface ab, coincide with the extraordinary rays E_1. Thanks to the half-wave plate, these coincident rays will be coherent. After their coincidence the rays follow the route already described in our first type of interferometer.

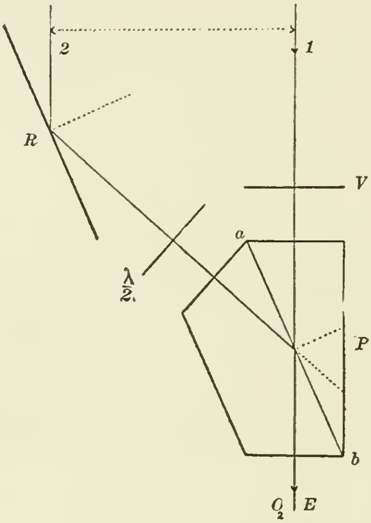

FIG. 4

V is a glass plate compensating the difference of path between O_2 and E_1, introduced by the interferometer itself. The optical power of our second interferometer will considerably surpass that of the first because the effective surface of the modified Glan-Thompson prism noticeably surpasses that of the rhombohedron of calc spar. In this case, one uses the same formulas (1) or (2).

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YERKES ACTINOMETRY

ZONE $+73^\circ$ TO $+90^\circ$

By J. A. PARKHURST

The problem of the co-ordination of visual and photographic magnitudes of stars has engaged the attention of the writer for some years. An opportunity for work under favorable conditions was presented by the acquisition by the Yerkes Observatory of a Zeiss doublet lens of 14.5 cm aperture and 81.4 cm focal length in 1905; and of an objective prism of the same aperture and 15° angle, in 1906; both of ultra-violet glass. This instrument proved to be extremely well adapted for photometric purposes, so that after some preliminary experiments it was decided to attempt the measurement of the stars of the *Potsdam Photometric Durchmusterung* in the zone from 73° north declination to the pole, and to include: (1) the photographic magnitudes; (2) the "visual" magnitudes (with color-sensitive plates and "visual-luminosity" filter); (3) the resulting "color-indices"; (4) the spectral class on the Harvard system. It was thought better to confine the investigation to the stars down to magnitude 7.5 in a limited zone, rather than to attempt an extended program before the methods of work had been thoroughly tested.

The dependence of photographic photometry on the colors of the stars, and the intimate relation between colors and spectra, seemed to make the determination of the above list of data of considerable value from the standpoint of astrophysics. A catalogue, including these data, of standard stars conveniently located for observers in the northern hemisphere would also have value as a foundation for further work. This relation between colors and spectra also furnished a useful check on both the spectra and the magnitudes.

ZERO POINT AND SCALE

The choice of zero point and scale is fundamental for a catalogue of stellar magnitudes. Fortunately, the selection of the zero point could be postponed till the completion of the reductions. In the meantime the proposal of the Committee on Magnitudes of the Astrographic Chart[1] gave what seemed to have a chance for acceptance as an "International System," and which was gladly adopted, namely—

> For the stars of spectral type Ao on the Harvard classification, and of magnitude 5.5 to 6.5, the photographic and visual magnitudes shall be assumed equal, and the visual magnitudes shall be reckoned on the Harvard scale.

The determination of an absolute scale for the photographic magnitudes was made possible by the excellent quality of the extra-focal images given by the Zeiss lens. The method was described in detail in a paper by Jordan and the writer,[2] but since the publication of that paper two improvements have been made in the sensitometer, one eliminating internal reflection in the cells, the other providing means of rotating the sensitometer during the exposure. An indirect, but satisfactory check on the scale obtained with the sensitometer, was given by Pleiades plates taken with the 24-inch reflecting telescope and a wire gauze screen, using the "Halbgitter" method suggested by Schwarzschild.[3] The value of the constant b in the equation

$$\text{Magnitude}=a-b\,\mathrm{l}\;\;\bar{D}$$

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[1] *Astronomische Nachrichten*, **186**, 40, 1910.

[2] "An Absolute Scale of Photographic Magnitudes of Stars," *Astrophysical Journal*, **26**, 244, 1907.

[3] *Astronomische Nachrichten*, **183**, 297, 1910.

is determined by the scale of the plate. The values found by the two methods were:

	Gingrich
From sensitometer magnitudes....	0.90±0.01
By "Halbgitter" method.........	0.91±0.01

The tabular values used in the reductions are given in the next section with the description of the "Mikrophotometer."

INSTRUMENTS

The camera was made in the shops of the Yerkes Observatory by providing the Zeiss doublet with a brass tube arranged for focusing with a vernier reading to 0.1 mm; a sliding carrier for the plate-holder so that a series of exposures could be placed on the plate; a guiding telescope of 5.3 cm aperture and 90 cm focal length, and an equatorial mounting originally made by Brashear for a 16 cm reflector. The objective prism fits over the cell of the lens, and can be put in place or removed in a few minutes' time. Fig. 1 shows the instrument without the prism.

The excellent quality of the extra-focal images produced by this lens is shown by Fig. 2, a negative of the Pleiades group taken 10 mm outside the focus, giving a practically uniform disk, capable of measurement with great precision. Images taken as near as 6 mm from the focus begin to show slight inequalities, but if the measuring machine is set a little out of focus they can be measured with sufficient accuracy. In practice the plates were taken 6 mm inside the focus, where the field was a little more nearly flat than on the outside. Within an angular distance of 2°5 from the axis of the lens, the illumination of the image is quite uniform, but farther out, the side away from the axis becomes denser, and the measurements, being less precise, were given a smaller weight. No images farther than 3°5 from the axis were measured.

The "Mikrophotometer," with which the extra-focal images were measured, is shown in Fig. 3. A detailed description by the inventor, Hartmann, was published in this *Journal*, **10**, 321, 1899. A single improvement, suggested by Hartmann, was made by mounting the lamp (a 16–20 candle-power incandescent) on a slide, so that a change in the position of the lamp, in a line parallel to the horizontal microscope, would vary the relative amounts of light sent into the two light-tubes; thus making it possible to bring any given opacity on the plate to a certain reading on the wedge-scale. With the opal glass, furnished with the instrument, covering the junction of the two light-tubes, this adjustment of the scale-readings was not possible, since the opal acted as a source of light; but on replacing the opal with a piece of ground glass the adjustment was readily made.

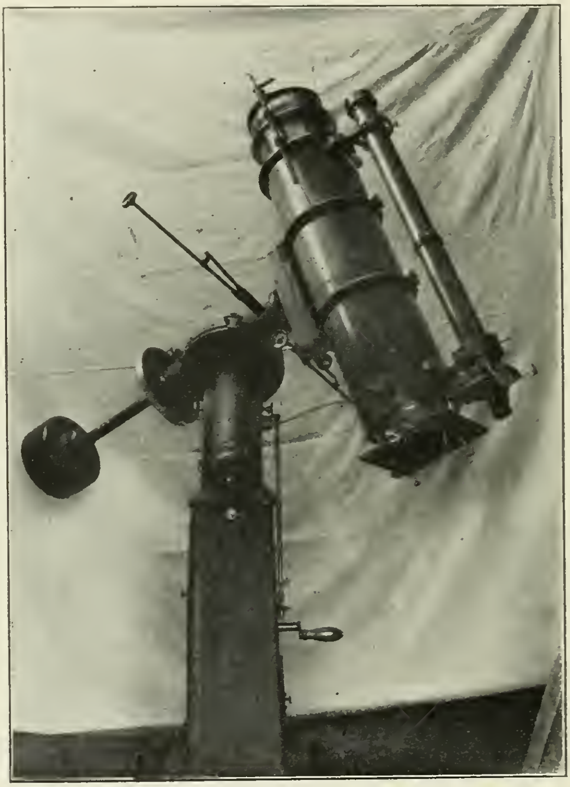

FIG. 1.—Zeiss camera, U-V glass

The wedge used in this photometer was one of a photographic set made by King and kindly furnished by Professor E. C. Pickering. Investigations of the visual absorption of these wedges have been repeatedly made,[1] and indicate a very nearly uniform absorption expressed in magnitudes. For use with photographic plates it was desired to calibrate the wedge directly in terms of stellar magnitude. The sensitometer used contained a nest of 20 light-tight cells, in four rows of five each. The end of the cells exposed to the light (north sky) was covered by a metal plate with a round hole opposite the middle of each cell. Thus the amount of light admitted to each cell was proportional to the area of the hole.

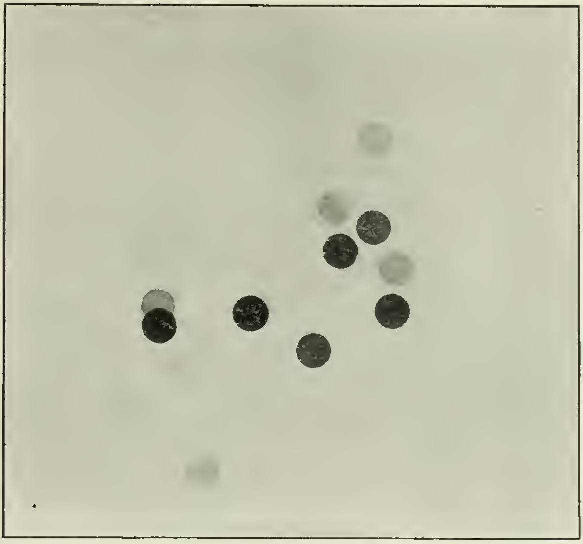

FIG. 2.—Extra-focal images of the *Pleiades* taken with Zeiss camera

The numerical data for the metal plate used (called Plate *D*) are given in Table I.

In this table the diameters of the holes are expressed in millimeters, and the Δ mag. represents the log. diam.[2] (the log. of the relative light) divided by 0.4 to reduce to relative magnitudes. The end of the nest of cells away from the light was covered by a metal plate provided with a hole 1 mm in diameter opposite the center of each cell. The sensitive plate was placed with the film in contact with this metal plate, and on exposure to daylight would be impressed by a series of small circles (smaller than the extra-focal star-images, and further distinguished by their regular arrangement) whose opacities would be the same as if caused by a series of stars having relative magnitudes represented by the data of Table I. This series was impressed on the zone plate after exposure to the sky and before development, thus giving an absolute scale on the plate.

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[1] King, *Harvard Annals*, 41, 237; Aitkin and Maddrill, *Astrophysical Journal*, 22, 147, 1905; Parkhurst, *Researches in Stellar Photometry*, pp. 8–10.

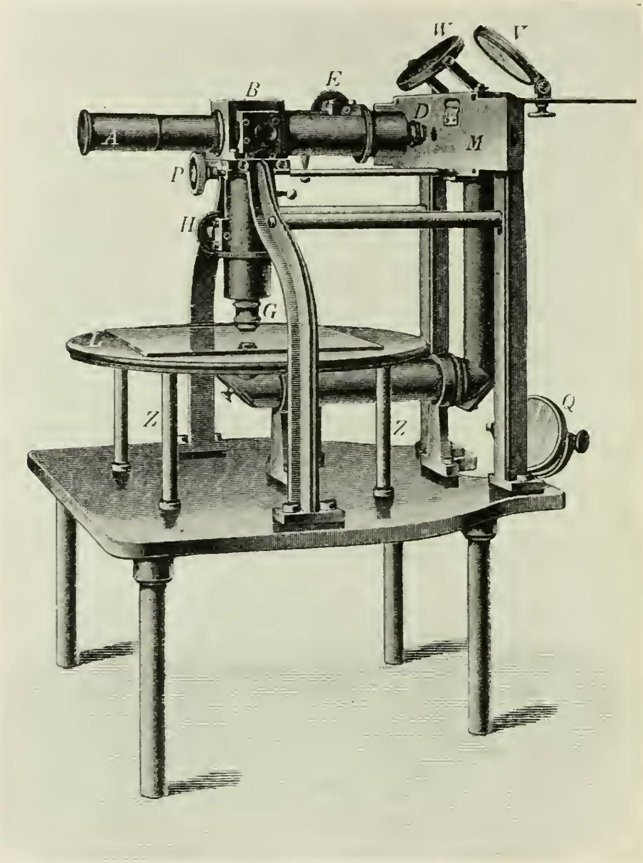

FIG. 3.—Hartmann "Mikrophotometer"

The machine for measuring diameters of the focal images used, in part of the work, is shown in Fig. 4. The slide carrying the plate has rack and pinion motion in two directions. The microscope has a Zeiss position micrometer with two screws at right angles, allowing measures of diameter in both co-ordinates with the same setting of the plate. The magnifying power used was 23.

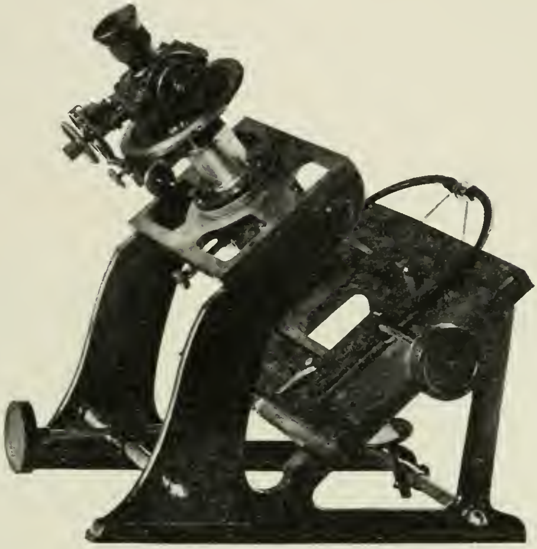

FIG. 4.—Micrometer for the measurement of disk diameters

TABLE I

Hole	Log. Diam.^{2}	Δ Mag.	Hole	Log. Diam.^{2}	Δ Mag.
1..........	0.0198	0.050	11..........	0.8420	2.105
2..........	0.0828	0.207	12..........	0.9024	2.256
3..........	0.1592	0.398	13..........	0.9760	2.440
4..........	0.2360	0.590	14..........	1.1130	2.782
5..........	0.3630	0.908	15..........	1.1508	2.877
6..........	0.4200	1.050	16..........	1.2432	3.108
7..........	0.5134	1.284	17..........	1.3344	3.336
8..........	0.5908	1.477	18..........	1.3972	3.493
9..........	0.6474	1.618	19..........	1.4664	3.666
10..........	0.7554	1.888	20..........	1.5950	3.988

PATROL CAMERA

In April 1911, the Observatory received from Brashear a doublet lens of 5 cm aperture and 40 cm focal length, with curves calculated by Hastings especially to give smooth extra-focal images. This was pointed toward the pole in a stationary box, and plates were exposed simultaneously with the zone plates, giving extra-focal trails 1 mm wide. Any change in the transparency of the air during the exposure would be registered on the trails and easily detected if exceeding 0.02 or 0.03 magnitudes. After the use of this camera (which perhaps should be called a "sentinel" rather than a "patrol") was begun, one night which seemed uniform to the eye was proved to be changing by the trails, and the zone plates were rejected. Fig. 5 is a specimen patrol plate.

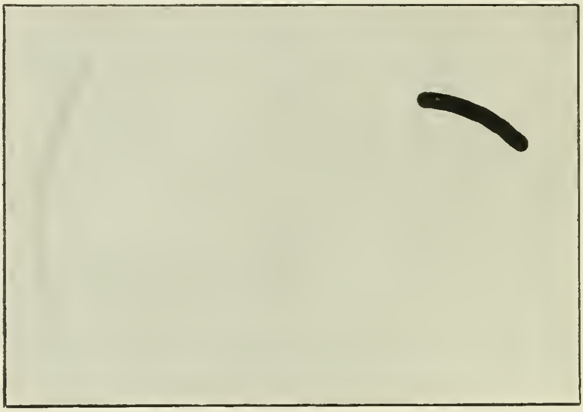

FIG. 5.—Patrol plate showing two polar trails

PROGRAM OF EXPOSURES

The useful field of the camera being about 6° in diameter, the program was arranged so that each star should appear on at least two plates. The setting in declination and the interval in right ascension were as follows:

Dec.	Interval	Dec.	Interval
75°	40^m	85°	4^h
79	1^h	87	4
83	2	90	8

As the space in this *Journal* does not permit the publication of the separate results for magnitude, there is no advantage in giving the journal of plates, further than to say that they were taken between 1908 May 22 and 1911 July 14. The number of plates used in the reductions is as follows:

Extra-focal for photographic magnitudes	122
Focal, for "visual" magnitudes	113
Objective-prism, for spectra	90
Total	325

PLATES, EXPOSURE, AND DEVELOPMENT

The plates used were 4×5 inches (10.1×12.6 cm) in size. For the photographic magnitudes, Seed "*Gilt Edge*" No. 27 (six No. 30 in 1911), and for the "visual" magnitudes, Cramer *Trichromatic* plates were used. An exposure of 30 minutes in the camera was usually sufficient to give images of the faintest stars needed. The exposures of the objective-prism plates ranged between 20 and 30 minutes. The plates were developed with hydroquinone for ten minutes (see exceptions noted below) in a metal tray floating in a tank of water which was kept at a temperature of +20° Centigrade. The development was carried on in the dark.

DETERMINATION OF PHOTOGRAPHIC MAGNITUDES

The photographic magnitudes were obtained from the images taken 6 mm inside the focus of the camera, on Seed 27 plates. The opacity of the images was measured with the Hartmann "Mikrophotometer," the measures being reduced to magnitudes by the aid of the "absolute scale" obtained from the sensitometer images impressed on the plate. Great care in securing uniformity in handling the plate, left two sources of error which were under only partial control: (1) sky-fog, which darkened the film, and affected the reduction curve for the fainter stars; and (2) local differences in the sensitiveness of the film. Fig. 6 shows specimen reduction curves for measures of plates in the "Mikrophotometer." The readings on the millimeter scale of the instrument are ordinates and differences in magnitude (from sensitometer images) abscissas.

The scale-readings increase from faint to dense images; the magnitudes increase in the usual manner from bright to faint stars. The solid curve corresponds to a clear film on the negative; but any sky-fog acts as a supplementary exposure, and makes an appreciable increase in the opacity of the fainter images, thus raising the lower end of the curve. The dotted lines show this effect for film readings 10 and 12 on the scale. It is evident that each plate must be reduced with a curve corresponding to the reading of its film. A check on the determination of this curve was made by putting a second exposure on the plate, giving pairs of star-images which must differ by the same number of magnitudes. Usually the exposure times were 25 and 5 minutes, respectively. The denser images would fall on the part of the curve unaffected by the sky-fog; and the fainter image of the pair, separated from the other by a constant difference in magnitude, would serve to locate the lower part of the reduction curve.

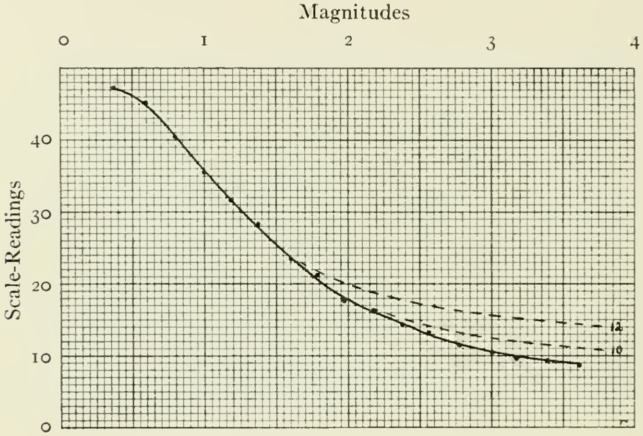

FIG. 6.—Reduction curves for extra-focal plates

As a check on the uniformity of sensitiveness of the plate, photometer readings were taken on the unexposed film near each star-image.

Correction to the center of the plate.—Images of *Polaris*, with constant exposure times, were taken at a series of positions on the plate, and from measures of their opacities a correction curve was deduced to reduce each image to the center of the plate. It was found that the correction, expressed in magnitudes, was the same for star-images of different densities. The amounts of the corrections were—

Distance from Center	Correction	Distance from Center	Correction
$0^\circ.0$	$+0^{\mathrm{M}}.00$	$2^\circ.0$	$+0^{\mathrm{M}}.14$
0.5	+0.01	2.5	+0.22
1.0	+0.03	3.0	+0.32
1.5	+0.08		

Correction for atmospheric absorption.—The values given by Wirz were used for the photographic magnitudes. These values were determined from extra-focal plates reduced with the absolute scale.[1] As in the preceding paragraph, the corrections were applied to reduce to the center of the plate. As the zenith distances were usually less than 50° the amount of this correction was seldom greater than 0.02 magnitudes.

Standard stars.—Before the reduction of the photometric plates the spectral types of the stars were estimated on the Harvard classification from the objective-prism plates. Those stars were selected for standards which were as nearly white as Class A5 F. To the Potsdam visual magnitude was added a correction amounting to 0.04 for each step from A toward F, and the corrected quantity taken as the photographic standard magnitude. This provisional correction was confirmed by the final results of the zone work.

A few fields contained only two or three standard stars, but the number averaged six per plate. The mean of the corrected curve readings for the standard stars, subtracted from the mean of the photographic magnitudes, gives M_0, the quantity to be added to each curve reading to give the magnitude.

Systematic corrections.—The methods outlined permit the construction of a provisional catalogue, and a comparison of the separate stars and plates with the means from this catalogue gives data for a number of systematic corrections.

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[1] *Astronomische Nachrichten*, 154, 349, 1900; *Astrophysical Journal*, 31, 23, 1910.

1) Correction to the reduction to the center. For stars having a distance from the center greater than 2°5, the following improvements to the reduction curve were found:

Dist.	Cor.	Dist.	Cor.
2°6	$-0^{\mathrm{M}}.01$	3°2	$0^{\mathrm{M}}.00$
2.8	−0.02	3.3	−0.04
3.0	−0.02	3.4	−0.07

This correction was applied to all the extra-focal plates.

2) Correction for scale and zero point. After the corrections for reduction to the center had been applied, the residuals for each plate were arranged in order of magnitude. (*a*) The slope of this curve showed the scale-error of the plate, as compared with the mean of all overlapping plates. (*b*) The distance of the curve, from the axis at a point corresponding to the mean magnitude of the standard stars used on the plate, showed the error of those standards as compared with the standards on all the overlapping plates. The application of these corrections to the individual stars therefore corrects for scale-error of the plate, and also refers the stars to the mean of the standards on all the overlapping plates, and in a less degree to the mean of all the standard stars in the entire zone.

Correction for color-error of the objective.—The color-curve of the Zeiss lens is flat in the region of the photographic rays, but in the visual part of the spectrum the curve is quite steep, the focal length for the wave-length of maximum visual sensitiveness being 2 mm longer than for the photographic maximum. As the plates were taken 6 mm inside the focus for the photographic rays, the light of the yellow stars would be even more out of focus, and their images would be larger and therefore less dense. The correction to be applied for this color-effect was determined in two ways: (1) by measuring the diameters of the images and calculating the change in magnitude from the change in area; (2) by making two exposures on the same plate, at equal distances inside and outside the focus, so that the color-effect would have the opposite sign. From the accordant results of these two methods the following table of corrections was found:

Uncorrected Color Index	Correction	Uncorrected Color-Index	Correction
0.25	$-0^{\mathrm{M}}.01$	1.25	$-0^{\mathrm{M}}.07$
0.50	−0.02	1.50	−0.09
0.75	−0.03	1.75	−0.11
1.00	−0.05	2.00	−0.14

This correction is the same in sign but only about one-fifth the amount of that found by Hertzsprung[1] for the Zeiss "UV" triplet at Potsdam.

Weights in the final catalogue.—The final weights were assigned from the following table, based on the progressive loss in uniformity of the images at increasing distances from the center.

Distance from Center	Weight	Distance from Center	Weight
0°0 to 1°5	6	2°5 to 2°7	3
1.6 to 2.0	5	2.8 to 3.0	2
2.1 to 2.4	4	3.1 to 3.4	1

A search was made for systematic differences in the magnitude results from the preceding and following sides of the plate (but with negative results). For the 75° zone the mean residuals were—

	Preceding Half	Following Half
Tel. East	−0.004	+0.003
Tel. West	−0.007	+0.001

If these errors were the result of faulty collimation, the general mean would be found by combining the preceding half Tel. East with the following half Tel. West, and vice versa. Weighted according to the number of plates these combined residuals become–

Preceding half	$-0^{\mathrm{M}}.003$
Following half	0.000

The systematic differences were therefore negligible.

DETERMINATION OF "VISUAL" MAGNITUDES

Experiments made by Jordan and the writer, with the 24-inch reflecting telescope, demonstrated the possibility of obtaining magnitudes on the visual scale with a color-filter and color-sensitive plates.[1] A filter 4×5 inches in size, and identical in composition with that used on the reflector, was made by Mr. R. J. Wallace in February 1907. The spectral intensity curve of this filter with Cramer Trichromatic plates is shown in Fig. 7. A comparison of this curve, with those given by Ives[2] for the sensibility of the eye to lights of high and low intensity, will show that the sensibility curve of the plate falls between the high and low intensity curves for the eye. Fig. 8 shows in a striking manner the capacity of the plate and filter to reproduce visual magnitudes of colored stars. The red star *U Cygni* (color-index 5.6 magnitudes), located by the short lines, is shown faint on the photographic (Seed) plate, but has its true visual magnitude on the filtered plate.

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[1] *Astronomische Nachrichten*, **186**, 180, 1910.

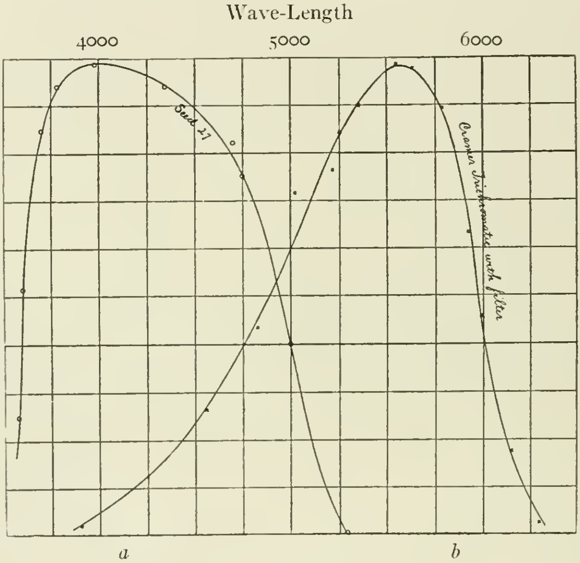

FIG. 7.—Spectral intensity curves

a, Seed 27, without filter

b, Cramer Trichromatic with "visual-luminosity" filter

Exposure and development.—As exposures of four or five hours would be required to take extra-focal plates with the filter, it was decided to take these plates in focus, at the risk of some loss in accuracy of measurement. However, the loss was less than expected, on account of the remarkable sharpness of the star-images, in spite of the fact that the portion of the spectrum passed by the filter lies in the steep part of the color-curve. The diameters of these images could be measured with considerable precision under the microscope; also these diameters were less sensitive to conditions of development than were the opacities of the extra-focal images; so that measures on different plates were nearly as consistent as those obtained by the extra-focal method. Previous to January 1910, the Cramer plates were developed in precisely the same manner as the Seed plates;[1] but unless the Cramer plates

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[1] "The Photographic Determination of Star-Colors and Their Relation to Spectral Type," *Astrophysical Journal*, **27**, 169, 1908.

[2] P. G. Nutting, *Outlines of Applied Optics*, p. 122.

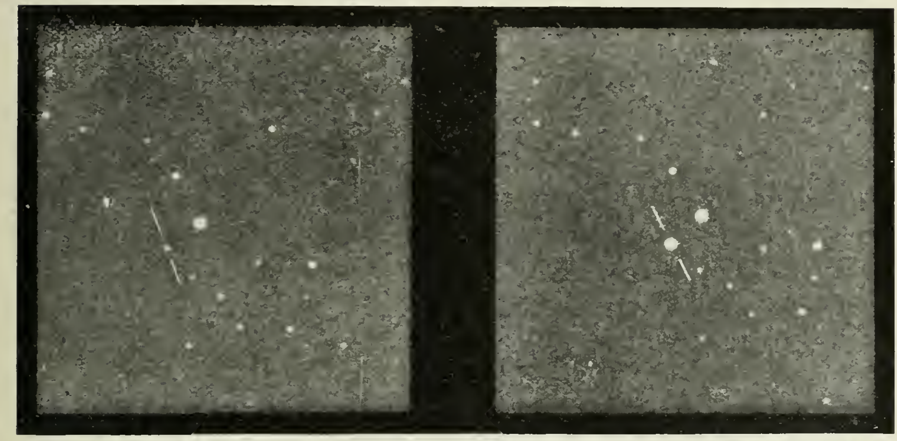
a *b*

FIG. 8.—The red star *U Cygni*

a. On Seed 27 plate, without filter

b. On Cramer Trichromatic plate with "visual-luminosity" filter

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[1] DEVELOPER FOR SEED PLATES

A		B	
Water	64 oz.	Water	64 oz.
Sodium sulphite, dry	2 oz.	Sodium sulphite, dry	2 oz.
Hydroquinone	2.5 oz.	Potassium carbonate	8 oz.
Sulphuric acid, c.p	2 dr.	Potassium bromide	200 gr.

Equal parts A and B

were less than three months old there was a tendency to fog with this treatment, so the remainder were developed with "Cramer Extreme Contrast"[1] developer, for six minutes, which gave identical values of the reduction constants.

Reduction to the absolute scale.—As the method by sensitometer images could not be applied to plates taken in focus, the reduction constants for a mean plate were obtained by reducing Pleiades[2] exposures with values of the magnitudes found by the absolute scale. As already mentioned, this scale was confirmed later by the "Halbgitter" method, and the deviations of the individual plates were eliminated by systematic corrections.

It was found by trial that the reduction formula

$$\text{Magnitude} = a - b\sqrt{D}$$

fitted the camera and plates within the accidental errors of measurement. If the logarithm of the diameter were substituted for the square root, the representation is much worse. In Fig. 9 the same values of magnitude and diameter of the Pleiades stars are used, but in the upper part the square roots of D are platted while in the lower part the logarithms are used. Exposures ranging from 3 to 1000 seconds agreed in showing that the reduction "curve" was a straight line. The value of b, the tangent of

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[1] DEVELOPER FOR CRAMER PLATES

A		B	
Water	64 oz.	Water	64 oz.
Sodium sulphite, dry	2 oz.	Sodium carbonate, dry	2 oz.
Hydroquinone	3 oz.	Sodium sulphite, dry	6 oz.
Sulphuric acid, c.p.	2 dr.	Potassium carbonate	6 oz.
		Potassium bromide	240 gr.

Equal parts A and B

[2] Following are the magnitudes of the Pleiades stars found from extra-focal camera plates. The notation is Bessel's, and the magnitudes are expressed on the "International System."

Star	Magnitude	Star	Magnitude	Star	Magnitude
a	3.04	*g*	5.49	24	6.79
f	3.75	28	5.51	29	6.96
b	3.75	*m*	5.63	40	6.98
c	3.92	34	6.18	19	6.93
d	4.24	*s*	6.77	22	7.10
e	4.29	38	6.77	10	7.26
h	5.06	12	6.73	39	7.36

the inclination of this line, ranged from 0.50 to 0.53 for the different individuals taking part in the measurements.

Sources of error.—The principal sources were two: (1) error in focusing the camera, causing errors in the reduction to the center of the plate; (2) unsteadiness in the air, making a change in the value of *b* in the formula.

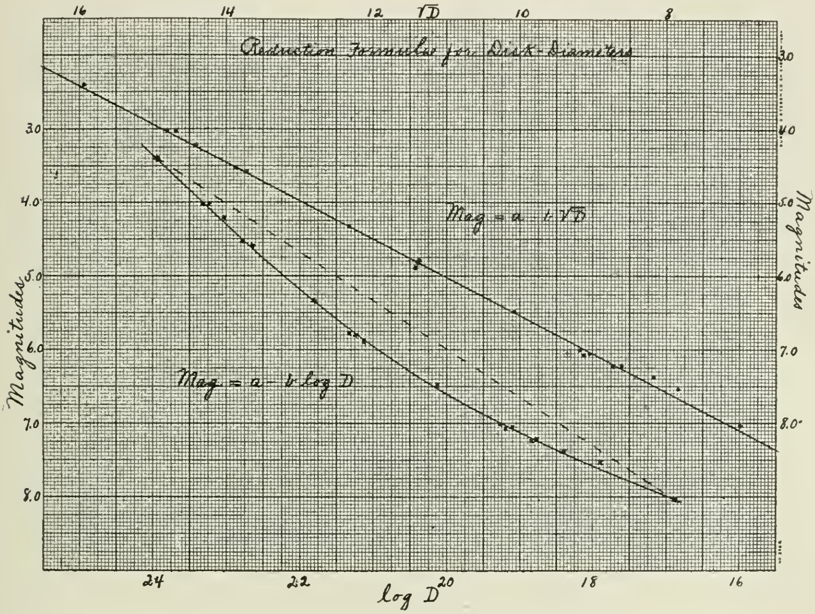

FIG. 9.—Reduction formulae for focal plates

Correction to the center of the plate.—This was determined in the same manner as the extra-focal corrections described on p. 179. The values were as follows:

Distance from Center	Correction	Distance from Center	Correction
0°0	$0^{\mathrm{M}}.00$	2°0	$+0^{\mathrm{M}}.10$
0.5	+0.02	2.5	+0.07
1.0	+0.07	3.0	0.00
1.5	+0.10		

No images at a distance greater than 3°0 were used. The above table applies directly to images for which the value of $b\sqrt{D}=5.00$.

For any image the correction is represented within the accidental errors by the correction just given, multiplied by the factor

$$1+0.82(b\sqrt{D}-5.00).$$

Correction for atmospheric absorption.—The values used were those given in Vol. 3 of the Potsdam publications, p. 285. They were applied to reduce to the center of the plate.

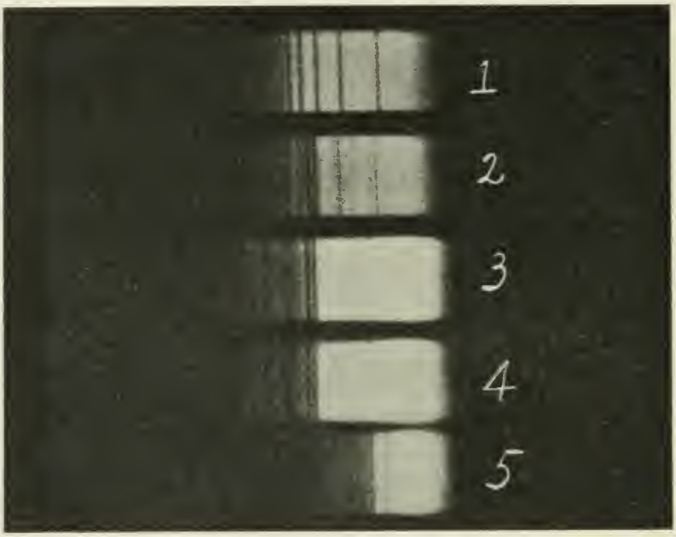

FIG. 10.—Classes of objective-prism spectra

1. *β Persei*, class B8
2. *α Persei*, class F5
3. *α Aurigæ*, class G0
4. *Saturn*, class G0
5. *α Tauri*, class K5

Standard stars.—These were chosen as for the photographic magnitudes, only in this case the Potsdam magnitudes were used as given in Vol. 17. The corrected value for $b\sqrt{D}$ for each standard star gave a value of the constant a, and the mean for all the standards gave the value of a for the plate.

Systematic corrections.—These were obtained in the same manner as described for the extra-focal plates on p. 180. The corrections to the reductions to the center for distances greater than 2°5 were as follows:

Distance from Center	Correction	Distance from Center	Correction
2°6	$-0^{\mathrm{M}}.02$	2°9	$-0^{\mathrm{M}}.11$
2.7	−0.07	3.0	−0.13
2.8	−0.10		

Weights.—Stars not farther from the center of the plate than 2°5 were given weight 2, those farther out were given weight 1.

SPECTRAL CLASSIFICATION

The objective prism of 15° refracting angle, fitting over the cell of the doublet, gives spectra with a scale of 3.0 mm from $H\beta$ to $H\theta$. The field is flat in the region of the photographic rays, but the focal length increases rapidly for the visual region, so that only the region of shorter wave-length than λ 5000 could be used in the classification. Between this point and λ 3500 the focus is sharp.

The spectra were broadened by the diurnal motion. The clock was left running, and by using the lost motion in the right ascension screw, the star was allowed to trail to a width of about 0.5 mm a number of times sufficient to give a faint image of a star of magnitude 8.5 photographic. If there were stars brighter than 7.0 in the field, a second exposure with a less number of trails was also given.

In the effort to approximate the classification as closely as possible to the Harvard system, spectra of the following typical stars were taken:

Class	Star	Class	Star
B0	ε *Orionis*		π *Tauri*
A0	*Vega*	K0	*Arcturus*
A5	*Altair*		δ_1 *Tauri*
F0	δ *Aquilae*		ε *Tauri*
F5	*Procyon*		1 *Hev. Draconis*
G0	*Capella*		τ *Draconis*
	η *Pegasi*	K5	*Aldebaran*
	η *Aquilae*		β *Ursae Minoris*
	10 *Camelopardalis*	M	α *Orionis*
G5	γ *Delphini*	N	19 *Piscium*
	ε *Ursae Minoris*		249a *Schjellerup*

In January 1911 Mrs. W. P. Fleming very kindly classified for me the spectra of 138 stars on three of the plates used in this work, Nos. OP 374, 377, and 410. Her favorable opinion of the quality of the plates,[1] in spite of the small scale, was given in her *Report to the Committee on the Classification of Stellar Spectra of the Solar Union.*

By constant reference to these standard spectra, taken under similar conditions, the classification was made as closely as possible on the Harvard system. There remains, however, one source of uncertainty as to what is meant by the "Harvard System." In *Harvard Circular No. 172*, p. 1, spectra like that of the sun are called Class G5, while in the previous volumes, for example **28**, pp. 35 and 158, they are called Class G. In this work the earlier classification is adopted.

The systematic difference between Mrs. Fleming's classification and my own is given in the following table, expressed in units which are tenths of the difference between Classes A and F, etc.

Mean Class	No. Stars	Diff. F−P
A5	31	+0.8
F5	12	+1.4
G5	14	+3.1
K5	9	−1.0

On the 90 objective-prism plates used, 1674 spectra of 654 stars were classified. The spectra of 134 stars were marked doubtful, leaving 520 stars available for a study of the relation between spectra and color-index.

REDUCTION TO "INTERNATIONAL SYSTEM"

The systematic difference between the magnitudes given in *Potsdam*, **17** and *Harvard*, **45** was found for the 124 stars in the present work, having spectra between Classes B8 and A2 inclusive. The result was:

124 stars . . . . Potsdam-Harvard = $+0^{M}.265$.

At Professor Schwarzschild's suggestion the same differences were taken out for the stars within the same spectral limits included in his "*Katalog der Polsterne*,"[2] with the result:

40 stars . . . . Potsdam-Harvard = $+0^{M}.268$.

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[1] *Astrophysical Journal*, **33**, 273, 1911.

[2] *Aktinometrie*, B, p. 42.

Schwarzschild's value[1] for this difference was $+0^{\mathrm{M}}.29$, so it was thought best to adopt the mean between his value and mine. The catalogue values are therefore:

Potsdam $-0^{\mathrm{M}}.28$.

These give the magnitude at the zenith, since the Potsdam values, used as the basis, were so expressed.

THE CATALOGUE

Column 1 gives the B.D. number of the star. This is in italics where the precession between 1855 and 1900 has changed the declination to the adjacent degree. An asterisk before the number indicates that the star is not in the Potsdam *Durchmusterung*, an "s" that it was used as a standard for magnitude.

Columns 2 and 3 give the position for 1900, taken from the Potsdam catalogue. The degree of declination, for column 3, is to be taken from column 1.

Column 4 gives the photographic magnitude on the "International System," based on the visual magnitudes of the white stars in the Potsdam *Durchmusterung*.

Columns 5 and 6 give the probable error of the catalogue magnitude and the number of plates.

Columns 7, 8, and 9 give the corresponding values for the "Visual" magnitudes, found from color-sensitive plates used with a "visual-luminosity" filter.

Column 10 gives the "Color-Index" = Photographic Magnitude minus Visual Magnitude. This is, in general, negative for stars of Class B, and positive for stars of Classes A to M.

Column 11 gives the spectral class, estimated from the objective-prism plates on the Harvard system.

Column 12, under "S—Pa" gives the differences between Schwarzschild's photographic magnitudes of polar stars (*Aktinometrie*, B, p. 42) and column 4.

Column 13 gives the differences between Müller and Kempf's visual magnitudes in *Potsdam*, 17 and column 7.

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[1] *Aktinometrie*, B, p. 13.

CATALOGUE OF ZONE +73° TO +90°

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
s73°	2	0	3	23	39′.4	7.33	.01	4	7.30	.05	3	+ .03	A1	......	+ .02
s78	1		3	47	9.6	6.09	.02	5	5.97	.05	4	+ .12	A3	......	+ .09
73	4		5	55	55.7	8.16	04	2	7.05	.03	3	+1.11	G±	......	− .07
75	4		8	29	28.3	8.12	.04	2	7.21	.02	3	+ .91	G±	......	− .21
s76	5		10	33	23.7	6.21	.03	7	6.46	.02	3	− .25	A0	......	− .15
s75	7	0	12	43	43.3	6.99	.01	3	7.15	.04	3	− .16	A0	......	− .03
s79	10		20	42	29.9	6.20	.02	4	6.40	.05	4	− .20	B8	......	+ .08
76	10		24	28	28.1	7.05	.04	5	6.18	.04	2	+ .87	G4	......	− .02
81	13		32	13	56.5	6.87	.02	10	6.25	.03	5	+ .62	F5	− .01	+ .07
76	14		33	23	19.3	7.85	.03	5	6.64	.08	3	+1.21	K0	......	− .18
*75	36	0	38	10	23.6	7.84	...	1	7.12	.03	2	+ .72	G0	......	......
s74	27		39	2	26.5	5.88	.04	5	5.67	.05	3	+ .21	A1	......	+ .03
s74	29		41	5	18.1	5.39	.06	5	5.51	.03	3	− .12	A0	......	− .09
77	25		41	33	55.3	8.05	.02	3	6.91	.06	4	+1.14	G3	......	+ .04
77	27		43	41	24.5	6.90	.05	8	6.59	.03	5	+ .31	A7	......	− .05
72	43	0	44	19	1.4	8.58	.04	2	7.69	.06	2	+ .89	G±	......	− .07
s82	20		45	30	9.9	5.62	.02	6	5.50	.03	4	+ .12	A1	− .02	+ .06
73	40		46	30	46.2	7.75	.06	2	7.55	.10	3	+ .20	A5	......	− .08
79	24		52	8	0.3	6.98	.02	7	6.64	.03	5	+ .34	F0	+ .04	− .06
s83	20		52	49	4.1	6.87	.01	8	6.66	.07	4	+ .21	A4	− .01	+ .04
85	19	0	55	1	43.2	5.47	.01	8	4.25	.03	13	+1.22	G4	+ .08	− .11
s88	4		55	37	29.3	6.43	.02	6	6.45	.02	10	− .02	A1	+ .01	− .03
s73	51		57	38	50.1	6.75	.04	3	6.73	.03	2	+ .02	A1	......	− .01
86	17		59	5	36.8	7.35	.02	6	6.14	.03	9	+1.21	G5	− .10	− .12

79	29	1	0	40	28.7	7.16	.03	5	6.17	.06	4	+ .99	G2	......	− .06
s78	*34*		3	36	8.5	5.32	.03	4	5.59	.04	4	− .27	B9	......	− .09
73	59		6	1	23.5	8.27	.00	2	7.58	.05	2	+ .69	G±	......	− .03
79	36		7	40	22.7	6.59	.01	4	6.18	.01	4	+ .41	F1	......	.00
*73	60		9	2	28.9	....	...	..	7.86	.02	2	......	..	......	......
73	61		9	6	29.9	6.97	.07	4	6.96	.04	2	+ .01	A±	......	+ .06
80	35	1	9	41	20.0	7.55	.06	4	7.14	.03	5	+ .41	F0	− .01	− .14
s80	36		10	3	22.0	6.63	.02	6	6.58	.03	5	+ .05	A1	− .05	+ .04
76	39		11	6	16.2	8.37	.03	3	7.12	.00	2	+1.25	G–K	......	− .04
76	*40*		11	59	2.5	7.23	.01	6	6.23	.04	5	+1.00	G4	......	− .08
78	36	1	13	9	30.1	8.57	.16	2	7.06	.05	3	+1.51	K5	......	− .12
73	*66*		13	46	3.4	7.29	.05	4	7.12	.04	2	+ .17	A4	......	− .04
75	*58*		13	48	11.2	7.89	.03	3	7.12	.05	3	+ .77	G2	......	− .04
75	59		13	51	42.9	6.62	.02	4	6.49	.07	3	+ .13	A5	......	− .19
77	*49*		14	57	12.2	6.31	.01	6	5.84	.02	3	+ .47	B5?	......	+ .11
73	75	1	21	0	41.4	7.74	.03	3	7.15	.04	3	+ .59	F8	......	− .05
*73	76		21	28	42.9	....	...	..	9.00	...	..	......	..	......	......
88	8		22	33	46.4	2.53	...	..	2.11	...	..	+ .42	..	......	......
s73	81		29	9	47.4	6.53	.04	4	6.62	.04	2	− .09	A0	......	− .14
s80	50		29	38	55.2	7.01	.01	6	7.01	.03	5	+ .00	A0	− .05	+ .01
s77	58	1	31	35	27.6	6.64	.01	7	6.64	.06	4	.00	A0	......	+ .04
73	*92*		35	19	6.0	7.52	.00	2	6.55	.06	2	+ .97	G3	......	− .13
s75	72		35	57	22.1	7.37	.03	3	7.18	01	2	+ .19	A4	......	− .02
s80	55		38	50	23.2	6.94	.01	6	6.86	.04	4	+ .08	A0	+ .14	+ .10
s80	57		39	46	52.6	7.52	.03	4	7.35	.03	4	+ .17	A0	− .07	.00
74	*84*	1	42	48	5.6	7.25	.00	3	6.86	.04	2	+ .39	F4	......	− .08
81	61		43	31	27.9	8.59	.08	4	6.70	.04	4	+1.89	K5	......	− .05
80	58		44	35	25.0	8.48	.06	3	7.10	.06	5	+1.38	K0	− .37	.00
77	65		44	44	42.2	7.75	.04	4	6.47	.08	4	+1.28	K0	......	+ .01
s75	76		46	12	43.9	7.20	.02	3	6.88	.01	2	+ .32	A5	......	.00

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
74°	87	1	48	48	50′8	7.52	.02	2	6.59	.03	2	+ .93	G2	......	− .13
75	83		51	17	28.0	8.62	.08	2	6.96	.04	3	+1.66	K±	......	− .22
77	73		52	49	25.9	7.23	.03	6	5.92	.09	3	+1.31	G9	......	− .14
s*74*	*91*		52	52	1.1	6.48	.02	5	6.59	.01	3	− .11	A0	......	− .07
s73	108		54	19	22.0	6.32	.01	3	6.11	.04	2	+ .21	A3	......	+ .05
76	63	1	55	5	48.1	5.58	.05	5	5.52	.04	3	+ .06	A5	......	− .38
s*79*	*61*		55	43	11.1	7.64	.03	4	7.51	.05	4	+ .13	A2	− .15	− .17
75	86		55	58	38.1	6.24	.00	3	5.45	.05	3	+ .79	G2	......	− .43
s*73*	*112*		56	55	6.2	7.53	.07	2	7.40	.02	3	+ .13	A1	......	− .05
s80	64		57	4	49.1	6.03	.01	6	5.95	.05	5	+ .08	A2	.00	+ .05
s74	95	1	57	53	23.5	7.59	.09	2	7.58	.01	3	+ .01	A2	......	+ .08
78	69		57	53	52.1	7.32	.03	5	6.98	.06	4	+ .34	A6	......	− .08
s*80*	*65*		57	53	0.3	6.96	.02	6	6.77	.05	5	+ .19	A4	+ .02	− .11
78	*71*	2	1	6	13.1 }	6.20	.02	8	7.19	.04	4	......	A2	......	− .29
s79	63		1	25	13.1 }				6.45	.03	4				− .05
82	*51*		1	25	5.5	7.28	.03	4	6.36	.06	3	+ .92	G2	......	− .12
73	121		4	8	33.4	7.16	.03	3	6.18	.04	3	+ .98	G3	......	− .06
s78	73		4	41	42.7	7.06	.03	4	6.99	.08	3	+ .07	A1	+ .31	+ .19
s75	94	2	14	39	40.6	7.60	.06	3	7.55	.03	2	+ .05	A0	......	+ .09
72	*128*		15	19	2.6	8.19	.09	3	8.11	.04	2	+ .08	A±	......	+ .13
73	139		18	22	15.7	7.54	.03	3	7.18	.02	2	+ .36	A7	......	− .14
77	84		22	31	13.1	8.33	.04	4	7.68	.04	3	+ .65	G±	......	− .06
79	75		23	41	17.3	8.45	.04	3	7.40	.07	2	+1.05	G2	......	− .02

76	81	2	23	44	16.6	8.55	.04	4	6.91	.01	2	+1.64	K±	......	− .35
74	*111*		27	48	5.5	8.14	.06	3	7.94	.04	2	+ .20	A–F±	......	− .04
74	112		27	51	56.7	8.46	...	1	8.20	.06	2	+ .26	......	......	− .04
80	*86*		33	21	1.5	7.01	.05	6	5.67	.02	3	+1.34	K2p	+ .15	− .09
s74	117		36	6	59.1	7.30	.01	3	7.13	.01	3	+ .17	A3	......	− .01
s79	86	2	41	47	41.6	7.27	.03	6	7.25	.01	2	+ .02	A2	......	− .17
s74	120		42	12	19.5	7.74	.01	4	7.57	.02	3	+ .17	A4	......	+ .13
75	*109*		44	15	7.1	8.57	.06	3	7.42	.03	3	+1.15	G5±	......	− .12
76	101		47	18	40.5	7.78	.01	3	6.90	.04	3	+ .88	G2	......	− .21
78	*103*		52	47	1.4	7.25	.04	6	5.38	.03	3	+1.87	M	......	− .18
s74	131	2	53	20	45.2	6.93	.02	3	7.08	.03	2	− .15	B8	......	− .04
s73	165		54	52	33.1	7.44	.02	3	7.34	.01	2	+ .10	A2	......	+ .12
s80	*97*		56	11	5.0	5.87	.02	8	5.71	.02	5	+ .16	A4	+ .13	− .01
75	124		56	16	24.8	8.51	.06	2	7.07	.01	2	+1.44	K±	......	− .28
75	127		58	28	41.3	7.83	.02	3	7.48	.01	2	+ .35	A–F	......	......
77	109	3	0	46	48.6	8.61	.03	2	7.13	.04	4	+1.48	K±	......	− .19
s73	*168*		1	5	0.9	4.46	.03	3	4.82	.01	2	− .36	Ao	......	+ .02
79	94		1	27	45.2	7.68	.05	4	6.59	.06	3	+1.09	G4	......	+ .10?
s73	170		2	36	55.0	7.66	.05	3	7.04	.02	2	+ .62	F2	......	+ .08
73	171		3	26	51.9	8.39	...	1	6.74	.01	2	+1.65	G–K±	......	− .09
77	*111*	3	3	45	7.6	8.62	.02	2	7.33	.02	3	+1.29	Ko	......	+ .39
s78	109		4	1	30.0	6.89	.02	4	6.74	.11	3	+ .15	A3	......	− .06
73	172		4	25	29.6	7.97	.03	3	6.45	.01	2	+1.52	K+	......	− .16
81	107		6	30	47.0	8.42	.06	3	6.82	.03	4	+1.60	K5	......	+ .03
s77	115		7	37	22.1	5.27	.07	6	5.45	.06	4	− .18	A3	......	− .10
82	*82*	3	8	7	10.1	8.58	.09	3	7.04	.07	3	+1.54	K5	......	.00
s74	144		8	34	52.2	7.24	.03	4	7.23	.03	3	+ .01	A1	......	+ .03
84	59		8	34	33.5	6.38	.02	8	5.45	.06	4	+ .93	Go	+ .16	+ .02
73	179		10	15	20.0	8.09	.03	4	6.66	.01	3	+1.43	G–K±	......	− .12
73	180		12	57	48.9	7.20	.04	4	6.74	.01	3	+ .46	F1	......	− .03

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
77°	119	3	14	38	7′.4	8.16	.03	2	7.47	.03	3	+ .69	F5	......	+ .14
*74	151		18	22	55.9	7.98	...	1	7.87	.07	3	+ .11	A3	......	......
77	123		18	31	39.9	8.58	.01	3	7.13	.06	3	+1.45	K0	......	+ .06
s72	178		24	14	0.5	6.45	.08	3	6.40	.04	2	+ .05	A1	......	+ .12
74	161		27	1	24.2	8.24	.11	2	7.33	.01	3	+ .91	G2	......	+ .05
75	143	3	27	21	24.4	7.15	.01	4	6.10	.06	3	+1.05	G3	......	.00
76	128		29	41	50.6	7.87	.03	7	7.68	.06	2	+ .19	A1	......	+ .17
74	168		33	29	13.3	7.66	.05	4	6.57	.01	2	+1.09	G5	......	+ .03
s83	91		33	43	14.1	7.25	.02	6	7.58	.05	3	− .33	B7	+ .16	+ .06
86	51		33	54	20.0	6.13	.03	8	5.78	.07	4	+ .35	F0	− .01	− .06
s77	133	3	34	56	48.3	6.88	.02	4	6.90	.06	4	− .02	A0	......	+ .08
74	174		42	4	21.8	8.56	...	1	7.62	.04	2	+ .94	G±	......	+ .11
73	204		44	57	46.9	8.16	.06	2	6.58	.00	2	+1.58	K0	......	− .16
77	138		49	2	55.0	7.95	.03	3	6.70	.06	4	+1.25	G5	......	+ .04
75	160		53	8	8.3	7.94	.03	4	7.61	.02	3	+ .33	A8	......	+ .09
80	125	3	53	17	25.4	5.44	.04	4	4.93	.04	4	+ .51	F4	+ 19	+ .11
*78	142		54	19	39.7	7.63	.03	4	....	...	..	......	A0	......	......
*74	184		54	53	54.7	7.44	.01	2	7.01	.05	3	+ .43	F2	......	......
74	186		55	19	21.8	7.66	.03	2	6.56	.01	3	+1.10	G5	......	+ .08
73	210		55	22	43.6	7.40	.02	2	6.26	.05	3	+1.14	G5	......	+ .18
78	146	3	57	8	45.7	8.09	.02	2	6.74	.04	3	+1.35	G7	......	− .02
73	212		57	44	17.9	7.16	.03	2	6.50	.03	3	+ .66	F5	......	+ .12

80	127	4	1	4	16.6	7.35	.02	6	6.20	.02	4	+1.15	G4	− .04	+ .03
s81	147		1	58	43.3	7.32	.02	4	7.35	.01	2	− .03	A0	+ .04	+ .05
s83	104		4	59	33.9	5.04	.07	6	5.50	.01	3	− .46	B6	+ .26	+ .09
85	63		5	5	17.4	6.91	.02	8	6.41	.04	4	+ .50	F6	+ .05	+ .05
77	150		5	36	49.7	7.70	.04	5	6.66	.04	5	+1.04	G3	......	+ .18
82	*113*	4	7	59	6.0	6.19	.02	4	5.47	.10	3	+ .72	G0	+ .10	− .07
s75	173		8	12	51.6	6.56	.02	5	6.58	.02	4	− .02	A0	......	+ .12
84	78		8	58	14.5	8.42	.05	4	7.13	.03	5	+1.29	K0	......	+ .01
80	133		9	37	35.2	6.57	.01	6	5.28	.06	4	+1.29	K2p	+ .16	.00
s80	134		12	0	41.9	7.20	.01	6	7.10	.03	4	+ .10	A2	+ .06	− .03
80	140	4	19	9	39.8	7.49	.01	6	7.22	.03	4	+ .27	A5	− .04	+ .12
83	114		21	32	49.6	8.09	.04	3	7.04	.03	3	+1.05	G2	− .42	+ .12
78	157		22	9	46.5	7.48	.01	4	7.16	.02	3	+ .32	A6	......	+ .04
s79	150		28	50	27.6	6.56	.01	6	6.51	.03	4	+ .05	A1	......	+ .15
76	174		32	8	25.4	6.93	.02	8	6.31	.01	3	+ .62	F5	......	+ .06
s75	189	4	35	22	45.6	6.22	.02	5	5.90	.02	3	+ .32	A4	......	+ .02
75	193		39	31	32.4	7.77	.02	3	6.87	.07	3	+ .90	G1	......	+ .13
80	*155*		41	37	1.7	6.47	.01	8	5.06	.04	5	+1.41	K4	+ .03	− .06
77	178		46	1	37.0	8.20	.03	4	7.64	.06	4	+ .56	F5	......	+ .06
74	229		49	38	6.9	7.69	.02	3	5.94	.04	2	+1.75	K5	......	− .10
73	264	4	51	45	36.9	7.86	.03	3	6.06	.00	2	+1.80	K5+	......	− .04
s73	265		52	3	55.2	6.03	.01	5	6.07	.01	2	− .04	A0	......	+ .06
85	74		56	16	49.8	6.67	.02	10	6.37	.03	3	+ .30	A5	+ .03	+ .07
s75	208		56	29	32.8	7.19	.02	4	7.08	.06	3	+ .11	A2	......	+ .19
s73	274		59	45	49.1	5.13	.02	4	5.52	.03	3	− .39	B8	......	− .13
s76	190	5	0	30	20.8	6.27	.02	8	6.29	.00	3	− .02	A0	......	+ .05
s74	238		4	34	25.1	7.47	.02	4	7.27	.03	3	+ .20	A4	......	+ .05
s73	280		5	53	9.2	5.68	.02	4	5.82	.02	3	− .14	A0	......	− .04
79	169		6	5	6.9	5.41	.04	4	5.26	.04	2	+ .15	F5	......	− .32
75	220		8	41	5.4	8.21	.02	5	7.90	.03	3	+ .31	F0	......	+ .10

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No . pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
s85°	78	5	9	52	35′.3	6.53	.01	11	6.64	.03	5	− .11	A0	− .02	− .10
78	183		10	25	18.8	7.74	.02	3	6.74	.02	2	+1.00	G2	......	− .28
s83	141		11	48	46.6	7.01	.02	6	7.00	.04	4	+ .01	A0	+ .09	− .08
73	285		12	24	36.5	7.97	.06	4	6.45	.01	3	+1.52	K5	......	− .15
78	187		13	6	12.6	7.03	.03	4	6.75	.00	2	+ .28	F2	......	− .11
s77	195	5	14	2	53.1	6.64	.03	6	6.48	.02	3	+ .16	A3	......	− .02
s74	241		14	37	27.7	7.29	.02	6	7.19	.04	4	+ .10	A0	......	− .22
s74	242		15	35	13.0	6.88	.02	6	6.91	.03	4	− .03	A0	......	− .03
74	249		23	56	15.1	7.21	.02	6	6.98	.04	3	+ .23	A6	......	− .14
74	250		24	24	37.5	7.99	.03	5	7.56	.05	3	+ .43	F0	......	− .04
74	252	5	26	21	58.6	7.76	.02	5	6.22	.01	3	+1.54	K5	......	− .25
85	80		29	54	8.8	7.65	.04	8	6.07	.02	3	+1.58	K5	− .11	− .16
s73	298		30	11	55.7	6.91	.04	4	6.54	.11	3	+ .37	A9	......	+ .16
74	257		33	14	34.1	7.84	.03	4	7.25	.02	3	+ .59	F8	......	− .03
75	236		36	37	40.5	7.87	.04	4	7.54	.05	3	+ .33	F2	......	+ .02
75	247	5	51	22	34.9	7.68	.05	4	6.20	.06	4	+1.48	K5	......	− .11
73	311		52	24	59.9	7.76	.07	4	7.29	.01	2	+ .47	F2	......	− .01
77	229		57	58	18.0	8.21	.03	5	7.31	.03	4	+ .90	G1	......	− .05
76	226	6	0	38	31.5	8.07	.03	5	7.65	.01	3	+ .42	F0	......	− .01
79	198		4	15	48.8	8.93	.03	2	7.88	.03	5	+1.05	G±	......	− .09
76	234		4	58	51.6	8.33	v?	5	7.80	.04	3	+ .53	G±	......	− .22
86	79		8	3	45.6	7.69	.03	5	6.45	.01	3	+1.24	K1	− .10	+ .03
73	328		13	13	29.0	8.60	.02	2	7.55	.10	3	+1.05	G±	......	− .05

78	225	6	13	33	20.3	8.97	.01	2	7.90	.00	2	+1.07	G±	......	− .08
79	201		15	52	31.7	8.33	.02	3	8.05	.01	2	+ .28	F0	......	− .10
79	202		16	33	2.4	7.86	.02	3	6.59	.01	2	+1.27	G8	......	− .14
80	210		16	57	38.2	8.54	.03	3	6.98	.04	3	+1.56	K5	......	+ .02
s78	226		17	41	13.8	7.01	.02	4	7.03	.02	2	− .02	B9	......	+ .07
76	244	6	20	58	56.2	8.44	.08	4	7.86	.01	2	+ .58	F–G	......	+ .22
s77	247		21	6	59.8	7.56	.03	5	7.49	.01	2	+ .07	A1	......	− .09
s79	208		23	6	40.6	6.74	.01	6	6.65	.03	2	+ .09	A1	......	+ .03
s82	177		23	23	12.0	6.68	.04	4	6.62	.06	3	+ .06	A3	− .01	− .06
78	227		24	57	4.5	7.25	.03	4	5.50	.01	2	+1.75	K5	......	+ .03
73	340	6	25	18	46.4	6.44	.03	3	6.24	.07	2	+ .20	F0	......	− .10
80	217		29	3	20.0	8.51	.00	3	7.23	.03	3	+1.28	K0	......	− .14
79	212		29	11	40.6	5.82	.03	6	5.39	.02	2	+ .43	F5	......	− .03
76	249		30	7	40.7	7.96	.04	5	7.00	.10	2	+ .96	G1	......	+ .03
78	234		38	44	7.4	8.12	.08	3	7.79	.05	2	+ .33	F1	......	− .21
75	273	6	45	5	19.3	8.07	.07	4	7.73	.04	3	+ .34	A8	......	+ .01
77	266		45	29	6.3	6.00	.03	6	4.62	.06	4	+1.38	K3	......	− .18
75	281		50	46	22.7†	7.22	.07	3	6.67	.04	2	+ .55	F5	......	......
75	280		50	48	33.0	7.57	.05	3	6.88	.08	2	+ .69	G2	......	+ .02
73	360		52	9	34.1	8.41	.02	2	7.39	.00	2	+1.02	G±	......	+ .01
87	51	6	53	45	12.3	6.70	.01	7	4.95	.06	9	+1.75	K5+	− .04	− .05
78	240		57	58	54.6	7.18	.01	4	6.81	.07	3	+ .37	A6	......	+ .13
73	366	7	3	46	29.4	7.99	.02	3	7.56	.01	2	+ .43	F0	......	− .14
80	230		5	45	48.3	8.29	.04	4	7.07	.03	6	+1.22	G5	− .94?	− .03
s81	242		6	24	26.4	6.22	.02	8	6.25	.04	6	− .03	B8	− .12	+ .11
78	240		7	44	4.6	8.23	.03	2	7.67	.02	4	+ .56	F5	......	− .17
s75	296		8	47	16.3	6.85	.01	6	6.82	.01	2	+ .03	A0	......	.00

† Σ 973, mean δ.

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
82°	201	7	10	4	36′.3	6.62	.01	6	4.90	.02	4	+1.72	M	+ .04	− .12
78	250		10	8	26.5	8.61	.06	2	7.11	.04	4	+1.50	K2	......	− .11
78	251		11	7	14.1	7.58	.02	4	7.51	.03	4	+ .07	A2	......	− .05
s74	322		13	33	2.5	7.42	.01	4	7.49	.00	2	− .07	Ao	......	− .15
*75	300		13	36	49	7.5	...	1	....	...	..	......	Ao?	......	.....
73	375	7	14	32	16.4	7.04	.01	4	7.14	.04	2	− .10	A–F	......	+ .08
*73	376		14	39	16.4‡	....	...	2	7.5	...	2	......	A±	......	......
81	252		16	27	6.0	7.62	.01	7	6.41	.05	4	+1.21	G5	− .18	− .01
77	286		17	3	9.3	8.32	.04	6	7.12	.05	4	+1.20	G+	......	+ .05
78	254		17	31	53.6	7.95	.01	2	7.49	.06	4	+ .46	Fo	......	− .27
76	284	7	25	47	0.5	8.05	.05	6	7.60	.03	3	+ .45	Fo	......	− .06
73	381		26	27	49.0	7.81	.02	3	7.38	.04	2	+ .43	F+	......	− .04
s82	*213*		27	41	54.8	7.75	.01	6	7.82	.02	4	− .07	Ao	− .37	− .34
74	335		36	23	17.2	8.20	.05	3	6.96	.01	2	+1.24	G–K±	......	− .03
81	257		38	56	36.5	8.31	.14	2	6.92	.05	4	+1.39	K2	......	+ .01
80	238	7	39	46	31.0	7.31	.03	6	6.52	.03	4	+ .79	G1	− .19	− .07
80	240		43	20	7.3	8.36	.11	3	6.79	.04	4	+1.57	K4	......	− .19
s77	303		43	29	50.1	7.23	.02	4	6.80	.06	3	+ .43	A3	......	− .10
74	338		48	14	11.1	6.85	.02	6	5.48	.00	2	+1.37	K2	......	− .31
s79	265		49	5	45.2	5.41	.03	5	5.41	.04	3	+ .00	Ao	......	− .11
86	*113*	7	52	28	59.4	8.12	.02	5	7.33	.03	3	+ .79	Go	− .25	− .13
s84	169		53	2	20.8	6.36	.01	11	6.36	.04	5	+ .00	Ao	− .03	+ .04
78	*278*		57	6	55.9	8.41	.10	3	7.73	.04	3	+ .68	F±	......	+ .23
s89	*13*		58	3	56.0	7.12	.02	8	6.96	.03	10	+ .16	A3	+ .02	+ .04

76	308	8	4	14	2.8	7.77	.01	4	7.50	.04	3	+ .27	Fo	......	+ .12
s82	235		5	12	44.5	6.25	.06	3	6.30	.07	2	− .05	B9	− .12	− .08
76	310		6	59	3.7	6.39	.01	7	5.68	.03	3	+ .71	Go	......	− .23
75	334		10	8	7.8	7.24	.00	2	6.42	.01	3	+ .82	G2	......	− .14
78	284		10	45	15.2	8.42	.01	2	8.40	.10	3	+ .02	Ao	......	− .19
77	327	8	11	44	16.9	8.25	.07	4	6.75	.04	4	+1.50	K±	......	− .03
78	287		16	58	33.4	7.96	.05	2	7.06	.02	3	+ .90	Go	......	− .17
73	416		20	58	1.5	8.54	...	1	7.30	.01	2	+1.24	Ko	......	− .18
73	420		23	44	49.0	8.17	.07	2	7.24	.02	2	+ .93	Go	......	+ .04
77	337		24	20	23.1	7.70	.04	6	7.11	.02	5	+ .59	F5	......	+ .01
s75	342	8	25	11	3.9	6.43	.01	3	6.39	.04	3	+ .04	A2	......	− .15
85	128		25	25	24.6	7.62	.01	6	7.26	.03	4	+ .36	F3	− .09	+ .06
s82	253		28	19	35.6	6.74	.02	6	6.73	.03	3	+ .01	Ao	− .09	+ .11
74	370		28	36	58.8	7.07	.05	2	6.24	.02	3	+ .83±	G5	......	− .24
s73	428		31	59	31.4	6.78	.03	3	6.76	.04	3	+ .02	Ao	......	+ .08
78	291	8	35	2	3.1	7.86	.04	3	7.70	.04	4	+ .16	F5	......	− .18
s73	430		35	5	39.1	7.18	.00	2	7.19	.03	4	− .01	Ao	......	+ .19
78	293		37	31	31.9	8.54	.05	2	6.93	.03	4	+1.61	K5–M	......	− .19
74	379		40	5	7.3	8.40	...	1	7.16	.02	4	+1.24	K±	......	+ .08
s80	272		40	52	24.2	7.50	.04	7	7.62	.06	5	− .12	B9	− .28	− .06
83	232	8	41	48	5.9	8.11	.04	3	7.08	.05	3	+1.03	G3	− .29	− .11
83	233		44	31	7.6	7.30	.02	5	6.99	.10	3	+ .31	F1	− .10	− .13
78	297		45	23	31.5	7.61	.02	4	7.24	.02	4	+ .37	Fo	......	− .04
76	335		51	44	47.6	8.25	...	1	7.74	.02	3	+ .51	F±	......	− .09
s79	294		51	47	44.3	7.83	.01	7	7.70	.05	4	+ .13	A3	......	+ .10
84	196	8	54	32	35.0	6.47	.02	9	6.26	.05	3	+ .21	A6	.00	− .06
81	282		56	18	13.8	6.76	.03	6	6.27	.01	5	+ .49	F1	− .24	+ .03

‡ 32″ fol. 73°.375.

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
75°	365	0	2	51	41′.2	8.30	...	1	7.35	.04	3	+ .95	G±	......	+ .11
s79	300		3	59	41.5	7.89	.01	6	8.21	.02	3	− .32	B4	......	− .06
s77	359		5	17	29.2	7.01	.01	8	6.94	.03	6	+ .07	A0	......	+ .06
s73	452		5	51	21.6	6.08	.03	4	5.92	.01	4	+ .16	A2	......	+ .06
74	393		9	37	26.2	7.40	.03	2	6.30	.04	4	+1.10	G5	......	+ .08
s75	370	0	11	26	20.1	7.49	.01	3	7.21	.03	4	+ .28	A2	......	+ .14
79	*305*		14	13	51.8	8.33	.02	5	8.38	.02	2	......	F±	......	− .08
79	*305*		14	13	51.8				8.83	.02	2			......	− .23
s75	377		17	22	31.7	6.22	.02	6	6.16	.02	5	+ .06	A2	......	− .10
83	256		20	30	21.9	7.33	.02	5	6.91	.03	3	+ .42	F2p	− .10	− .01
81	302	0	22	51	46.1	5.83	.03	8	4.49	.03	5	+1.34	K2p	+ .09	− .35
s74	402		26	8	45.8	6.19	.01	5	6.43	.02	3	− .24	B9	......	− .10
s83	*262*		26	27	48.8	7.55	.01	4	7.49	.05	3	+ .06	A3	− .18	− .09
73	470		28	24	31.7	6.62	.01	5	6.29	.03	3	+ .33	A5	......	du
75	386		28	28	39.1	7.48	.06	3	6.58	.04	3	+ .90	G+	......	− .19
s73	471	0	30	5	11.7	7.15	.01	3	7.05	.02	3	+ .10	A3	......	+ .05
79	316		30	28	16.8	8.30	.03	4	8.28	.05	3	+ .02	A2	......	− .14
76	*360*		30	38	53.0	8.25	...	1	7.22	.03	3	+1.03	G+	......	+ .01
78	317		34	11	35.4	7.76	.01	5	6.20	.04	3	+1.56	K2	......	− .04
s75	389		34	39	2.6	6.86	.03	4	6.73	.02	3	+ .13	A5	......	+ .01
79	319	0	35	27	35.7	6.33	.01	6	6.14	.06	3	+ .19	A7	......	.00
78	327		46	5	24.6	8.50	.02	4	7.57	.04	3	+ .93	G±	......	− .17
81	*319*		46	52	51.2	8.56	.07	3	7.89	.05	4	+ .67	G±	......	− .04
77	389		47	44	5.2	8.48	.04	3	7.74	.04	6	+ .74	G±	......	− .01
73	478		49	27	21.3	6.99	.02	4	5.94	.04	3	+1.05	K2p	......	− .14

84	225	9	52	37	24.1	7.87	.04	6	6.13	.02	4	+1.74	K5	− .23	− .01
75	399		52	39	14.4	7.75	.09	3	6.84	.03	4	+ .91	G2	......	− .06
83	*280*		59	34	53.3	8.31	.07	2	7.26	.08	2	+1.05	G±	......	− .07
s79	328	10	5	49	26.5	6.95	.01	8	6.85	.03	4	+ .10	A2	......	+ .01
73	489		9	36	34.4	6.76	.03	5	6.51	.14	2	+ .25	A6	......	− .01
83	287		11	43	18.3	7.80	.03	2	6.73	.07	2	+1.07	G±	− .19	− .01
79	*330*		13	31	50.8	8.46	.03	3	7.22	.03	4	+1.24	G–K±	......	− .03
84	234		15	9	45.6	5.57	.02	10	5.39	.02	4	+ .18	A4	+ .07	+ .02
83	297	10	18	55	4.1	5.39	.02	6	5.07	.12	2	+ .32	A8	+ .16	+ .07
85	*161*		20	46	54.7	7.99	.01	7	7.15	.03	4	+ .84	G0	− .19	.00
78	349		22	53	0.6	8.89	.00	2	7.83	.04	3	+1.06	G±	......	+ .13
74	*436*		23	15	50.6	7.93	.04	2	6.90	.05	2	+1.03	G2	......	− .04
81	343		25	44	0.6	7.47	.02	6	6.19	.04	3	+1.28	K0	− .13	+ .23
s74	438	10	26	7	21.0	7.49	.03	3	7.36	.01	2	+ .13	A2	......	+ .02
76	393		26	36	13.7	5.87	.02	7	5.11	.09	4	+ .76	G±	......	− .39
74	440		33	15	17.7	7.82	.06	4	7.52	.04	3	+ .30	F2	......	+ .23
s*81*	*349*		33	38	56.9	6.71	.01	6	6.49	.03	3	+ .22	A1	− .16	+ .17
s*78*	*359*		34	31	56.0	7.57	.04	4	7.63	.04	3	− .06	A±	......	+ .09
76	402	10	43	28	31.5	7.27	.04	2	7.01	.07	2	+ .26	A4	......	+ .11
80	*338*		45	47	52.6	8.56	...	4	7.59	.09	2	+ .97	G±	......	− .04
77	412		47	11	37.2	8.40	.06	5	6.74	.06	3	+1.66	M±	......	− .14
*80	347		50	41	19.4	7.68	.06	4	....	...	..	......	F±	......	......
76	406		51	30	15.4	7.81	.05	4	7.42	.02	2	+ .39	A8	......	+ .08
78	367	10	51	58	18.4	7.20	.03	4	6.12	.04	2	+1.08	G3	......	− .06
79	348		53	14	25.3	8.38	.01	2	7.38	.09	3	+1.00	G±	......	.00
82	325	11	2	13	16.7	7.76	.03	7	7.10	.07	6	+ .66	F8	......	− .03
78	375		2	26	19.8	8.75	.08	2	6.92	.10	2	+1.83	K±	......	− .14
s86	161		2	33	11.0	7.36	.00	7	7.17	.04	4	+ .19	A5	− .05	− .03
s88	64		4	14	11.0	7.27	.02	10	7.45	.02	9	− .18	B9	+ .01	+ .01

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
74°	456	11	8	41	1′.0	8.34	.09	2	7.24	.04	2	......	G–K±	......	+ .33
74	456a		8	43	1.0				7.68	.00	2	......		......	+ .16
79	*356*		8	48	51.3	8.14	.13	2	6.87	.02	3	+1.27	G5	......	− .11
75	*438*		10	36	53.1	8.55	.06	3	7.46	.01	2	+1.09	K0	......	+ .02
78	*385*		15	58	55.3	7.84	.01	3	7.31	.04	3	+ .53	F5	......	− .01
85	183		24	23	15.5	7.98	.02	3	7.07	.02	4	+ .91	G1	− .13	− .03
s81	373	11	24	48	40.7	6.24	.03	8	6.10	.02	6	+ .14	A2	+ .02	− .02
86	170		28	20	10.1	7.48	.01	10	7.12	.02	3	+ .36	A8	− .01	+ .06
78	392		31	26	8.9	8.07	.16	2	6.34	.03	3	+1.73	K5	......	− .06
73	532		39	0	42.1	8.47	.03	2	6.82	.04	2	+1.65	K5	......	.00
76	*434*		45	23	54.8	8.23	.02	5	7.84	.01	2	+ .39	F±	......	+ .08
74	*475*	11	47	56	50.9	8.42	.06	4	7.44	.06	4	+ .98	G3	......	− .03
74	476		48	19	19.0	7.23	.03	3	6.65	.00	2	+ .58	F6	......	− .01
80	370		54	32	9.1	8.22	.11	3	7.70	.06	5	+ .52	F–G±	− .20	+ .04
81	389		55	6	24.7	7.88	.06	3	6.24	.05	4	+1.64	K8	− .16	− .07
86	176		59	43	8.5	6.68	.01	9	6.15	.03	4	+ .53	F5	+ .01	+ .11
77	460	12	0	2	19.3	7.64	.04	5	7.18	.05	4	+ .46	F2	......	+ .04
77	461		0	9	27.9	6.83	.03	6	5.68	.02	3	+1.15	G3	......	− .04
74	484		0	12	0.6	8.53	.04	4	7.24	.04	3	+1.29	G±	......	+ .04
75	469		4	56	13.1	6.60	.02	5	6.21	.02	3	+ .39	F5	......	+ .08
78	*406*		5	6	56.7	7.77	.05	6	6.74	.04	4	+1.03	G1	......	+ .03
82	356	12	6	30	16.0	7.29	.05	3	6.06	.02	3	+1.23	K2	+ .05	− .16
*82	357		7	3	16.3	....	...	..	8.29	...	1	......	....	......	......
s*78*	*411*		7	6	59.8	6.54	.04	6	6.64	.06	4	− .10	A0	......	+ .07
s78	412		7	31	10.3	5.40	.03	4	5.20	.06	4	+ .20	A4	......	− .14
84	269		8	47	3.6	8.44	.06	4	8.16	.01	4	+ .28	F±	+ .18	.00

73	549	12	11	0	6.5	7.39	.06	3	6.24	.03	2	+1.15	G3	......	− .12
87	*107*		13	54	59.5	6.54	.01	13	6.12	.03	10	+ .42	Fo	+ .05	+ .08
s75	470		14	21	42.9	5.28	.03	4	5.50	.03	2	− .22	B7	......	− .06
88	71		14	23	15.3	6.36	.03	7	6.07	.01	8	+ .29	Fo	+ .03	+ .15
84	*274*		16	32	55.8	8.26	.12	3	7.79	.02	4	+ .47	G±	......	− .13
77	475	12	27	3	14.3	8.60	.03	6	7.22	.06	5	+1.38	G±	......	− .04
75	473		28	2	21.8	8.39	.03	7	7.22	.02	3	+1.17	Go	......	+ .08
73	559		29	23	1.9	7.97	.03	7	7.28	.02	2	+ .69	F±	......	+ .10
76	*454*		29	48	45.8	8.32	.02	6	6.87	.03	3	+1.45	G5±	......	+ .01
81	*400*		31	7	48.1	7.45	.03	5	7.23	.03	6	+ .22	A6	+ .08	− .19
80	*389*	12	34	8	45.9	7.50	.04	4	7.00	.03	4	+ .50	F–Gp	......	− .08
s86	182		34	37	16.9	7.28	.01	10	7.12	.04	4	+ .16	A3	− .04	+ .01
73	561		35	32	33.1	8.24	.02	6	7.08	.02	3	+1.16	Go	......	+ .08
76	462		36	57	27.3	8.48	.03	8	7.08	.02	3	+1.40	K±	......	+ .05
84	286		37	47	11.6	7.72	.02	5	7.24	.01	4	+ .48	F5p	− .05	− .06
s81	402	12	41	54	10.2	6.39	.01	6	6.28	.03	7	+ .11	A3	− .03	+ .02
80	395		42	19	28.2	8.80	.11	3	7.43	.02	6	+1.37	Ko	......	− .01
84	*289*		48	16	57.7 }	4.62	.05	5	5.64	.13	3	......	A±	+ .16	{ + .19
84	*290*		48	23	57.4 }				5.14	.06	2				{ + .22
76	*470*		51	7	57.1	7.78	.01	9	6.84	.00	2	+ .94	Go	......	− .04
79	407	12	52	45	2.5	9.20	.04	2	7.51	.06	3	+1.69	K–M	......	+ .01
83	369		53	9	3.6	8.11	.07	4	6.83	.04	5	+1.28	G–K±	− .10	+ .11
76	473		55	50	.07	7.05	.02	10	6.08	.00	2	+ .97	G2	......	− .22
81	412		58	38	24.8	7.91	.03	4	7.15	.02	6	+ .76	Go	+ .10	+ .03
86	187		59	43	25.4	8.07	.05	7	7.70	.04	4	+ .37	F2	− .24	− .01
s73	583	13	1	44	33.6	6.57	.01	9	6.52	.04	2	+ .05	A3	......	− .28
s73	587		10	40	19.8	6.65	.01	9	6.54	.01	2	+ .11	Ao	......	− .06
80	404		11	12	10.9	8.70	.07	3	7.36	.01	5	+1.34	Ko	......	− .10
81	416		11	32	0.0	7.21	.01	8	6.16	.03	7	+1.05	G3	− .06	− .05
79	418		13	36	13.7	8.46	.04	3	7.45	.03	4	+1.01	Go	......	− .03

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		PLACE 1900		PHOTOGRAPHIC			VISUAL			COLOR-INDEX	SPECTRUM	DIFFERENCES	
		R.A.	Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h m s								M		M	M
85°	222	13 18 40	16′.6	7.68	.02	7	7.28	.03	4	+ .40	F5p	− .07	− .20
79	422	26 6	9.6	6.42	.02	4	5.75	.04	3	+ .67	G1	......	− .12
84	*311*	26 42	49.4	7.82	.03	5	7.12	.03	4	+ .70	F9	− .05	+ .02
75	507	27 43	23.8	8.57	.02	4	7.39	.07	4	+1.18	G±	......	− .01
76	491	31 14	34.5	8.09	.02	6	7.01	.04	4	+1.08	G0	......	+ .09
86	*193*	13 32 26	47.1	8.03	.03	7	7.96	.03	4	+ .07	F5p	+ .07	− .12
s76	492	32 36	18.5	7.72	.02	8	7.75	.04	4	− .03	A0	......	+ .01
77	*515*	32 36	48.4	8.75	.06	5	7.81	.03	5	+ .94	G±	......	+ .09
77	516	33 24	3.4	8.10	.02	7	6.31	.02	5	+1.79	K5	......	− .11
80	*417*	36 39	51.7	8.69	.15	2	7.30	.02	4	+1.39	K±	......	+ .02
s77	519	13 39 42	20.9	6.67	.01	11	6.58	.03	5	+ .09	A3	......	+ .07
78	466	42 13	33.9	6.86	.01	4	5.79	.03	3	+1.07	G8	......	− .03
80	421	42 16	42.4	8.53	.05	3	6.86	.05	4	+1.67	K5	− .45	− .07
83	397	45 10	15.3	6.85	.02	6	5.90	.03	4	+ .95	G3	+ .04	− .11
76	500	46 21	4.9	8.49	.01	5	7.99	.06	3	+ .50	A–F	......	+ .13
s76	502	13 48 27	3.7	7.67	.02	6	7.44	.01	3	+ .23	A5	......	.00
80	422	49 55	24.9	8.17	.04	4	7.12	.05	4	+1.05	G5	− .10	− .06
79	431	50 22	29.3	7.72	.03	4	6.48	.03	3	+1.24	G8	......	− .05
85	234	51 29	0.6	8.71	.12	5	7.65	.02	4	+1.06	G4	......	− .08
81	452	52 27	15.6	7.80	.03	4	6.50	.08	4	+1.30	K2	− .12	.00
74	*560*	13 52 50	48.2	8.53	.03	2	7.60	.04	3	+ .93	G±	......	.00
77	*523*	54 38	59.4	9.1 ±	...	2	7.76	.08	3	+1.3±	K±	......	− .02
74	*563*	58 30	52.9	8.24	.01	3	7.61	.02	3	+ .63	F–G	......	+ .18
86	201	59 25	14.1	7.59	.03	6	7.12	.03	3	+ .47	F2	+ .08	+ .10

75	527	14	4	40	11.8	8.47	.03	3	7.25	.04	4	+1.22	G5	......	+ .05
s75	529		6	9	4.0	6.43	.02	6	6.29	.04	4	+ .14	A2	......	+ .03
78	478		9	14	1.0	6.37	.04	4	4.78	.01	2	+1.59	K5	......	− .12
75	532		21	11	30.9	8.60	.04	2	7.57	.04	3	+1.03	G0	......	− .09
76	527		27	44	8.4	5.66	.03	7	4.31	.02	3	+1.35	K5	......	− .22
79	*447*	14	28	18	56.5	7.50	.03	4	7.11	.01	2	+ .39	F3	......	+ .11
81	482		32	57	15.2	7.65	.02	4	6.73	.04	5	+ .92	G0	− .14	+ .01
80	448		36	23	5.5	7.59	.01	4	6.11	.02	4	+1.48	K3	.00	+ .04
80	451		41	57	12.8	8.73	.09	3	7.05	.03	4	+1.68	K5±	−1.37	.00
s76	536		48	9	27.4	7.48	.02	6	7.32	.05	3	+ .16	A3	......	+ .02
86	217	14	49	39	21.8	8.03	.03	6	7.01	.05	3	+1.02	G3	− .04	− .03
74	595		51	0	33.8	....	...	..	2.0†	...	..	......	......	......	......
s81	495		54	58	9.3	7.16	.01	8	7.13	.03	5	+ .03	A1	− .07	− .01
78	501		55	25	34.8	7.52	.03	3	6.45	.03	3	+1.07	G5	......	− .08
75	545		55	41	17.0	8.60	.08	3	6.86	.01	2	+1.74	K–M	......	− .08
83	*431*	14	57	3	55.4	6.20	.01	6	5.49	.02	4	+ .71	F8	+ .04	+ .10
75	547		57	31	18.0	7.89	.05	4	6.76	.01	2	+1.13	G0	......	− .12
84	335	15	1	41	20.3	8.31	.04	5	6.72	.03	4	+1.59	K2	− .21	− .02
s74	602		5	39	16.5	7.68	.02	3	7.49	.01	2	+ .19	A3	......	− .03
87	143		9	21	37.1	8.37	.04	4	6.74	.03	8	+1.63	K5	− .36	+ .06
74	609		18	15	24.4	7.87	.03	5	6.60	.05	3	+1.27	G5	......	− .10
78	510	15	21	30	45.2	8.62	.01	2	7.12	.15	2	+1.50	K±	......	.00
73	672		22	18	49.7	7.84	.03	5	7.34	.02	3	+ .50	F+	......	− .04
73	680		31	33	16.5	8.72	...	1	7.72	.01	2	+1.00	......	......	+ .17
77	592		34	23	40.9	6.63	.04	9	4.84	.03	4	+1.79	K5	......	− .02
80	480		34	59	46.8	6.88	.02	6	6.56	.08	3	......	G2	− .12	+ .03
80	481		35	12	46.8				7.30	.03	3	......			......

† β *Ursae Minoris*, too bright

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900		Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.	Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h m s								M		M	M
81°	517	15 35 57	6′.2	7.92	.03	3	6.86	.04	3	+1.06	G2	− .18	− .02
s76	563	36 50	46.7	7.62	.02	7	7.51	.02	2	+ .11	A±	......	+ .17
82	463	38 8	35.8	7.25	.03	7	7.01	.03	2	+ .24	F0	− .04	− .17
85	263	42 32	9.4	7.88	.03	7	6.79	.03	4	+1.09	G1	− .09	− .09
81	*523*	15 42 56	56.0	7.83	.08	4	6.85	.04	3	+ .98	G5±	− .04	− .11
80	487	45 7	17.8	7.20	.02	6	6.78	.02	3	+ .42	F0	− .14	+ .01
78	527	47 37	6.1	4.11	.14	2	4.09	.08	2	+ .02	A1	......	+ .15
81	531	51 24	14.2	8.30	.04	3	6.79	.05	3	+1.51	K5	......	− .11
s83	453	53 46	15.0	7.61	.03	7	7.41	.06	3	+ .20	A3	− .04	+ .04
75	579	15 57 23	51.7	7.82	.02	7	6.73	.08	3	+1.09	G5	......	− .03
s85	269	57 25	35.3	7.07	.02	10	6.82	.02	5	+ .25	A5	− .12	+ .06
76	580	16 0 19	22.1	8.67	.06	5	7.90	.04	3	+ .77	G±	......	+ .12
74	650	1 56	12.6	8.95	.05	2	7.70	.03	2	+1.25	K±	......	+ .09
73	707	5 24	24.8	7.35	.04	6	6.81	.05	2	+ .54	F5	......	− .08
s77	616	6 50	3.6	5.59	.02	6	5.73	.01	3	− .14	A1	......	− .05
81	*541*	7 29	53.8	8.41	.01	4	7.44	.03	3	+ .97	G±	− .66?	+ .01
s76	594	16 13 40	7.8	5.18	.02	5	5.52	.02	2	− .34	A0	......	− .14
75	586	15 3	27.5	7.79	.03	4	6.16	.01	2	+1.63	K5	......	.00
s73	713	16 12	38.3	5.94	.02	8	6.08	.02	2	− .14	A±	......	− .06
76	*596*	20 26	59.0	5.18	.04	8	5.10	.05	2	+ .08	A6	......	− .30
77	623	22 39	47.1	8.97	.04	3	7.42	.06	3	+1.55	K±	......	− .04

s76	600	16 22 59	22.5	7.23	.02	10	6.92	.01	2	+ .31	A3	......	+ .07
76	606	25 58	40.2	9.0±	...	2	7.28	.03	2	+1.7±	..	......	− .08
79	498	31 19	10.6	5.67	.01	4	5.61	.01	2	+ .06	A3	......	− .04
s*84*	*361*	33 37	55.1	7.29	.02	6	7.03	.03	6	+ .26	A4	− .06	+ .08
77	627	34 57	38.6	7.43	.03	8	6.41	.05	3	+1.02	G4	......	− .27
80	*519*	16 37 45	59.7	7.53	.01	5	6.90	.04	3	+ .63	F–G	......	− .03
78	*562*	39 55	57.1	7.49	.02	7	6.98	.03	3	+ .51	F5	......	− .36
79	511	43 34	6.4	7.50	.03	3	6.33	.07	2	+1.17	K2p	......	− .17
s74	680	44 10	4.2	6.99	.01	11	6.74	.07	2	+ .25	A2	......	+ .08
77	634	47 32	41.1	6.36	.02	8	6.13	.06	3	+ .23	F1	......	− .13
75	605	16 52 37	32.6	7.87	.03	6	7.34	.05	2	+ .53	F±	......	+ .08
79	517	54 6	40.1	8.08	.01	2	6.77	.05	4	+1.31	K1	......	− .16
s75	608	56 1	33.0	7.03	.01	7	6.78	.05	2	+ .25	A5	......	+ .04
82	498	56 12	12.1	5.05	.03	8	4.46	.04	4	+ .59	G2	+ .15	− .26
s77	639	56 49	0.5	7.61	.03	9	7.36	.01	2	+ .25	A5	......	− .05
s73	751	16 58 16	16.8	6.47	.01	6	6.30	.01	2	+ .17	A5	......	− .08
74	695	58 49	26.1	7.53	.03	4	6.98	.08	2	+ .55	F3	......	+ .14
78	573	17 0 52	6.4	8.42	.07	2	7.59	.02	2	+ .83	G±	......	− .31
s77	641	0 55	48.0	6.60	.02	10	6.86	.00	2	− .26	B9	......	− .11
73	754	2 31	20.1	8.60	.02	3	7.23	00	2	+1.37	..	......	+ .06
73	755	3 24	27.1	9.0±	...	1	7.54	.01	2	+1.4±	..	......	− .03
75	612	3 31	22.0	7.29	.02	7	6.84	.01	2	+ .45	F0	......	+ .06
81	568	17 4 46	0.1	7.70	.02	6	6.59	.03	5	+1.11	G5	− .09	− .07
s75	613	4 49	26.2	6.37	.01	7	6.24	.02	2	+ .13	A5	......	− .11
78	580	5 15	14.2	8.40	.03	2	7.06	.06	3	+1.34	G–K±	......	− .20
75	616	9 7	13.8	7.89	.02	5	7.36	.08	3	+ .53	F±	......	− .04
s75	617	13 55	12.7	8.14	.03	5	8.06	.07	3	+ .08	A±	......	+ .12
77	654	17 19 5	27.3	8.70	.04	5	7.86	.03	3	+ .84	G±	......	− .08
s74	710	25 36	44.9	7.76	.02	5	7.49	.03	3	+ .27	A5	......	+ .04
s79	540	25 49	23.8	7.35	.02	3	7.30	.03	2	+ .05	A1	......	− .01
80	544	27 12	13.5	7.32	.04	5	5.56	.05	4	+1.76	K5+	+ .03	− .08
76	647	27 22	8.4	8.56	.03	6	7.19	.03	3	+1.37	K±	......	− .17

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900		Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.	Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h m s								M		M	M
83°	512	17 32 37	24′.9	8.11	.02	3	7.52	.07	4	+ .59	F+	− .19	− .18
74	717	35 27	17.4	7.94	.02	3	6.77	.04	2	+1.17	K±	......	− .19
74	720	38 47	3.8	8.30	.02	2	7.72	.08	2	+ .58	F-G±	......	− .10
75	639	42 31	57.7	9.1±	...	1	8.06	.03	2	+1.00±	..	......	− .01
79	556	44 9	15.5	7.67	.02	2	7.40	.04	2	+ .27	A8	......	− .20
s73	791	17 44 26	30.1	7.83	.01	2	7.76	.00	2	+ .07	A2	......	− .04
75	640	46 8	34.6	8.76	...	2	7.82	.02	2	+ .96	..	......	− .11
73	796	49 50	9.8	8.35	.06	2	7.75	.08	2	+ .60	F±	......	− .13
80	555	50 5	18.9	8.59	.03	2	6.91	.04	5	+1.68	K5+	......	− .03
77	670	52 59	3.0	7.98	.02	5	7.57	.04	3	+ .41	F±	......	+ .10
76	667	53 55	58.5	5.32	.02	8	5.01	.01	2	+ .31	F5?	......	+ .13
74	740	17 54 22	35.2	6.92	...	1	6.77	.04	3	+ .15	A5	......	− .05
79	564	55 35	21.0	7.77	.00	2	7.52	.02	2	+ .25	A8	......	− .03
78	616	55 49	19.4	7.72	.03	2	6.16	.04	3	+1.56	K5	......	− .12
s76	668	56 8	0.8	7.62	.03	5	7.33	.02	3	+ .29	A6	......	− .03
75	647	57 45	10.8	7.42	.02	4	6.56	.03	3	+ .86	G5	......	− .28
78	621	18 0 6	41.2	7.81	.03	2	7.62	.06	3	+ .19	F±	......	.00
s86	269	4 33	36.8	4.25	.04	6	4.41	.02	17	− .16	B9	+ .09	− .01
s75	655	6 31	46.6	6.91	.01	5	6.98	.03	3	− .07	A1	......	− .03
85	294	7 11	40.9	7.84	.03	6	7.59	.02	4	+ .25	A7	− .13	+ .04
79	570	7 31	59.2	5.39	.02	8	5.58	.16	3	......	F±	......	+ .44
79	571	7 37	59.4				5.79	.04	2	......		......	− .11
s86	272	18 7 48	59.6	5.91	.03	4	5.73	.01	18	+ .18	A0	+ .01	+ .08
76	678	9 58	6.7	8.12	.03	4	7.66	.05	2	+ .46	F±	......	− .19
77	681	12 24	34.1	8.32	.10	2	7.41	.04	4	+ .91	G±	......	− .15
74	757	14 53	18.1	>8.7−	...	2	7.84	.03	2	......	..	......	− .08

81	622	18	22	26	26.0	8.75	.02	3	7.18	.02	4	+1.57	K±	......	− .14
s82	540		22	56	53.9	7.25	.01	6	7.45	.05	3	− .20	B8	− .15	− .15
84	412		24	34	37.1	7.71	.01	6	7.27	.03	4	+ .44	F2	− .07	− .09
*86	275		26	18	33.4	8.81	.03	3	7.65	.01	4	+1.16	G5	......	......
77	696		29	2	29.7	8.03	.01	3	6.98	.04	5	+1.05	G5	......	− .06
79	587	18	30	37	9.2	7.79	.11	3	6.45	.05	4	+1.34	K0	......	− .02
79	590*a*		33	4	33.3	7.58	.09	4	7.20	.04	3	+ .38	A7	......	− .08
77	699		34	35	28.1	6.92	.04	6	5.59	.09	5	+1.33	K2	......	− .07
83	535		36	27	17.8	8.33	.09	4	7.20	.07	3	+1.13	G3	− .34	− .04
s83	536		37	22	6.1	6.14	.02	8	6.17	.02	4	− .03	A0	.00	− .02
73	831	18	37	57	11.5	8.28	.02	2	7.36	.02	3	+ .92	G±	......	− .04
77	702		42	17	35.4	7.25	.06	5	6.87	.04	5	+ .38	A7	......	− .03
s75	678		44	37	11.6	7.73	.04	4	7.46	.02	3	+ .27	A6±	......	+ .04
86	282		47	45	34.8	8.29	.01	5	6.60	.04	3	+1.69	M	− .37	− .24
73	835		48	16	58.2	6.21	.02	6	5.44	.05	2	+ .77	G±	......	− .26
75	682	18	49	36	19.0	5.28	.01	7	5.49	.02	3	− .21	B8	......	− .12
74	792		49	45	36.4	7.94	.02	4	7.18	.01	2	+ .76	G±	......	− .10
s79	604		52	41	49.3	6.71	.05	8	6.39	.06	4	+ .32	A4	......	− .07
78	660		55	43	42.5	8.53	.06	3	7.99	.08	4	+ .54	F±	......	− .03
78	661		56	8	50.2	8.22	.07	3	7.84	.05	3	+ .38	A6	......	− .11
75‡	683	18	56	54	39.2	6.24	.02	5	6.39	.01	2	− .15	A0	......	......
s73	845	19	1	2	59.5	7.31	.02	4	7.09	.01	2	+ .22	A5	......	− .08
76	712		2	10	54.5	7.03	.04	7	6.65	.00	2	+ .38	A9	......	− .21
s83	547		4	1	46.2	6.92	.02	9	6.83	.04	5	+ .09	A2	− .11	+ .03
s82	572		4	41	13.6	6.85	.03	5	6.88	.02	4	− .03	B9	+ .02	+ .06
80	604		6	4	17.9	8.97	.06	2	7.90	.06	4	+1.07	G±	......	− .02
77	715	19	6	26	31.4	9.13	.08	2	7.54	.03	4	+1.59	K±	......	+ .04
79	614		11	23	29.1	8.80	.06	3	7.96	.01	2	+ .84	F–G±	......	− .06
76	717		12	50	23.7	5.29	.04	8	5.43	.04	2	− .14	A5	......	− .43
80	607		14	22	33.6	8.35	.05	4	7.28	.06	4	+1.07	G0	− .66	− .03
s80	609		15	32	34.9	7.77	.01	4	7.57	.04	3	+ .20	A2	+ .14	+ .07

‡ Σ 2452, dist. = 5″.6.

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
73°	857	19	17	29	10′.2	5.84	.02	7	4.92	.11	2	+ .92	Ko	......	− .78
s73	860		20	51	21.5	7.09	.05	7	7.04	.02	2	+ .05	A5	......	− .11
88	112		22	30	59.3	8.09	.03	7	6.23	.02	12	+1.86	M	− .30	− .04
76	732		22	36	36.0	8.55	.02	5	7.30	.01	2	+1.25	G5	......	− .21
76	734		25	7	21.7	9.06	.03	3	6.34	.03	2	+2.72	N	......	− .38
s79	628	19	27	45	24.2	6.05	.01	4	6.17	.03	3	− .12	A2	......	− .04
s83	552		27	57	16.1	6.60	.02	7	6.43	.03	4	+ .17	A2	− .07	+ .07
73	863		28	44	9.4	7.87	.06	5	7.68	.01	2	+ .19	A5±	......	− .12
81	666		28	46	35.8	9.02	.17	2	7.62	.03	4	+1.40	K±	......	+ .12
77	734		34	8	2.9	7.44	.04	6	7.36	.05	4	......	A6±	......	+ .36
									7.99	...	1	......		......	+ .36
74	831	19	37	16	9.0	8.11	.06	4	6.96	.04	2	+1.15	G5	......	− .17
79	638		39	59	56.0	8.75	.02	2	8.46	.06	4	+ .29	F±	......	− .18
79	645		49	15	17.4	7.77	.05	3	7.96	.04	4	− .19	Ao	......	− .18
79	648		51	42	12.4	8.38	.02	4	7.32	.06	4	+1.06	G+	......	− .12
78	694		55	35	21.6	9.1±	...	3	7.62	.02	5	+1.5±	..	......	− .16
75	718		59	23	26.4	8.72	.15	2	7.22	.02	3	+1.50	K5±	......	− .14
76	771	20	2	25	12.2	7.95	.04	7	6.17	.07	3	+1.78	K5	......	− .12
73	897		5	51	37.2	7.77	.02	4	6.82	.00	2	+ .95	G2	......	+ .04
79	660		7	37	23.5	7.63	.04	5	6.76	.02	5	+ .87	Go	......	− .21
s77	764		12	16	24.6	4.22	.03	4	4.63	.04	7	− .41	B5	......	− .28
85	340		13	38	28.3	9.7±	...	2	7.72	.06	3	+2.0±	K+	......	− .20
s84	451	20	13	59	22.6	6.96	.03	6	6.79	.06	4	+ .17	A2	− .09	.00
81	699		15	33	54.8	7.83	.05	3	7.35	.05	2	+ .48	Fp	+ .05	+ .05
79	668		18	11	20.5	8.24	.03	3	6.81	.04	4	+1.43	K±	......	− .22
s80	650		20	9	13.1	6.69	.01	9	6.80	.03	5	− .11	B9	+ .04	+ .03
84	462		24	28	13.7	7.47	.01	5	7.19	.06	4	+ .28	A8	+ .01	− .15

84	463	20	24	33	48.7	7.62	.02	4	7.03	.05	4	+ .59	F5	− .05	− .04
75	739		24	50	43.0	9.15	.05	2	7.68	.03	3	+1.47	G–K±	......	− .01
81	*706*		28	43	2.5	7.43	.02	5	6.89	.05	2	+ .54	F2	− .01	+ .03
79	675		30	34	53.1	7.40	.03	7	7.54	.05	4	− .14	Ao	......	− .06
s74	872		32	50	36.7	5.24	.06	5	5.12	.11	4	+ .12	A±	......	− .06
80	*657*	20	33	9	5.7	7.59	.04	6	6.49	.01	4	+1.10	G4	+ .07	− .04
82	617		34	23	49.7	7.72	.01	4	6.67	.06	3	+1.05	G2	− .08	− .10
80	*659*		34	32	4.8	6.53	.01	10	5.48	.04	4	+1.05	G4	− .07	− .38
80	660		35	15	44.5	6.97	.02	7	5.97	.05	4	+1.00	G2	− .03	+ .01
75	752		38	39	13.7	8.35	.08	3	7.34	.03	4	+1.01	G+	......	+ .08
s83	588	20	39	5	16.8	6.37	.01	6	6.21	.02	5	+ .16	A2	− .10	+ .06
78	*716*		39	57	4.6	6.65	.02	8	6.97	.04	4	− .32	B2	......	+ .04
s76	809		40	10	28.5	7.38	.01	9	7.03	.02	4	+ .35	F±	......	+ .06
81	712		41	35	39.3	8.11	.02	5	6.92	.03	4	+1.19	K±	......	− .08
s*81*	*718*		49	51	9.7	5.78	.02	10	5.81	.05	4	− .03	B8	− .07	+ .07
80	672	20	52	8	10.6	6.67	.02	10	5.49	.03	3	+1.18	G8	− .09	− .37
74	890		52	34	15.6	7.29	.04	4	7.17	.02	3	+ .12	A2±	......	+ .01
75	764		55	55	32.3	6.98	.02	3	5.91	.02	4	+1.07	G2	......	+ .12
75	765		57	13	19.8	8.03	.02	2	6.62	.02	4	+1.41	K5	......	+ .16
73	922	21	0	4	53.5	>8.9±	...	2	8.01	.03	3	>+ .9 ±	G±	......	− .17
s77	800		7	30	43.2	5.83	.04	10	6.03	.04	6	− .20	Ao	......	− .13
80	679		8	6	45.4	7.17	.02	8	6.92	.03	6	+ .25	A6	− .03	− .05
74	907		10	6	49.1	7.44	.01	2	6.80	.02	3	+ .64	F5	......	− .08
80	682		11	4	36.7	8.35	.06	3	6.99	.05	5	+1.36	G–K±	......	− .15
78	742	21	11	22	15.1	7.44	.02	5	7.46	.02	3	− .02	Ao	......	− .02
s75	778		12	57	53.7	6.85	.01	6	7.00	.02	3	− .15	B8	......	− .04
78	744		13	49	33.6	7.04	.01	5	7.17	.04	3	− .13	B5	......	− .19
80	688		16	45	23.2	7.37	.05	6	7.36	.02	3	+ .01	A1	......	− .12
76	833		16	48	35.5	7.63	.02	7	5.92	.03	3	+1.71	K5–M	......	− .15

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900		Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.	Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h m s								M		M	M
s80°	690	21 17 31	48′.7	6.21	.01	10	6.05	.03	6	+ .16	A2	− .06	+ .07
77	*811*	17 33	10.6	7.13	.04	7	7.43	.03	3	− .30	A0	......	+ .03
s86	319	19 35	37.4	7.41	.02	5	7.43	.05	4	− .02	A2	− .12	− .05
83	603	21 31	50.3	8.10	.03	5	6.80	.04	3	+1.30	K±	− .41	− .06
81	*737*	22 59	5.2	8.24	.04	5	7.80	.07	3	+ .44	F±	......	− .01
s76	836	21 23 18	39.6	6.62	.02	14	6.55	.03	4	+ .07	A0	......	− .05
75	*787*	23 27	7.2	7.43	.00	3	6.79	.03	4	+ .64	F5	......	− .05
75	788	24 46	32.3	8.35	.06	3	7.19	.02	4	+1.16	G5	......	− .15
79	*707*	27 46	5.4	6.79	.03	7	6.01	.04	3	+ .78	G1	......	− .03
75	791	28 54	57.9	7.97	.05	4	7.66	.11	4	+ .31	A7	......	− .21
77	823	21 29 55	29.7	8.33	.05	6	7.19	.03	5	+1.14	G5	......	− .08
s*72*	*991*	30 3	2.1	8.54	.02	2	8.03	.06	3	+ .51	A–F±	......	− .19
s74	926	37 9	46.2	8.13	.03	2	7.92	.01	3	+ .21	A2	......	+ .04
s77	834	46 33	46.1	7.02	.01	9	7.02	.02	3	+ .00	A0	......	+ .08
s83	618	50 21	34.2	7.35	.06	3	7.08	.02	5	+ .27	A4	− .23	− .04
77	836	21 50 57	17.9	8.7±	...	2	7.77	.07	4	+ .9±	G±	......	− .09
s*72*	*1003*	51 37	13.7	6.68	.01	4	6.73	.05	2	− .05	A0	......	− .17
78	*768*	53 19	4.7	7.95	.03	4	6.46	.02	2	+1.49	K5	......	− .10
79	721	55 54	50.0	7.88	.02	5	6.39	.02	3	+1.49	M	......	− .11
74	946	56 54	31.1	8.05	.03	2	6.47	.07	2	+1.58	K5	......	− .33
75	808	21 57 20	36.6	8.66	...	1	7.75	.09	3	+ .91	G±	......	+ .05

82	673	22	1	52	23.3	7.04	.02	6	7.02	.02	3	......	F5	......	− .11
82	674		1	58	23.4				7.39	.09	3			......	− .15
75	818		11	28	57.9	8.34	.03	3	6.88	.08	3	+1.46	G–K±	......	− .18
s75	820		17	8	59.1	6.50	.02	6	6.62	.08	2	− .12	B7	......	− .02
s75	822		17	43	30.6	7.62	.07	2	7.42	.01	2	+ .02	A1	......	+ .02
s83	*630*	22	20	54	0.1	7.50	.02	2	7.46	.02	4	+ .04	A1	− .09	+ .02
s85	383		21	18	36.3	5.06	.04	9	5.24	.02	20	− .18	B8	+ .08	− .15
85	384		21	41	43.2	7.87	.01	4	6.49	.02	8	+1.38	K0	− .26	− .05
s77	860		22	52	44.1	6.65	.01	8	6.68	.07	4	− .03	B8	......	+ .10
81	775		23	42	25.7	7.43	.03	4	6.90	.04	3	+ .53	F3	......	+ .12
s76	859	22	23	59	55.4	7.49	.02	5	7.36	.07	2	+ .13	A5	......	− .14
s87	205		24	16	34.4	7.48	.02	7	7.43	.06	6	+ .05	A2	− .11	− .17
78	796		25	57	16.6	6.02	.01	5	5.63	.07	2	+ .39	A2	......	+ .21
79	*739*		26	8	11.4	7.75	.01	4	6.56	.07	3	+1.19	G6	......	− .06
84	509		27	29	33.2	8.08	.01	4	7.16	.07	3	+ .92	G2	− .21	− .14
78	801	22	29	0	18.7	5.58	.03	5	5.31	.00	2	+ .27	A2	......	− .01
s75	836		30	31	42.7	5.58	.03	6	5.71	.03	2	− .13	B8	......	− .11
73	982		30	37	33.4	8.36	.02	2	6.80	.04	2	+1.56	K5	......	− .04
72	*1049*		33	17	7.4	5.43	.04	4	5.15	.02	2	+ .28	F5	......	− .13
74	978		35	4	51.1	7.55	.04	3	5.72	.06	4	+1.83	K5+	......	− .10
80	731	22	39	12	52.2	7.50	.02	4	6.72	.05	3	+ .78	F5	− .05	+ .02
76	*870*		39	48	5.5	7.46	.01	4	7.22	.02	4	+ .24	A4	......	− .02
73	989		40	4	48.5	8.27	.06	2	7.54	.02	3	+ .73	F5±	......	+ .16
73	991		41	51	54.1	>8.7±	...	3	7.54	.02	3	>1.2±	G5±	......	+ .04
s77	871		42	15	59.5	7.17	.03	4	7.31	.08	3	− .14	B8	......	− .03
73	*994*	22	43	4	1.7	8.53	...	3	7.20	.06	3	+1.33	K±	......	− .02
79	750		43	10	54.6	7.88	.03	4	7.06	.00	2	+ .82	G0	......	− .16
82	703		47	53	37.4	6.12	.02	7	4.62	.06	4	+1.50	K2	− .02	− .09
79	756		48	25	50.2	7.93	.05	5	6.98	.04	2	+ .95	G1	......	− .11
84	513		50	3	14.7	8.11	.03	4	7.01	.05	4	+1.10	G5	− .26	− .07

CATALOGUE OF ZONE +73° TO +90°—*Continued*

B.D.		Place 1900				Photographic			Visual			Color-Index	Spectrum	Differences	
		R.A.			Dec.	Mag.	p.e.	No. pl.	Mag.	p.e.	No. pl.			S−Pa	MK−Pa
		h	m	s								M		M	M
73°	*998*	22	51	18	4′.8	7.86	.01	2	7.28	.02	3	+ .58	F–G	......	+ .17
84	517		53	29	50.3	7.25	.02	5	5.74	.04	5	+1.51	K4	+ .02	+ .01
77	879		54	37	57.7	8.25	.10	2	7.53	.04	3	+ .72	G±	......	+ .35
83	640		55	12	48.7	6.19	.02	6	4.57	.07	4	+1.62	K6	+ .03	− .09
79	759		57	24	48.4	8.25	.05	3	7.22	.08	3	+1.03	G5	......	− .34
75	867	22	58	11	35.1	8.28	...	1	7.32	.00	2	+ .96	F–G±	......	− .02
s 76	892		58	25	19.9	7.94	.03	3	7.70	.03	2	+ .24	A+	......	+ .10
79	*761*		59	32	14.6	6.91	.02	7	6.69	.02	3	+ .22	A5	− .13	− .07
s 74	*1002*	23	3	11	2.4	6.64	.03	3	6.62	.04	3	+ .02	A0	......	+ .05
74	1006		4	43	50.8	5.11	.02	4	4.36	.04	3	+ .75	F8	......	− .10
79	*769*		5	29	1.7	7.34	.05	4	7.22	.06	3	+ .12	A3	− .13	− .12
s 73	1023		11	4	41.1	5.75	.01	6	5.78	.02	3	− .03	A0	......	+ .10
78	825		12	46	41.2	7.73	.09	2	8.04	.10	2	− .31	B8±	......	− .24
77	896	23	13	9	36.3	7.61	.02	4	7.35	.03	4	+ .26	A4	......	− .01
s 74	1016		13	47	45.2	6.32	.03	6	6.19	.03	3	+ .13	A0	......	+ .07
72	*1096*		14	17	8.5	8.35	.04	2	7.05	.01	2	+1.30	G–K±	......	− .06
73	1035		23	43	34.1	7.49	.03	4	7.10	.08	3	+ .39	F1	......	− .02
78	*835*		23	57	14.6	7.75	.03	2	7.57	.08	2	+ .18	A2	......	+ .10
85	399	23	24	24	52.1	6.79	.02	5	6.50	.05	5	+ .29	A6	+ .08?	+ .04
s 74	1022		25	2	40.5	6.56	.01	6	6.59	.02	4	− .03	A0	......	− .03
s 77	908		26	50	20.5	7.04	.01	7	7.10	.02	5	− .06	A0	......	− .09
s 86	344		27	48	45.4	5.62	.03	6	5.54	.01	24	+ .08	A4	+ .08	− .02
s 77	909		27	50	16.0	7.06	.02	7	7.03	.01	4	+ .03	A0	......	− .09

82	728	23 33 17	39.0	8.44	.05	2	7.35	.03	3	+1.09	K±	......	− .06
s74	1032	34 54	44.3	5.95	.04	4	5.79	.07	3	+ .16	A3	......	+ .08
73	1047	35 0	26.9	6.94	.05	3	5.91	.02	3	+1.03	G2	......	− .03
76	*928*	35 15	4.4	4.28	.03	4	3.56	.08	5	+ .72	G0	......	− .41
76	*934*	47 8	2.8	6.89	.02	5	6.56	.03	5	+ .33	F1	......	− .10
74	1047	23 47 30	59.2	7.29	.01	2	6.33	.05	3	+ .96	G2	......	− .03
s73	1063	49 58	51.2	6.57	.02	4	6.47	.03	3	+ .10	A0	......	+ .02
74	1051	50 24	19.0	8.63	...	1	7.62	.02	2	+1.01±	G±	......	− .18
73	*1068*	54 10	15.0	8.10	.03	3	7.80	.05	2	+ .30	A8	......	− .05
s*72*	*1135*	56 32	3.3	6.66	.02	3	6.40	.00	2	+ .26	A±	......	+ .07
s82	748	23 57 34	25.0	7.27	.03	2	7.16	.01	3	+ .11	A0	− .05	− .04
79	799	57 36	43.5	7.91	.01	5	7.82	.08	3	+ .09	F0	......	− .31

DISCUSSION OF CATALOGUE RESULTS

Probable error and number of plates.—The photographic images occurred on the average on 4.7 plates. In this reckoning the short duplicate exposures are counted as separate plates. The "visual" images averaged 3.4 plates, there being no duplicate exposures. The probable errors of the catalogue mean of the magnitude results are given in detail in the two following tables.

TABLE II

PROBABLE ERRORS CLASSIFIED BY NUMBER OF PLATES

No. of Plates	Photographic		"Visual"	
	No. of Stars	Probable Error	No. of Stars	Probable Error
2	89	±0.046	177	±0.031
3	116	±0.042	241	±0.041
4	131	±0.033	146	±0.040
5	79	±0.029	49	±0.036
6	90	±0.024	14	±0.035
7	44	±0.024	3	±0.033
8	34	±0.022	3	±0.020
9	11	±0.016	3	±0.037
10	16	±0.018	2	±0.030
11	4	±0.010	..	....
12	...	....	1	±0.020
13	1	±0.010	1	±0.030
14	1	±0.020	..	....

TABLE III

PROBABLE ERRORS CLASSIFIED BY MAGNITUDES

Limiting Magnitudes	Photographic			"Visual"		
	No. of Stars	No. of Plates	Probable Error	No. of Stars	No. of Plates	Probable Error
Brighter than 6.0	33	5.2	±0.035	58	3.5	±0.039
6.00 to 6.49	28	6.4	±0.020	61	4.0	±0.033
6.50 to 6.99	66	6.6	±0.021	104	3.6	±0.037
7.00 to 7.24	30	6.2	±0.024	87	3.3	±0.037
7.25 to 7.49	54	5.5	±0.023	107	3.4	±0.038
7.50 to 7.74	60	4.6	±0.024	73	3.2	±0.032
7.75 to 7.99	74	4.7	±0.030	61	3.1	±0.040
Fainter than 8.0	256	3.7	±0.042	44	3.1	±0.047
Mean of all	...	4.7	±0.032	...	3.4	±0.036

For the photographic magnitudes the stars brighter than 6.0 and fainter than 8.0 have large probable errors. Excepting these two groups the mean probable error is ±0.024. For the "visual" magnitudes the dependence on brightness is not so evident. Making due allowance for the occurrence of the "visual" images on a less number of plates, the precision of the two methods is about equal.

RELATION BETWEEN SPECTRUM AND COLOR-INDEX

In Table IV the stars are arranged according to spectral type and the mean color-index of each group is taken. Fig. 11 is a graphic representation of the same data. It will be seen that a straight line fits the platted points as well as any simple curve. True, there is a systematic clustering of the points below the curve between B8 and A8, above between A8 and G, and below between

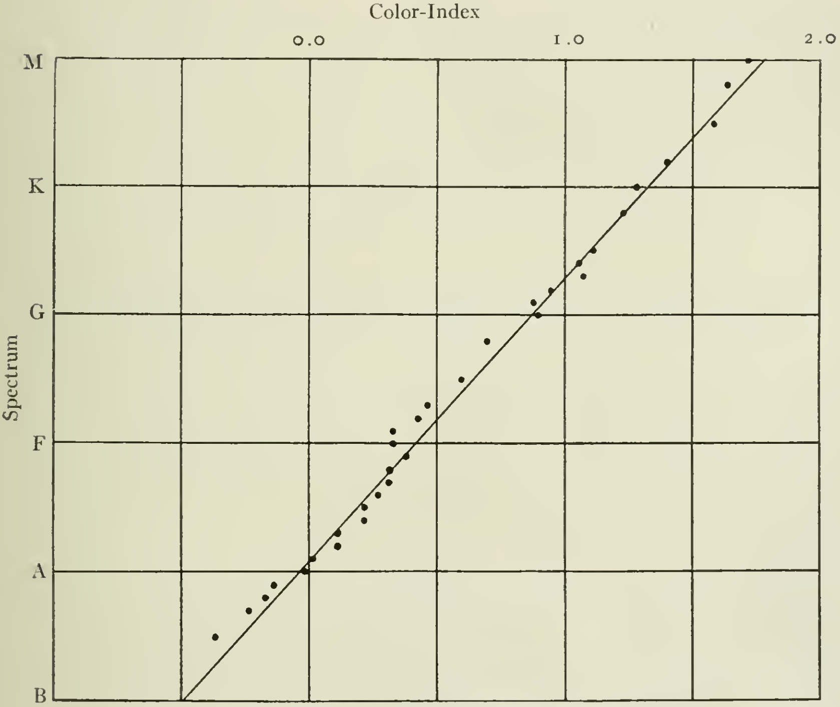

FIG. 11.—Relation between spectrum and color-index

G and K, but the divergence is slight and the data too meager to warrant the use of a complex curve to represent the relation.

TABLE IV

Limits	No. Stars	Mean Spectrum	Mean Color-Index	Limits	No. Stars	Mean Spectrum	Mean Color-Index
B2 to B6...	5	B5	−0.36	B2 to B6...	11	F2	+0.43
	3	B7	−0.23		4	F3	+0.46
	12	B8	−0.16	F4 to F6	24	F5	+0.60
	10	B9	−0.14	F8 to F9	6	F8	+0.70
					22	Go	+0.90
	55	Ao	−0.01		11	G1	+0.88
	22	A1	+0.02		25	G2	+0.95
	30	A2	+0.12		13	G3	+1.08
	23	A3	+0.12		9	G4	+1.06
	17	A4	+0.22		25	G5	+1.12
	20	A5	+0.22	G6 to G9	7	G8	+1.24
	12	A6	+0.27		16	Ko	+1.29
	7	A7	+0.31	K1 to K3	13	K2	+1.40
	9	A8	+0.31	K4 to K6	32	K5	+1.59
	1	A9	+0.39		1	K8	+1.64
	20	Fo	+0.33		5	M	+1.73
	9	F1	+0.33		1	N	+2.74

There are 492 stars used in Table IV. Their average magnitude is 7.3 visual = 7.7 photographic. Table V shows a comparison between ten points from the curve in Fig. 11 and the corresponding points from King's curve from 153 bright stars, given in *Harvard Annals*, **59**, 180; also from Schwarzschild's curve from his "Aktino-

TABLE V

SPECTRUM	COLOR-INDEX		
	Parkhurst	King	Schwarzschild
	M	M	M
B5	−0.28	−0.17	−0.50
Ao	−0.06	0.00	−0.30
A5	+0.18	+0.18	−0.10
Fo	+0.40	+0.32	+0.10
F5	+0.63	+0.52	+0.30
Go	+0.86	+0.71	+0.54
G5	+1.08	+0.90	+0.80
Ko	+1.32	+1.16	+1.05
K5	+1.55	+1.42	+1.50
M	+1.77	+1.67	......

metrie" stars in the *Harvard Revised Photometry*, given in Part B, p. 19.

It will be noticed that in the range of color-index included between the extremes of spectra, the present work lies between the other two. The difference of about 10 per cent remaining between King's values and mine seems to be due mainly to the estimates of spectral type. The larger differences with Schwarzschild depend somewhat on the earlier Draper Catalogue estimates of spectra, and are therefore more uncertain.

COMPARISON WITH SCHWARZSCHILD'S PHOTOGRAPHIC MAGNITUDES

There are 151 stars in common with the list of polar stars in the *Aktinometrie*, B, p. 42. These were arranged in four groups according to magnitude, and each group in four or five divisions according to color-index. Table VI gives the differences in the photographic magnitudes in the sense Parkhurst minus Schwarzschild.

TABLE VI

Mean Mag.	Division 1			Division 2			Division 3			Division 4			Division 5		
	Color	Δ Mag.	No.	Color	Δ Mag.	No.	Color	Δ Mag.	No.	Color	Δ Mag.	No.	Color	Δ Mag.	No.
6.50	−.05	+.03	10	+.15	+.01	10	+ .33	+.01	11	+1.23	−.03	..	....	.....	..
7.35	−.03	+.05	10	+.16	+.04	9	+ .33	+.09	10	+ .79	+.05	5	1.30	+.01	5
7.78	+.07	+.05	9	+.41	+.04	10	+1.05	+.08	8	+1.46	+.14	7	....	.....	..
8.16	+.37	+.06	7	+.59	+.14	9	+1.12	+.23	9	+1.58	+.32	9	....	.....	..

Fig. 12 is a graphic representation of the same data showing clearly the systematic differences. In the two brighter groups but slight differences appear. In group 3, mean magnitude 7.78, the agreement is good for the white stars, but for the colored stars the differences become evident, though not large. The faintest group contains only colored stars, on account of the method of selection of the observing list. In this group the differences increase rapidly with the color. It should be noted, however, that the exposures were not designed to give stars as faint as the last two divisions of the group, so that their magnitudes are somewhat uncertain. In response to an inquiry Professor Schwarzschild kindly informed me that the faintest stars (those that would give a difference with a negative sign) were not measured at Göttingen

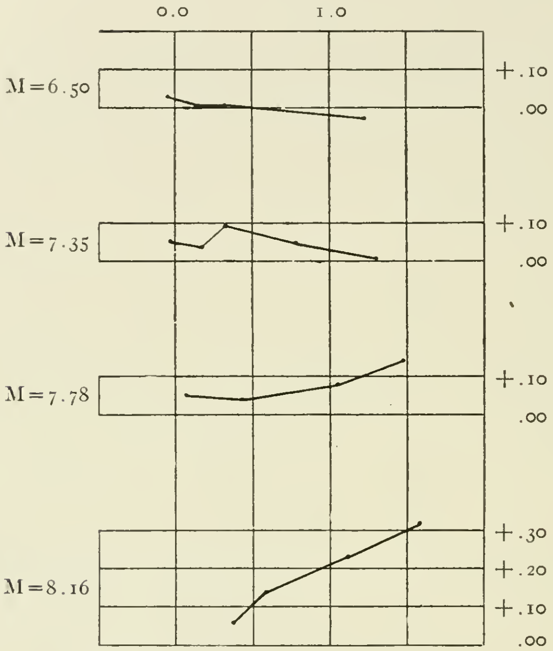

FIG. 12.—Comparison with Schwarzschild

on account of their faintness, thus leaving a systematic difference from the method of measurement, rather than from any peculiarity of the stars themselves.

COMPARISON WITH MÜLLER AND KEMPF'S VISUAL MAGNITUDES

The 598 stars in common with *Potsdam*, 17 were arranged in eight groups of four or five divisions each. Table VII gives the differences in the sense, P minus M&K.

The curves in Fig. 13, representing the above data show a very satisfactory agreement, indicating not only that the scale is nearly the same as that of Müller and Kempf, but that the sensitiveness to color of the plate behind the filter agrees closely with the eyes of the Potsdam observers. Here, also, the first and last exposures are entitled to less weight, as they include extremes in brightness. Omitting the extreme groups, the difference in color-perception introduces a difference in magnitude of 2 per cent. Including the extremes, it becomes 3 per cent. The sign of the difference

TABLE VII

Mean Mag.	Division 1			Division 2			Division 3			Division 4			Division 5		
	Color	ΔMag.	No.	Color	ΔMag.	No.	Color	ΔMag.	No.	Color	ΔMag.	No.	Color	ΔMag.	No.
5.47	−.23	+.09	11	+.12	+.08	12	+ .56	+.05	12	+1.16	+.09	12	+1.64	.00	12
6.30	−.03	+.01	13	+.25	−.05	13	+ .73	−.02	11	+1.11	−.05	12	+1.61	−.04	14
6.75	−.04	+.01	20	+.17	+.01	20	+ .43	−.03	16	+1.03	−.01	19	+1.50	−.01	24
7.11	+.07	−.02	20	+.41	−.02	15	+ .66	.00	16	+1.26	.00	20	+1.66	.00	14
7.38	−.01	+.01	20	+.24	.00	25	+ .57	.00	20	+1.05	.00	26	+1.44	−.02	21
7.63	−.02	+.03	15	+.23	+.02	16	+ .46	.00	15	+ .95	−.04	16	+1.30	−.03	14
7.85	+.03	+.02	15	+.21	−.04	13	+ .44	+.03	13	+ .75	+.02	10	+1.26	−.08	9
8.21	−.06	+.09	10	+.27	+.07	12	+ .52	+.01	10	+ .90	−.05	12	....	......	..

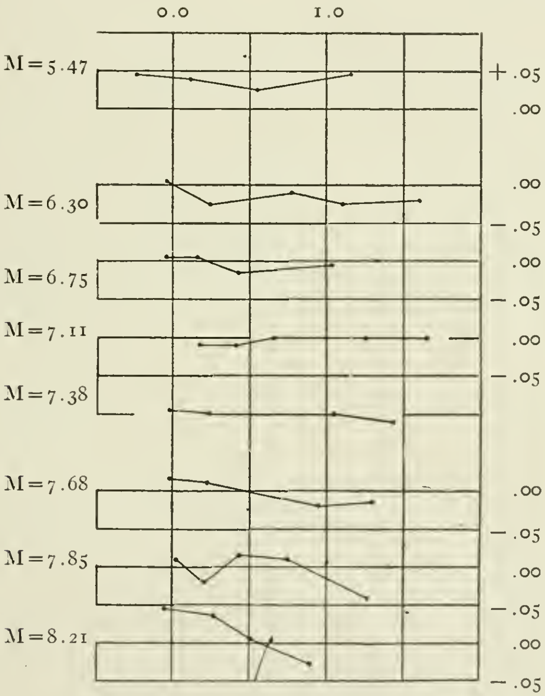

Fig. 13.—Comparison with Müller and Kempf

is such that if the eyes of the Potsdam observers are taken as the standard, the color-correction of the plate and filter is overdone, so that the faint colored stars appear too bright on the plate.

COMPARISON WITH MÜLLER AND KEMPF'S COLOR-ESTIMATES

The same grouping of stars used for Müller and Kempf's visual magnitudes serves for the comparison with their estimates of color. In Fig. 14 the Potsdam color-steps are platted as uniform increments in abscissas, while the ordinates are values of the color-index. The points representing the mean values for each magnitude-group are connected by lines, with the mean magnitude at its upper end. For the white stars at the lower left, the lines of the difference groups lie close together, but for the colored stars at the upper right, the lines diverge. The line representing the brightest group, mean magnitude 5.47, lies well to the right of the rest. As the stars become fainter the lines move to the left. This is a demonstration of the law of physiological optics, that the apparent maximum of intensity shifts toward the shorter wave-lengths as the light is reduced in brightness; in other words, a given color is less evident in a faint star than in a bright one.

The massing of the platted points, near the GW (yellowish-white) line, seems to indicate a preference of the Potsdam observers for that color.

COMPARISON WITH THE HARVARD VISUAL MAGNITUDES

The tables given by Müller and Kempf[1] furnish the best means of comparing the magnitudes obtained with the "visual-luminosity" filter with the visual magnitudes found at Harvard. Applying these tabular differences to the quantities given in Table VII, we have the values in Table VIII; where each magnitude group is divided according to values of the color-index into four or five divisions.

It follows from the above differences, which are in the sense Parkhurst minus Harvard: first, for the white stars the scale is very nearly the same; second, the colored stars are made a little

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[1] *Potsdam Photometric Durchmusterung*, p. xxxiv.

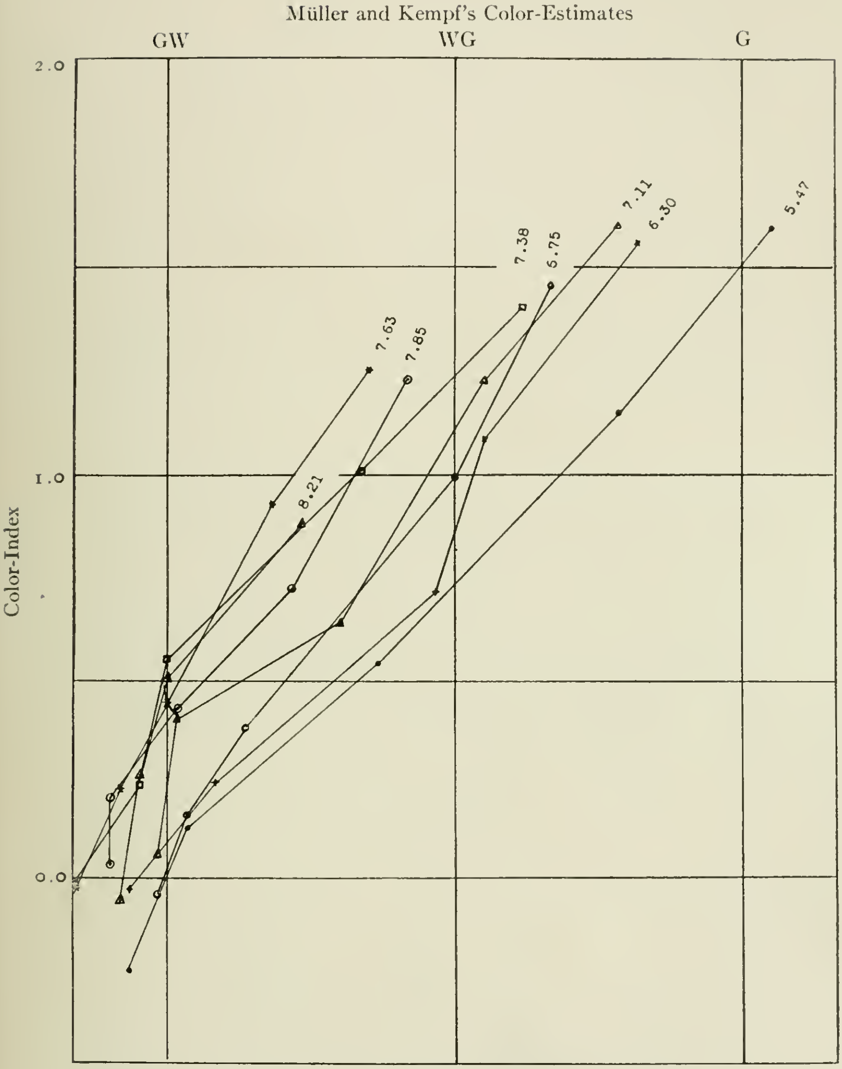

FIG. 14.—Comparison of color-index with Müller and Kempf's color-estimates

brighter than in the Harvard catalogues, for the extremes the differences being about one-quarter of a magnitude.

TABLE VIII

COMPARISON WITH THE HARVARD VISUAL MAGNITUDES

MEAN MAG.	DIVISION 1		DIVISION 2		DIVISION 3		DIVISION 4		DIVISION 5	
	Color	ΔMag.	Color	ΔMag.	Color	ΔMag.	Color	ΔMag.	Color	ΔMag.
5.47......	−0.23	+.08	+0.12	+.08	+0.56	−.09	+1.16	−.03	+1.64	−.25
6.30......	−0.03	+.02	+0.25	−.12	+0.73	−.15	+1.11	−.24	+1.61	−.35
6.75......	−0.04	.00	+0.17	.00	+0.43	−.10	+1.03	−.20	+1.50	−.24
7.11......	+0.07	−.02	+0.41	−.10	+0.66	−.14	+1.26	−.20	+1.66	−.28
7.38......	−0.01	+.06	+0.24	.00	+0.57	.00	+1.05	−.13	+1.66	−.24
7.63......	−0.02	+.06	+0.23	+.04	+0.46	+.01	+0.95	−.10	+1.30	−.16
7.85......	+0.03	+.05	+0.21	−.03	+0.44	+.04	+0.75	−.04	+1.26	−.24
8.21......	−0.06	+.13	+0.27	+.10	+0.52	+.03	+0.90	−.11	......	.....

COMPARISON WITH GREENWICH MAGNITUDES OF STARS NEAR THE NORTH POLE

The list given in *Monthly Notices*, **72**, 695, June 1912, contains 42 stars in common with this zone. The magnitudes there given are based directly on the Harvard Polar Sequence. The stars were arranged in seven groups of six stars each, and showed the following residuals, in the sense P−G.

Group	Mean Mag.	Δ Mag.
1....................	5.43	−0.02
2....................	6.57	+0.06
3....................	6.79	+0.15
4....................	7.19	+0.20
5....................	7.41	+0.28
6....................	7.63	+0.25
7....................	7.95	+0.18

The stars were too few in number to admit of forming subgroups according to color-index, but the residuals were evidently greater for stars of more intense color. This is clearly shown by averaging the values of the color-index for the stars whose residuals are respectively below and above the mean in each group.

Group	Average Color-Index	
	Below Mean	Above Mean
1	−0.17	+0.17
2	+0.14	+0.72
3	+0.32	+0.27
4	+0.18	+0.26
5	+0.78	+1.24
6	+0.56	+1.05
7	+0.85	+1.66
Mean	+0.43	+0.77

In all the groups except the third, a large residual corresponds to a large value of the color-index. To determine the numerical value of the difference due to color, a greater number of stars would be required.

Excluding *Polaris*, which is too bright for this work, there are eleven stars of the Harvard Polar Sequence in the above list. They range from 4.47 to 7.73 in magnitude. The above tables therefore furnish a rough comparison with the Harvard magnitude scale. A careful comparison is reserved for a subsequent investigation.

DISTRIBUTION OF SPECTRAL CLASSES

The spectra of 492 stars were clear enough on the plates to admit of good classification. Just half of them were included in the limits B2 to F0, and half between F1 and M. The numbers in the separate classes are as follows:

Class	No.	Percentage
B2 to B9	30	6
A0 to A9	196	40
F0 to F9	78	16
G0 to G9	112	23
K0 to K9	70	14
M	5	1
N	1	..

It would seem to be unnecessary to go into details in regard to the distribution with respect to the Galaxy. The difference in the number of white standard stars per field was, however, very noticeable. At the nearest approach to the Galaxy, 10° at right ascension 1 hour, the number of white stars per field reached 10 or 12; but at the greatest distance, 44° at 13 hours, the number of standards fell to 2 or 3.

SUMMARY

The main characteristics of the present work are as follows:

1. The determination, with the same instrument, of the three photometric elements, photographic magnitude, visual magnitude, and spectrum (and therefore the relation between spectrum and color-index).
2. The use of extra-focal images and an absolute scale for the photographic magnitudes.
3. The use of color-sensitive plates and a filter especially designed to give, with the particular plates used, magnitudes on the visual scale.
4. The statement of the spectral intensity curve of each kind of plate used, in the condition used; thus defining the meaning of the results obtained.
5. The correction of the photographic magnitudes for the color-curve of the lens.
6. The comparison with visual and photographic magnitudes found at Harvard, Potsdam, and Greenwich, showing systematic differences but a reasonable degree of agreement.

ACKNOWLEDGMENTS

My thanks are especially due to Dr. Anne S. Young, professor of astronomy in Mt. Holyoke College, who measured and reduced in duplicate all the focal plates in the 75° zone, determined part of the systematic corrections, and furnished expert advice and aid in many questions arising during the work; to Miss Eudora Magill, formerly of the computing staff of Columbia College Observatory, for careful and accurate work in measuring and reducing focal plates and arranging the catalogue; to the Trustees of the Rumford Fund of the American Academy, for aid in purchasing the Hartmann "Mikrophotometer"; and to the Trustees of the Bache Fund

of the National Academy, for a grant to pay for assistance in measuring and reducing plates.

Acknowledgments have already been partially made of the important part taken by Professor F. C. Jordan in devising and perfecting the "absolute scale," which forms the basis of so much of the work.

Yerkes Observatory
August 1912

THE ABSORPTION SPECTRUM OF TELLURIUM VAPOR AND THE EFFECT OF HIGH TEMPERATURE UPON IT

By E. J. EVANS

The absorption spectrum of tellurium vapor was first investigated by Gernez,[1] but, as far as the present author is aware, the effect of high temperatures on the spectrum has not been studied. Gernez heated tellurium in an atmosphere of carbon dioxide in a hard glass tube, and obtained a banded absorption spectrum extending from yellow to violet. The emission spectrum of the vapor at high temperatures was studied by Paterno and Mazzucchelli.[2] They found that the yellow tellurium vapor became green with increase of temperature, and finally blue at the highest temperatures employed. The emission spectrum showed the presence of a large number of bands.

The present research is a continuation of previous work carried out by Antonoff and the author[3] on the absorption spectrum of selenium vapor. The vapor-densities of several substances including selenium and tellurium were investigated by Deville and Troost.[4] In the case of selenium they found a diminution of vapor-density with increase of temperature, pointing to the conclusion that complex molecules were being dissociated into Se_2. They were unable in the case of tellurium to observe any change in the vapor-density with change of temperature, but this they regarded as being due to impure material. According to Deville and Troost the vapor-density of tellurium at temperatures above the boiling-point (1390° C.) is 130 ($H=1$). The molecule of tellurium at these high temperatures can be represented by Te_2. From analogy with sulphur and selenium there is reason to believe that there are complex molecules present in tellurium vapor especially at low temperatures. The boiling-point of tellurium in a cathode vacuum is about 500° C. according to Krafft,[1] and it would be interesting to determine the vapor-density at this comparatively low temperature. The present paper is mainly concerned with the changes produced in the absorption spectrum of the vapor by increase of temperature. These changes are probably connected with the dissociation of complex molecules into simpler ones. At present it is difficult to give a precise meaning to the results obtained, because our knowledge of the vapor-density of tellurium at different temperatures and pressures is so meager.

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[1] *Comptes rendus*, **74**, 1190, 1872.

[2] *Accad. Lincei Atto*, **17**, 428, 1908.

[3] *Astrophysical Journal*, **34**, 277, 1911.

[4] *Comptes rendus*, **56**, 891, 1863.

EXPERIMENTAL ARRANGEMENT

The experimental arrangement was similar to the one previously employed in the investigation of the absorption spectrum of iodine,[2] and consequently only a short description is necessary. The tellurium vapor was heated in a quartz tube placed inside an electric furnace, whose temperature was measured by a Pt Pt–Rh thermocouple. The quartz tube was also provided with a side tube of 3 or 4 mm bore in which the tellurium was placed, and after the apparatus had been evacuated to a pressure of $\frac{1}{10}$ mm or less, the side tube was sealed off in the oxyhydrogen flame. The pressure of the vapor could be varied by sending currents of different magnitudes through a nickel wire wound round the side tube. The temperature of the coldest part of the side tube was determined, as the vapor pressure inside the apparatus depended upon it. When the furnace and side tube had been adjusted to their respective temperatures, the light from the positive pole of the carbon arc was passed through the vapor and focused on the slit of a concave grating of 1 meter radius and 15,000 lines to the inch. The absorption spectrum was usually photographed with Wratten and Wainwright's panchromatic films. Two different specimens of tellurium were employed, and the spectra obtained under similar conditions were found to be identical. One of the specimens was known to have been carefully prepared from the pure oxide, but the other was suspected of containing a small quantity of an impurity (probably copper). Since the maximum temperature of the side tube in the present experiments was about 700° C., any metallic impurities such as copper—which has a high boiling-point—would not be present in sufficient quantity to produce an absorption spectrum. A careful examination of the photographs also showed that the tellurium did not contain an appreciable quantity of selenium as an impurity. This could be easily tested, as selenium is much more volatile than tellurium, and absorbs strongly in the violet and ultra-violet. In all experiments with tellurium vapor the quartz tube was evacuated to a pressure of $\frac{1}{10}$ mm or less, for tellurium combines readily with oxygen, forming a white oxide. This oxide melts at red heat, forming a yellow liquid, and at a still higher temperature boils. It was found that if the oxide was heated in a quartz tube for some time at a high temperature, a white incrustation was formed on the clear ends of the tube, preventing the transmission of the light from the electric arc to the slit of the grating spectroscope. The quartz tubes became porous after long-continued exposures to high temperatures, and consequently such tubes had to be frequently replaced by new ones. The absorption spectra were examined for the region λ 2400 to λ 7000, and for temperatures varying from about 500° C. to 1350° C. In one series of experiments the pressure was kept constant and the temperature varied, while in another series a known weight of tellurium was placed in a quartz tube and the absorption spectrum photographed at different temperatures.

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[1] *Berichte der Deutschen chemischen Gesellschaft*, **36**, 4344, 1903.

[2] *Astrophysical Journal*, **32**, 1, 1910.

EXPERIMENTAL RESULTS

THE DIOXIDE SPECTRUM

As tellurium dioxide might be present as an impurity in the experiments on the absorption spectrum of tellurium, a brief examination of the absorption spectrum of the former was made. A few grams of the dioxide were prepared by dissolving tellurium in nitric acid and evaporating to dryness. About 0.004 gm of the oxide was then sealed in a quartz tube and the absorption spectrum photographed at temperatures ranging from 700° C. to 1050° C. In the first series of experiments glass lenses were employed, and the absorption spectrum could be investigated from λ 3500 to λ 7000. The light from the positive pole of the arc after passing through the vapor contained in the tube at a temperature of 700° C. gave a continuous spectrum extending from λ 3500 to λ 7000, and no definite absorption bands could be detected. The temperature of the vapor was raised to 800° C., and the spectrum again photographed. The spectrum extended from λ 7000 to λ 3800, and there was complete absorption between the latter wave-length and λ 3500. The photograph also showed the presence of faint, ill-defined bands between λ 3880 and λ 4100. The temperature of the vapor was finally raised to 1050° C., and the spectrum of the transmitted light photographed. A continuous spectrum was obtained between λ 7000 and λ 4100, but no definite absorption bands could be detected in that region. There was complete absorption between λ 4100 and λ 3500. A number of photographs were also taken at various temperatures of the vapor in order to study the ultra-violet part of the spectrum. For this purpose the glass lenses were replaced by those of quartz. At 600° C. or 700° C. a continuous spectrum extending from λ 3885 to λ 2800 was obtained, and no absorption bands appeared on the photographs. Bands were, however, obtained when the temperature of the vapor was higher (900° C.). These bands, which are found between λ 3100 and λ 3400 are not due to tellurium vapor, although they may possibly be due to some impurity in the oxide. Experiments on the absorption spectrum of the oxide at high temperatures were rendered difficult by its action on the clear ends of the quartz tube.

GENERAL DESCRIPTION OF THE ABSORPTION SPECTRUM OF TELLURIUM VAPOR

A small quantity of tellurium was placed in a quartz tube, which was evacuated to a pressure of $\frac{1}{10}$ mm and the absorption spectrum studied for temperatures of the furnace ranging from about 500° C. to 1000° C. In the first series of experiments glass lenses were employed and the region of the spectrum that could be photographed was comprised between λ 3500 and λ 7000. When the temperature was about 500° C. faint absorption bands extending from λ 3900 to λ 4200 were obtained. No bands could be detected in the region of higher wave-length or in the region between λ 3900 and λ 3500. The temperature of the furnace was then raised to 700° C., and the absorption bands in the above region (λ 3900–λ 4200) increased in intensity and extended toward the red end of the spectrum. The absorption bands now extended from λ 3900 to λ 5200, and the photographs also showed the presence of continuous absorption between λ 3900 and λ 3500. The temperature was then successively raised to 800° C., 900° C., and 1000° C., and the absorption spectra, photographed, gave the following results. At 800° C. the general absorption in the ultra-violet increased, complete absorption was obtained between λ 3885 and λ 4400, and an increase in the intensity of the absorption bands between λ 4400 and λ 5400. When the temperature was 900° C. complete absorption was obtained in the region λ 5000–λ 3500, and the absorption bands increased in intensity toward the red end of the spectrum. In this case they could be recognized as far as λ 5700. At 1000° C. absorption bands could be detected as far as λ 6100, and this was the limit attained with the small amount of substance used in the present work. The glass lenses were now replaced by others made of quartz, so that the absorption spectrum could be studied in the ultra-violet (λ 4000–λ 2500).

When the temperature of the furnace was 700° C. a continuous spectrum extending from λ 3885 to λ 2500 and showing no absorption bands was obtained. There was, however, in this region a general absorption which decreased in intensity with diminishing wave-length. At a higher temperature (900° C.) there was complete absorption from λ 4000 to λ 3450, and no absorption bands were visible in the region λ 3450–λ 2600. Finally the temperature of the furnace was raised to 1100° C., and the absorption spectrum photographed in the visible part as well as in the ultra-violet. Absorption bands were obtained extending from λ 6000 to λ 5300, complete absorption from λ 5300 to λ 3400, and a continuous spectrum showing no absorption bands extending from λ 3400 to λ 2600.

EFFECT OF TEMPERATURE ON THE ABSORPTION SPECTRUM

CONSTANT MASS OF VAPOR IN TUBE

About 0.002 gm of tellurium was introduced into a quartz tube, which was evacuated to a pressure of a fraction of a millimeter and sealed. The tube was placed inside a carbon-tube furnace, and the absorption spectrum was examined visually at temperatures ranging from 1000° C. to 1200° C. The initial temperature was taken as high as 1000° C. to insure that all the tellurium in the tube was in a state of vapor. The visual observations showed scarcely any difference in the appearance of the absorption spectrum as the temperature was raised, and this was confirmed by the careful examination of photographs taken at 1000° C., 1250° C., and 1350° C. All the photographs were taken with the same exposure and showed the presence of absorption bands extending from λ 6000 to λ 5390, and increasing in intensity with diminishing wave-length. There was in each case complete absorption of all wave-lengths from λ 5390 to λ 3500—the limit of photography with the glass lenses employed. The absorption bands at 1000° C. and 1250° C. are shown in Plate X, Fig. 1. The two spectra overlap slightly.

CONSTANT-PRESSURE EXPERIMENTS

In these experiments the vapor pressure of the tellurium was kept constant while the absorption spectrum was photographed at a low and a high temperature. The actual value of the pressure was not determined, but the temperature of the coldest part of the apparatus was always observed. The vapor pressure was varied over a range corresponding to a change of temperature from about 500° C. to 750° C. The absorption bands in the region λ 3900–λ 4200 could be detected when the temperature of the coldest part of the apparatus was only 20° or 30° C. above the melting-point (450° C.) of tellurium, and as the normal boiling-point of tellurium is 1390° C. the vapor pressure required to detect the bands must be small. The experimental results for a few values of the vapor pressure will now be given.

Experiment I.—In this experiment the temperature of the coldest part of the apparatus was 540° C., and two photographs were taken with the same exposure, one at a temperature of 600° C. and the other at 1200° C. The absorption bands which could be easily detected at the low temperature were much less intense at the high temperature. Their diminution in intensity is shown in Plate X, Fig. 2. The carbon arc lines are also prominent in the photograph.

Experiment II.—The vapor pressure was greater than in the first experiment and corresponded with a temperature of 580° C. Two photographs were taken, one at a temperature of 700° C. and the other at 1100° C. The absorption bands in the region λ 3900 to λ 5000 were much reduced in intensity at the higher temperature. The result of this experiment is shown in Plate X, Fig. 3.

Experiment III.—The vapor pressure was again increased and corresponded with a temperature of 700° C. The absorption spectrum was photographed at 830° C. and 1100° C. At the lower temperature absorption bands were obtained in the region λ 5300 to λ 4500 and the light from the positive pole of the arc comprised between λ 4500 and λ 3500 was completely absorbed. The absorption bands at the higher temperature were not as intense as at the lower temperature, but they extended farther toward the violet (to λ 4300) before complete absorption set in.

Experiment IV.—In this case the temperature of the coldest part of the apparatus was not measured directly, but from the current passing through the nickel wire it was estimated to be about 670° C. Two photographs were taken at 800° C. and 1115° C., but the effective exposure at the higher temperature was slightly greater than at the lower temperature. The photograph taken at 800° C. showed the presence of absorption bands extending from λ 5300 to λ 4500; the intensity of the absorption increasing with diminishing wave-length. As in experiment III, there was complete absorption between λ 4500 and λ 3500. At the higher temperature the bands extended from λ 5300 to λ 3900, and the difference in the spectra was too great to be explained by the slight difference in exposure. The region comprised between λ 4800 and λ 6000 is shown in Plate X, Fig. 4.

EXPERIMENTS IN THE ULTRA-VIOLET

Although previous experiments indicated that tellurium vapor did not give any absorption bands in the ultra-violet, it was considered advisable to perform a few experiments at a high and a low temperature when the vapor pressure was kept constant. The temperature of the coldest part of the apparatus was kept constant at 530° C. and the ultra-violet portion of the spectrum photographed when the temperature of the vapor was 600° C. and also 1000° C. In each case no absorption bands could be detected in the region λ 3885–λ 2800. The same result was obtained when the vapor pressure was higher (corresponding to a temperature of 680° C.).

DISCUSSION OF RESULTS

The experimental results show that tellurium vapor gives a banded absorption extending from λ 6100 to λ 3900, and also a general selective absorption. It is possible that the absorption bands may extend still farther toward the red if greater vapor pressures are employed. Previous work on the absorption spectrum of selenium vapor shows that the bands extend over the region λ 5300 to λ 3200, and according to Graham[1] the absorption bands of sulphur vapor in the region λ 4775–λ 3985 are due to S_8 molecules and bands in the ultra-violet (λ 3415–λ 2600) are due to S_2 molecules. A comparison of the absorption spectra of these three substances, which belong to the same group in the periodic classification of the elements, shows that the banded absorption spectrum is shifted toward the red with increasing atomic weight. If the absorption spectra of tellurium dioxide and selenium dioxide are compared, it is found that while the latter gives well-defined bands in the violet and blue, the former does not give any bands in the visible spectrum. One would expect from analogy between the behavior of tellurium and selenium that tellurium dioxide would give bands in the green and yellow. The absorption spectra of tellurium and selenium, both in their appearance and behavior with change of temperature and pressure, show many points of resemblance. When tellurium is heated in a quartz tube absorption bands first make their appearance at λ 3900, and as the pressure increases the bands extend toward the red end of the spectrum. Another consequence of the increased pressure is that the region in which the bands appear at a lower pressure is completely absorbed at the higher pressure. The same general result is obtained in the case of selenium, the only difference being that the bands first make their appearance at λ 3200. The resemblance is again evident when the effect of temperature on the absorption spectra of the two vapors at constant pressure is considered. When the pressure of the tellurium vapor is small, absorption bands are obtained in the region λ 3900–λ 4500, and the intensities of these bands diminish with increase of temperature until they almost disappear at 1200° C. A similar result is obtained with selenium vapor; the absorption bands in the region λ 3200–λ 3800 diminish in intensity with increase of temperature and almost disappear at 1200° C. The disappearance of the absorption bands with increase of temperature can be explained on the hypothesis that complex molecules of tellurium are dissociating into simpler ones, and that the presence of the more complex molecules is necessary for the production of these bands. At present the nature of the complex molecule responsible for the absorption bands in the region λ 3900–λ 4500 is unknown; also, the number of bands belonging to a particular complex molecule is difficult to determine.

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[1] *Proceedings Royal Society*, A, 84, 311, 1910.

In experiment II (constant pressure) the bands between λ 3900 and λ 5000 are diminished in intensity at the higher temperature, and possibly all the bands between λ 3900 and λ 5300 would disappear at temperatures above 1500° C. It is interesting to consider experiment IV (constant pressure), paying special attention to the region λ 3900–λ 4500. At 800° C. there is complete absorption in that region, but at 1115° C. distinct absorption bands are obtained. The pressure of the vapor in this experiment is large compared with that employed in experiment I, and consequently the bands are not very faint even at 1100° C. It would require a much higher temperature to bring about the complete disappearance of the bands. The appearance of the high-temperature spectrum in the above region suggests that a general selective absorption is superimposed on the band absorption, and it is interesting to inquire into the probable origin of this general absorption. Koenigsberger,[1] as the result of investigations on the absorption spectra of several substances, comes to the conclusion that continuous selective absorption is due to the normal state of gas molecules, and that band spectrum absorption is due to an intermediary non-stationary condition. In the absorption spectrum of tellurium the two types of absorption are present, but the hypothesis cannot be proved or disproved by means of the experimental results. The band absorption is undoubtedly connected with dissociation, but from the results obtained, it is impossible to say whether the bands are produced by the molecules in their non-stationary condition or by molecules in their natural state. As to the origin of the general absorption several hypotheses are possible; the continuous absorption may be due to the more complex molecules in their normal state or due to the diatomic molecule. Also both types of molecules may produce general selective absorption in the same region of the spectrum. Further researches on the variation of the intensity of the general absorption with increase of temperature, when the vapor-density of tellurium over a wide range of temperature and pressure is known, may throw light on this question. In conclusion, the experiments on a constant mass of tellurium sealed in a quartz tube will be considered. The photographs taken at temperatures above 1000° C. seem to indicate that all (0.002 gm) the tellurium is in the state of vapor, and consequently the pressure of the vapor at any temperature above 1000° C. can be estimated by means of the equation $pv=R\theta$. On the assumption that all the molecules are diatomic the pressure at 1300° C. is found to be about 5 cm of mercury. The photographs of the absorption spectrum at 1000° C., 1200 °C., and 1350° C. show the presence of bands in the region λ 5300–λ 6000, and complete general absorption between λ 5300 and λ 3500. Further, the intensities of the absorption bands are not affected by a change of temperature from 1000° C. to 1350° C. It is unfortunate, however, that quartz tubes become porous and opaque at higher temperatures, making it impossible to obtain information which would have an important bearing on the question of the origin of the bands. The nature of the molecule responsible for these bands is left undetermined, but it is interesting to note that these bands are present when the vapor is at a high temperature, and probably almost completely diatomic.

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[1] *Astrophysical Journal*, **35**, 139, 1912.

SUMMARY OF RESULTS

1. The bands in the absorption spectrum of tellurium extend from λ 3900 to about λ 6000. In addition to a band spectrum the vapor gives a general selective absorption. The absorption

spectrum of tellurium is similar to that of selenium, but compared with the spectrum of the latter is displaced toward the red.

2. For small pressures absorption bands are first photographed in the extreme violet (λ 3900), and as the pressure increases, absorption bands are also photographed in regions of greater wave-length. At pressures sufficient to show the presence of bands in the region λ 5300–λ 6000, there is complete absorption in the violet and blue.

3. When the pressure of the vapor is low, the absorption bands in the region λ 3900–λ 4500 diminish in intensity with increase of temperature until they almost disappear at 1200° C. This result may be explained on the hypothesis that the absorption bands are due to complex molecules, which are present in tellurium vapor at low temperatures. These molecules are completely dissociated at high temperatures and hence the absorption bands are not visible.

4. The absorption spectra of a constant mass (0.002 gm) of tellurium vapor at 1000° C. and 1350° C. do not show any difference. Both spectra show the presence of bands between λ 5300 and λ 6000 and a continuous absorption from λ 5300 to λ 3500. The intensities of the bands are not affected by a change of temperature from 1000° C. to 1350° C. From the experimental results it is impossible to determine whether these bands are due to diatomic molecules or more complex ones.

I take this opportunity of thanking Professor Rutherford for the interest he has taken in the work and for placing the necessary facilities at my disposal.

Physics Laboratory
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May 1912

ON DARKENING AT THE LIMB IN ECLIPSING VARIABLES. I

By HENRY NORRIS RUSSELL and HARLOW SHAPLEY

1. *Solution for total eclipses of completely darkened stars.*—In the problem of determining the orbital elements of an eclipsing binary from the light-curve, the nearly universal custom has been to assume that the stars are without appreciable absorbing atmospheres and that the stellar disks, therefore, are uniformly bright from the center to the limb. The present accuracy of photometric observations has been considered insufficient to justify introducing into the theory the complications which a contrary hypothesis would admit; moreover, the few reliable orbits so far computed on the assumption of uniform disks have represented the observations very satisfactorily. The extreme unlikelihood, however, of uniform disks has been generally recognized, the analogy of the sun being definite evidence in the case of stars of advanced spectral type, and attempts have been made by Rödiger[1] and Blažko[2] to determine the elements of *β Persei* and *U Cephei*, respectively, on the assumption of a differential darkening on the stellar disks equal to that of the sun. These investigations were not prompted by the discrepancy between observations and the uniform-disk theory but rather by the reasonableness of the opposing assumption.

The general problem of darkening at the limb has already been discussed by one of us in his second article on the determination of the orbital elements of eclipsing variable stars,[3] and it is the purpose of the present paper to carry out the investigation there outlined, constructing tables to facilitate the solution of the orbits of darkened stars and discussing the errors which the occurrence of darkening at the limb may produce in elements computed on the assumption that it does not exist. The notation and fundamental equations are those of the papers just mentioned.

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[1] *Untersuchungen über das Doppelsternsystems Algol,* Königsberg, 1902; cf. Stebbins, *Astrophysical Journal*, **32**, 200 and 207, 1910.

[2] "Sur les étoiles variable du type Algol," *Annales de l'observatoire astronomique de Moscou*, Série II, **5**, No. 9, 1911.

[3] Russell, *Astrophysical Journal*, **36**, 69 ff., 1912.

If we take

$$J=J_0\,(1-x+x\cos i)\tag{1}$$

as a sufficiently general law of variation of the surface brightness, J, on a star's disk, where i is the angle between the line of sight and the normal, we may consider x the darkening coefficient. For $x=0$ the surface is uniformly illuminated; for $x=1$ the star is completely darkened at the limb. Denoting by α the fraction of the light of the eclipsed star that is intercepted by the companion (in the case of a uniform disk α is equivalent to the fraction of the area obscured), we see that this quantity depends on the darkening coefficient, the radii of the component stars, r_1 and r_2, and the apparent distance of centers, δ—or, in fact, on x and the ratios of the last three quantities. For fixed values of the ratios $\frac{r_2}{r_1}=k$ and $\frac{\delta}{r_1}$, α becomes a linear function of x, which, for $x=0$, is a familiar function of the remaining variables, $f\left(k, \frac{\delta}{r_1}\right)$, and for $x=1$ may be written $f^1\left(k, \frac{\delta}{r_1}\right)$, so that in general

$$\alpha^x=(1-x)f\left(k, \frac{\delta}{r_1}\right)+xf^1\left(k, \frac{\delta}{r_1}\right)=f^x\left(k, \frac{\delta}{r_1}\right).\tag{2}$$

Now the function $f\left(k, \frac{\delta}{r_1}\right)$, which is a fundamental relation in the uniform-disk theory, has been computed in the previous paper and its inverse tabulated.[1] If we compute $f^1\left(k, \frac{\delta}{r_1}\right)$, which is a similar but much more complicated function, for a system of stars darkened to zero at the limb, we can then, from (2), by interpolation readily determine $\alpha^x=f^x\left(k, \frac{\delta}{r_1}\right)$ for any desired value of the darkening coefficient.

The computation by mechanical quadratures of α^1 for several different values of the geometrical variables, k and $\frac{\delta}{r_1}$, will be facilitated if we replace δ by r_1+r_2p, where p, a function of k and α^1, is the apparent distance, measured along the line of centers in units of the radius of the smaller star, from the circumference of the larger star to the center of the smaller one; so that we may write

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[1] *Astrophysical Journal*, **35**, 333, 1912.

$$\alpha^1 = F^1(k, p), \tag{3}$$

and proceed to determine the amounts of light lost, for given values of k, as p varies from $+1$ at the beginning of eclipse to -1 at the second contact (we are considering here only total eclipses).

For the purpose of the quadrature a standard disc of 16 inches diameter was first constructed, representing the apparent surface of the eclipsed star, and was divided radially into 64 equal sectors. These were then cut by 24 circles concentric with the disk, thus dividing it into 1600 parts, the lengths of the radii of the successive circles being so determined that an equal amount of light is received from each part. Since the total light received from the inside of a concentric circle of radius $\sin\theta$, drawn upon the disk completely darkened at the limb, whose radius is taken as unity, is

$$L = \int_0^\theta J d\sigma$$

where $J = C\cos\theta$ is the apparent surface brightness of the projected element of surface, $d\sigma = 2\pi\sin\theta\cos\theta d\theta$, that is, since

$$L = \int_0^\theta 2\pi C\cos^2\theta\sin\theta d\theta = \tfrac{2}{3}\pi C(1-\cos^3\theta)\,, \tag{4}$$

we see that the light varies directly with the cube of the cosine of the angle subtended at the center of the star by radii drawn on the apparent disk. Therefore, by computing the radii of the concentric circles, $r = \sin\theta$, from 24 equidistant values of $\cos^3\theta$(0.04, 0.08, 0.12, ... 0.96), 25 circular segments are obtained from which equal amounts of light are received. Upon this completed diagram of 1600 equally potential divisions was superposed a sheet of transparent paper on which circles of radius $\frac{1}{k}$ were drawn for k equal to 1.0, 0.8, 0.6, 0.4, and 0.2, that is, of 16 inches, 20 inches, $26\frac{2}{3}$ inches, etc., diameter. These circles, representing the eclipsing star with different values of the radius, were superposed so that the circumference cut the radius of the diagram normally at the distances p from the center. By simply counting and tabulating the divisions and fractions of divisions included between the intersecting circumferences, the quadrature for α^1 was rapidly effected and checked by difference curves. From this tabulated material, curves, represented by equation (3), were drawn for fixed values of k. For $k=0$ the relation between α^1 and p is algebraic,

$$4\alpha^1=(1+p)^2(2-p),$$

and the smoothness of the differences of the other curves from the plot of this equation afforded a further check on the quadrature.

Having assigned definite values to k, it is possible to invert the α^1-function just determined and read off from the curves p as a function of α^1 and k. This has been done to form Table Ix of this paper, which is analogous in every way to Table I of the former article.[1] The two tables are fundamental for the uniform disk and completely darkened disk hypotheses, respectively, and interpolation between them, or rather between the inverted functions which precede them, forms a ready basis for the investigation of intermediate degrees of darkening.

From the relation $\delta=r_1+r_2p(k, \alpha^1)$, we have

$$\frac{\delta}{r_1}=1+kp(k, \alpha^1)\ , \tag{5}$$

and since, geometrically,

$$\delta^2=\sin^2\theta+\cos^2 i\cos^2\theta\ , \tag{6}$$

where i is the inclination of the orbit to the tangent plane, $\theta=\frac{2\pi t}{P}$ is the orbital angle of the eclipsing star counted from minimum, and t is the corresponding time expressed in the same units as the period, P, we obtain the fundamental relation

$$r_1^2\{1+kp(k, \alpha^1)\}^2=\cos^2 i+\sin^2 i\sin^2\theta\ , \tag{7}$$

and, further, by a simple reduction,

$$\frac{\{1+kp(k, \alpha_1^1)\}^2-\{1+kp(k, \alpha_2^1)\}^2}{\{1+kp(k, \alpha_2^1)\}^2-\{1+kp(k, \alpha_3^1)\}^2}=\frac{\sin^2\theta_1-\sin^2\theta_2}{\sin^2\theta_2-\sin^2\theta_3}=\psi(k, \alpha^1)\ , \tag{8}$$

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[1] *Astrophysical Journal*, **35**, 333, 1912.

TABLE Ix

RELATION BETWEEN k, α^1 AND $p(k, \alpha^1)$ FOR TOTAL ECLIPSES OF DISKS COMPLETELY DARKENED TOWARD THE EDGE

α^1	$k=1.0$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
0.00	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000
.02	0.796	0.800	0.804	0.808	0.812	0.816	0.820	0.823	0.827
.05	.685	.689	.694	.698	.703	.708	.712	.717	.721
.10	.543	.550	.558	.564	.569	.576	.583	.591	.599
0.15	+0.428	+0.437	+0.447	+0.455	+0.462	+0.471	+.0480	+0.492	+0.503
.20	.328	.338	.349	.359	.369	.379	.389	.402	.415
.25	.241	.251	.263	.273	.283	.295	.306	.320	.334
.30	.160	.169	.181	.192	.203	.216	.229	.244	.259
0.35	+0.081	+0.091	+0.104	+0.116	+0.128	+0.142	+0.156	+0.172	+0.187
.40	.004	.017	.032	.045	.057	.072	.085	.101	.117
.45	−0.070	−0.055	−0.039	−0.025	−0.011	.003	.018	.033	.049
.50	.143	.126	.108	.093	.078	−0.064	−0.049	−0.034	−0.018
0.55	−0.214	−0.196	−0.176	−0.161	−0.145	−0.130	−0.115	−0.100	−0.084
.60	.284	.265	.245	.229	.213	.197	.182	.167	.151
.65	.353	.334	.314	.298	.282	.265	.249	.235	.219
.70	.424	.403	.383	.367	.351	.334	.318	.304	.290
0.75	−0.495	−0.475	−0.454	−0.438	−0.423	−0.406	−0.390	−0.377	−0.365
.80	.570	.549	.528	.513	.498	.482	.467	.454	.443
.85	.650	.628	.607	.592	.578	.563	.549	.538	.528
.90	.737	.716	.697	.682	.668	.656	.644	.635	.626
0.95	−0.837	−0.818	−0.800	−0.784	−0.770	−0.761	−0.753	−0.747	−0.741
.98	.910	.889	.876	.865	.855	.850	.846	.840	.836
.99	.945	.930	.917	.907	.898	.894	.892	.888	.885
1.00	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000

when we follow the method of the uniform-disk theory and give three definite values to a^1 and the corresponding definite values to θ. The function $\psi(k, a^1)$, which enables us to make use of the whole light-curve in determining k, is exactly the same as before except that it depends now on the new values of the p-function. By assigning to a_2^1 and a_3^1 the permanently fixed values 0.6 and 0.9, respectively, $\psi(k, a^1)$ becomes a function of k and a_1^1 only, and by equation (8), with the aid of the p-table described above, it was readily computed for the values of k and a_1^1 entered in Table II*x*. This table corresponds to the second one of Russell's paper.

A third table, analogous to Table II*a* of the former paper, was constructed to aid in the derivation of the remaining elements of a system of completely darkened stars after k and the final light-curve have been determined in the usual way. Setting $A = \sin^2 \theta_2$, $B = \sin^2 \theta_2 - \sin^2 \theta_3$, so that, from (8),

$$\sin^2 \theta_1 = A + B\psi(k, a_1^1) \; , \tag{9}$$

we can derive from (7) the convenient relations

$$r_1^2 \operatorname{cosec}^2 i = \frac{B}{\phi_1^1(k)} \; , \tag{10}$$

and

$$\cot^2 i = \frac{B}{\phi_2^1(k)} - A \; , \tag{11}$$

where

$$\phi_1^1(k) = \frac{4k}{\psi(k, 0) - \psi(k, 1)} \text{ and } \phi_2^1(k) = \frac{4k}{(1-k)^2\psi(k, 0) - (1+k)^2\psi(k, 1)} \, . \tag{12}$$

These last two quantities are tabulated for convenient intervals of k.

We are now in a position to work as easily with a system of stars darkened according to our adopted law to zero at the limb as with a pair assumed to be uniformly bright, provided that the eclipse is total. The above formulae and tables complete the working theory only in those cases where there is a constant phase at minimum and the eclipsing star is the larger. During the annular phase of an eclipse of a darkened disk, the light will not be constant. As a consequence, for those systems where the secondary minimum has been observed, the relative depths of the two minima can no longer be used to determine k, as is possible on the assumption of uniform

TABLE IIx

Relation between k, a_1^1 and $\psi(k, a_1^1)$ for Total Eclipses of Disks Completely Darkened toward the Edge

a_1^1	k=1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
0.00	+7.865	+6.676	+5.752	+5.050	+4.476	+3.979	+3.571	+3.189	+2.853
.02	6.115	5.238	4.558	4.042	3.618	3.240	2.928	2.642	2.383
.05	5.248	4.509	3.936	3.483	3.135	2.828	2.566	2.322	2.104
.10	4.215	3.649	3.206	2.875	2.583	2.344	2.142	1.958	1.789
0.15	+3.440	+3.000	+2.651	+2.394	+2.170	+1.983	+1.820	+1.674	+1.546
.20	2.825	2.473	2.196	1.993	1.821	1.669	1.542	1.430	1.328
.25	2.314	2.034	1.810	1.649	1.513	1.395	1.298	1.210	1.130
.30	1.869	1.649	1.473	1.347	1.238	1.149	1.076	1.011	0.946
0.35	+1.475	+1.306	+1.173	+1.078	+0.994	+0.927	+0.871	+0.822	+0.772
.40	1.115	0.994	0.901	0.831	0.770	0.721	0.679	0.644	0.609
.45	0.794	0.711	0.651	0.603	0.563	0.528	0.500	0.475	0.452
.50	0.504	0.453	0.418	0.391	0.368	0.346	0.329	0.314	0.300
0.55	+0.240	+0.217	+0.201	+0.190	+0.181	+0.170	+0.163	+0.156	+0.149
.60	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
.65	−0.217	−0.199	−0.188	−0.180	−0.173	−0.165	−0.159	−0.154	−0.150
.70	0.411	0.385	0.365	0.352	0.340	0.328	0.318	0.309	0.302
0.75	−0.585	−0.556	−0.534	−0.518	−0.505	−0.490	−0.478	−0.468	−0.460
.80	0.740	0.714	0.695	0.680	0.667	0.654	0.643	0.634	0.628
.85	0.879	0.861	0.850	0.840	0.832	0.823	0.815	0.810	0.807
.90	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.95	−1.097	−1.125	−1.147	−1.161	−1.173	−1.188	−1.203	−1.216	−1.227
.98	1.138	1.191	1.237	1.272	1.305	1.336	1.369	1.388	1.419
.99	1.149	1.220	1.276	1.323	1.362	1.403	1.477	1.474	1.501
1.00	−1.156	−1.257	−1.347	−1.424	−1.495	−1.559	−1.617	−1.669	−1.715

disks, and for a like reason the relation $a_0 = 1 - \lambda_1 + \frac{1-\lambda_2}{k^2}$, which for uniform brightness is of basic importance in the partial eclipse theory, fails to be valid. In addition to the failure of this previously rigorous equation on account of the dependence of the depth of the minima upon the darkening coefficient as well as upon k and the area obscured, a brief investigation has shown that the remarkable approximate relations upon which were based the empirical ω-functions, which fortunately saved the "uniform" partial eclipse problem from a long trial and error process, likewise fail to hold for darkened stars. We shall soon see, however, that the problem may be solved in another way for partial eclipses.

TABLE II*ax*

FOR COMPUTING THE INCLINATION AND THE RADIUS OF LARGER STAR

k	$\phi_1^1(k)$	$\phi_2^1(k)$
1.00	0.444	0.865
0.95	.451	.824
.90	.454	.782
.85	.454	.740
.80	.451	.698
0.75	0.444	0.656
.70	.433	.614
.65	.420	.572
.60	.403	.530
0.55	0.383	0.488
.50	.361	.445
.45	.336	.402
.40	.308	.359
0.35	0.278	0.317
.30	.246	.274
.25	.212	.230
0.20	0.176	0.186

2. *Comparison of the results obtained on the two hypotheses.*—A comparison of the two tables of the ψ-function indicates how the orbital elements will differ on the two assumptions. The solutions for an orbit will be identical up to the point where we enter the ψ-table for k. For any chosen value of the loss of light, a^1, we find that a given value of $\psi(k, a^1)$ will correspond to a larger k on the assumption of darkening than for uniform brightness.

The mean "darkened" k from determinations all along the curve will be about two-tenths greater than the mean "uniform" k, depending on the magnitude of the latter. If we denote the former by k_d, and the latter by k_u, and adopt a similar notation to distinguish other quantities belonging to the two solutions, we may write the relation between them in the form

$$\psi_u(k_u, \alpha_1) = \psi_d(k_d, \alpha_1). \tag{13}$$

We may therefore regard k_u as a function of k_d and α_1 (or vice versa). Keeping k_d constant, and varying α_1, we find that the values of k_u are nearly constant except at the extreme ends of the range. A typical example is the following, in which $k_d = 0.700$:

α_1	k_u	α_1	k_u	α_1	k_u
0.00	0.670	0.40	0.506	0.95	0.538
0.10	0.572	0.50	0.495	0.98	0.544
0.20	0.546	0.70	0.480	0.99	0.514
0.30	0.528	0.80	0.473	1.00	0.450

It follows that a light-curve computed on the assumption of uniformly illuminated disks, with the mean of these values of k (which, taking account of weights, is 0.525), will agree very closely with one computed on the assumption of disks completely darkened at the edge, with $k = 0.700$. In many cases the fit can be improved by slight alterations in the other constants A and B used in computing the curves.

How good an agreement can be attained in this way can best be shown by numerical examples, of which the three that are here given are typical. The table gives the values of sin θ_1, computed from the equation $\sin^2 \theta_1 = A + B\psi\ (k, \alpha_1)$, in which B has been taken as 0.01 for the darkened stars, and A so determined that the duration of totality shall be about half its maximum value, thus presenting a typical case. The values of A, B, and k for the uniform disks have been chosen so as to get as good a representation as possible of the general course of the "darkened" curve between $\alpha_1 = 0.1$ and $\alpha_1 = 0.99$.

	Darkened	Uniform	Darkened	Uniform	Darkened	Uniform
k	1.000	0.850	0.700	0.525	0.400	0.200
A	0.01156	0.01145	0.01500	0.01500	0.01800	0.01810
B	0.01000	0.00980	0.01000	0.01000	0.01000	0.01020
$\sin\theta_1$ for $\alpha_1=$						
0.0	0.3003	0.2795	0.2560	0.2374	0.2317	0.2148
0.1	.2316	.2304	.2092	.2055	.1985	.1942
0.2	.1995	.2003	.1871	.1860	.1828	.1814
0.3	.1738	.1742	.1666	.1664	.1696	.1694
0.4	.1507	.1511	.1526	.1529	.1574	.1580
0.5	.1289	.1287	.1375	.1379	.1460	.1466
0.6	.1076	.1070	.1224	.1224	.1341	.1345
0.7	.0864	.0857	.1071	.1068	.1218	.1219
0.8	.0645	.0639	.0901	.0905	.1074	.1071
0.9	.0395	.0406	.0707	.0707	.0895	.0889
0.95	.0243	.0257	.0582	.0580	.0772	.0763
0.98	.0134	.0131	.0468	.0462	.0656	.0658
0.99	.0084	Max. eclipse for $\alpha_1=0.987$	.0421	.0426	.0594	.0602
1.00	.0000		.0276	.0332	.0427	.0504

Computations for other values of k give similar results, which may be summarized as follows:

Given any light-curve arising from the total eclipse of a star, completely darkened at the limb, by another (not more than three times its diameter), it is possible to find a light-curve due to the total eclipse of one uniformly bright disk by another which will agree with the former almost perfectly between obscurations of 10 per cent and 98 per cent of the whole light of the eclipsed star. The extreme phases, at the beginning and end of eclipse, will be much slower for the "darkened" curve. The light changes in the immediate vicinity of totality will be slower for the "darkened" curve unless k for this curve exceeds 0.83, when they will be more rapid.

The values of A and B for the two curves will be very nearly the same, but that of k will be much smaller for the "uniform" curve. In the extreme case of a central eclipse of equal, darkened stars (which must obviously be of very rare occurrence), the corresponding elements for uniform disks lead to an eclipse which is not quite total.

The only difference between a total and a partial eclipse is that in the latter case the constant A in equation (9) is smaller, so that $\sin^2\theta$ vanishes for some value of α_1 less than unity which we have previously called α_0. It follows at once that also in this case a "uniform" light-curve and set of elements may be substituted for the "darkened" ones, which differ from the latter only in the less gradual character of the extreme phases. The constant α_0 will be identical for the two curves; the others, A, B, and k, will be related as above.

We may proceed further and tabulate the relations between the systems of elements obtained on the two hypotheses. In doing so it is well to throw them into as general a form as possible. The previous comparison of light-curves shows that the percentage change in B, in passing from the "darkened" to the "uniform" curve, depends only on k, and that the change in A is one-half that in B. We may therefore write

$$B_u = B_d + \delta B_d\,, \qquad A_u = A_d + \tfrac{1}{2}\delta B_d\,, \tag{14}$$

where δ depends only on k. Introducing these into the equations (10) and (11), we find readily

$$\left.\begin{aligned} r_{1u}\operatorname{cosec}^2 i_u &= \alpha r_{1d}\operatorname{cosec}^2 i_d \\ r_{2u}\operatorname{cosec}^2 i_u &= \beta r_{2d}\operatorname{cosec}^2 i_d \\ \cot^2 i_u = \cot^2 i_d + \gamma B &= \cot^2 i_d + \gamma'^2 r_{1u}^2 \operatorname{cosec}^2 i_u \end{aligned}\right\} \tag{15}$$

where α, β, γ, γ' are functions of k_d and the corresponding k_u (and hence of either one alone), which may be written down at once. If i_d is to be real, $\cot^2 i_u$ must not be less than γB. Hence if we set $\gamma B = \cot^2 i_0$, i_0 is the nearest value to 90° which i_u may have, if the "uniform" curve is to be capable of representation by a "darkened" curve with the assigned value of k_d. We may then put the last of equations (15) in the form

$$\left.\begin{aligned} \cot^2 i_u &= \cot^2 i_d + \cot^2 i_0 \\ \cot i_0 &= \gamma' r_{1u} \operatorname{cosec} i_u \end{aligned}\right\} \tag{16}$$

The quantities α, β, γ', δ are tabulated below. With their aid it is possible (within certain limits) to pass at once, by means of equations (15) and (16), from any given set of elements derived on one hypothesis to one which, on the other hypothesis, will give an almost identical light-curve (with the small differences already specified).

The depth of the secondary minimum, computed from these two sets of elements, will be different. If L_1 is the light of the larger star (which at principal minimum eclipses the other), and a'' denotes the fraction of this which is obscured at secondary minimum, the loss of light at this time is $a''L_1$. If one star is completely projected on the other's disk, we have for uniform disks $a''_u = k^2_u$. For darkened disks a'' depends on the apparent position of the smaller disk in front of the larger, being greatest when it is central—then $a''_d = 1-(1-k^2_d)^{\frac{3}{2}}$—and least when it is internally tangent (when computation by quadratures is necessary). These values are given in the table under the headings a''_d and a'''_d.

TABLE C

For the Transformation of "Uniform" into "Darkened" Elements, or Vice Versa

k_d	1.000	0.900	0.800	0.700	0.600	0.500	0.400
k_u	0.850	0.735	0.630	0.525	0.420	0.310	0.200
α	1.019	1.027	1.040	1.065	1.107	1.168	1.283
β	0.865	0.840	0.821	0.798	0.775	0.724	0.642
γ'	0.180	0.254	0.314	0.366	0.431	0.528	0.650
δ	−0.020	−0.010	0.000	0.000	0.000	+0.010	+0.020
a''_u	0.722	0.541	0.397	0.276	0.176	0.096	0.040
a''_d	1.000	0.917	0.784	0.636	0.488	0.350	0.229
a'''_d	1.000	0.904	0.750	0.581	0.427	0.289	0.175

Various interesting conclusions can be drawn from this table. From the values of α, β, γ', it appears that the "uniform" solution always gives the large star larger, the small star smaller, and the inclination farther from 90° than the "darkened" solution; and hence if darkening really exists, the solution made without regarding it will lead to a density too low for the large star, too high a density for the smaller one, and too great a ratio of the surface brightness of the two (in the case here discussed where the faint star is the larger). The most important effect is on the computed density of the smaller star, which on the average will be about twice too great.

The numerical data of a paper which will appear soon[1] show that the difference in form between the light-curves at principal minimum derived on the two hypotheses is almost at the limit of detection, even by the most numerous and accurate observations so far published, and is certainly too small to justify the hope of distinguishing between different degrees of darkening toward the limb.

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[1] In the *Astrophysical Journal* for November, 1912.

The only chance of this seems to be by means of the secondary minimum. It appears from Table C that, in the case of complete darkening at the limb, the depth of secondary minimum, corresponding to a given form of the light-curve of primary minimum, will exceed the corresponding depth in the case of uniform disks by about three-tenths of the light of the fainter star. Even with a range of $2^{\mathrm{mg}}.5$ at principal minimum, the secondary should be $0^{\mathrm{mg}}.03$ deeper on the first hypothesis than on the second. With a range at principal minimum of $1^{\mathrm{mg}}.0$ the difference would be $0^{\mathrm{mg}}.13$. The secondary minimum lasts as long as the primary, and, if equally well observed, its depth should be determinable within less than $0^{\mathrm{mg}}.01$—provided that the non-eclipse part of the curve has also been so well observed as to determine and eliminate the ellipticity and radiation effects. It should therefore be possible in this way to determine conclusively, not only whether certain stars are darkened toward the limb, but the extent of the darkening. The best stars for the purpose are those with a total eclipse of relatively small depth at primary minimum. Unfortunately, no sufficient data for the whole light-curve of such a star are at present available.

Unless the observations upon which a star's light-curve is based are of unusual number or accuracy, the transition from the elements computed on the assumption of uniform disks to those which assume complete darkening at the limb may be made with the aid of Table C. When the light-curve is very accurately determined, a direct solution by means of Table IIx is preferable. This may give results differing slightly from those of Table C, as the detailed solution will give a light-curve which fits the observed curve most closely where the course of the latter is determined with the greatest weight, while the comparisons upon which Table C is based aim at obtaining a good general fit all over the curve.

One exceptional case needs further discussion. If the value of

TABLE IV

VALUES OF $\Phi(k, \alpha_1)$ FOR UNIFORMLY ILLUMINATED DISKS

For Computing the Values of k and the Light-Curve in the Case When the Eclipse Is Central. $\sin^2 \theta = \Phi(k, \alpha_1) \sin^2 \theta_6 = A\Phi(k, \alpha_1)$

α	k=1.0	0.9	0 8	0.7	0.6	0.5	0.4	0.3	0.2
0.00	9.887	7.345	5.683	4.500	3.608	2.920	2.368	1.925	1.556
.10	6.407	4.922	3.940	3.233	2.695	2.266	1.919	1.627	1.391
.20	4.668	3.700	3.047	2.572	2.209	1.914	1.673	1.472	1.294
.30	3.368	2.761	2.363	2.062	1.826	1.635	1.470	1.332	1.210
.40	2.399	2.056	1.822	1.648	1.522	1.398	1.300	1.215	1.136
.50	1.609	1.471	1.371	1.299	1.239	1.189	1.144	1.104	1.067
.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.70	0.549	0.631	0.692	0.744	0.789	0.829	0.870	0.901	0.935
.80	0.244	0.351	0.443	0.525	0.599	0.670	0.738	0.804	0.869
.90	0.061	0.152	0.245	0.337	0.428	0.518	0.610	0.702	0.798
.95	0.019	0.080	0.151	0.248	0.341	0.438	0.540	0.645	0.751
.98	0.002	0.045	0.112	0.191	0.281	0.381	0.490	0.604	0.727
1.00	0.000	0.020	0.070	0.140	0.225	0.324	0.436	0.558	0.692

TABLE IVx

VALUES OF $\Phi(k, \alpha_1)$ FOR DISKS COMPLETELY DARKENED TOWARD THE EDGE

For Computing the Values of k and the Light-Curve in the Case of Central Eclipse

α	k=1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2
0.00	7.805	6.220	5.015	4.100	3.373	2.770	2.281	1.874	1.530
.10	4.648	3.856	3.238	2.763	2.369	2.044	1.769	1.536	1.333
.20	3.446	2.917	2.532	2.223	1.966	1.742	1.554	1.392	1.247
.30	2.617	2.290	2.028	1.826	1.656	1.512	1.386	1.277	1.176
.40	1.965	1.778	1.629	1.510	1.408	1.321	1.244	1.177	1.113
.50	1.435	1.354	1.292	1.240	1.195	1.154	1.118	1.086	1.056
.60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
.70	0.644	0.698	0.746	0.784	0.820	0.854	0.886	0.915	0.944
.80	0.360	0.442	0.515	0.583	0.646	0.709	0.769	0.826	0.883
.90	0.135	0.218	0.302	0.386	0.470	0.555	0.641	0.726	0.814
.95	0.051	0.120	0.200	0.287	0.378	0.471	0.568	0.667	0.772
.98	0.016	0.069	0.137	0.219	0.309	0.406	0.509	0.620	0.736
1.00	0.000	0.017	0.061	0.127	0.209	0.307	0.419	0.544	0.681

cot i determined on the assumption of uniform disks is less than cot i_0 (as defined by equation (16) and Table C), there will be no real "darkened" solution with the value of k_b resulting from this table, and it will be necessary to represent the observations, as well as may be done, by a set of elements for which cot $i=0$, i.e., on the hypothesis of central transit. In this case, by (9) and (11), $B=A\phi_2(k)$, $\sin^2\theta=A\{1+\phi_2(k)\psi(k, \alpha_1)\}$. To simplify the process of getting the best fit for the observed curve under these circumstances, we may set $1+\phi_2(k)\psi(k, \alpha_1)=\Phi(k, \alpha_1)$ and tabulate this function. Then $\sin^2\theta=A\Phi(k, \alpha_1)$, and the solution is made with the aid of the new table, just as in the general case with the ψ-table. The computation of the elements is immediate, for $i=90°$, and, if θ' and θ'' correspond to the beginning of eclipse and of totality,

$$r_1=\tfrac{1}{2}(\sin\theta'+\sin\theta'') ,$$
$$r_2=\tfrac{1}{2}(\sin\theta'-\sin\theta'') .$$

As no similar table has so far been published, the Φ-function is tabulated here both for uniform disks and for those completely darkened at the limb.

The full discussion of total eclipses of darkened stars is now completed. When elements have been derived for a partial eclipse on the hypothesis of uniform disks, and the larger star is found to be in front at principal minimum, Table C makes it possible to find at once (for values of k within its range) a set of "darkened" elements which will give sensibly the same light-curve for this minimum, except for the greater extent of the extreme phases at beginning and end. These elements will, however, give a decidedly deeper secondary minimum than the corresponding "uniform" elements.

It is obvious that by computing "uniform" elements for the same principal minimum, but a shallower secondary, a set of "darkened" elements can be found which will represent both observed minima; but the detailed discussion of this, and of annular eclipses, must be postponed to another communication to avoid undue delay of the present publication.

PRINCETON UNIVERSITY OBSERVATORY
August 6, 1912

ON CONVERGENCE FREQUENCY IN SPECTRAL SERIES

By FERNANDO SANFORD

It is the purpose of this paper to show some relations between the convergence frequencies (A, in Kayser and Runge's formula) in the spectral series of certain atoms and some of their other physical properties.

Professor Kayser says in his *Handbuch der Spectroscopie*, Band II, S. 592:

> Man sollte erwarten, da die Lage des Spectrums sich vom Atomgewicht abhängig erweist, dass z. B. auch die erste Constante in einer mathematisch ausdrückbaren Beziehung zum Atomgewicht stehen werde. Aber es ist bisher nicht gelungen, eine solche Beziehung zu finden; man kommt hier nicht weiter, als bis zum Ausspruch, dass die Lage des Serienspectrums eine periodische Function der Atomgewichte sei, ebenso, wie man für die übrigen Eigenschaften der Elemente nicht weiter gelangt ist.

I am not aware that such a relation has been observed since the above was written, but I am not familiar enough with the literature of spectroscopy to know whether it has or not. However, I am sure that the particular relation to which I wish first to call attention has not been observed.

Since the quantities to which I refer, and which I have called the characteristic charges of the atoms, are not generally recognized, it will be necessary for me to explain briefly what I mean by the term, and how these quantities are determined.

In a paper entitled "On Positive Atomic Charges," published in *Physical Review*, **32**, 512, May 1911, and in a paper entitled "A Physical Theory of Electrification," *Leland Stanford Junior University Publications, University Series,* No. 6, I gave reasons for believing that the positive sub-atoms of the elements have characteristic charges which are not simple multiples of the charge on a positive hydrogen atom. An attempt was made to calculate the relative magnitude of these charges on the ions in electrolysis on the assumption that these ions are dissociated for only very brief periods and move but short distances in the solution before recombining with another ion or with a water molecule. Under these conditions their motion through the solution will be accelerated according to the equation $F=ma$, where F is the electromotive force acting upon the ion, and m is its ionic mass. Since the force acting upon an ion in a given electric field is proportional to its charge, we may substitute e, the ionic charge, for F in the above equation. The charges of different ions in a given electric field (assuming them to be free for the same periods of time) would be proportional to the products of their ionic masses into their ionic speeds.

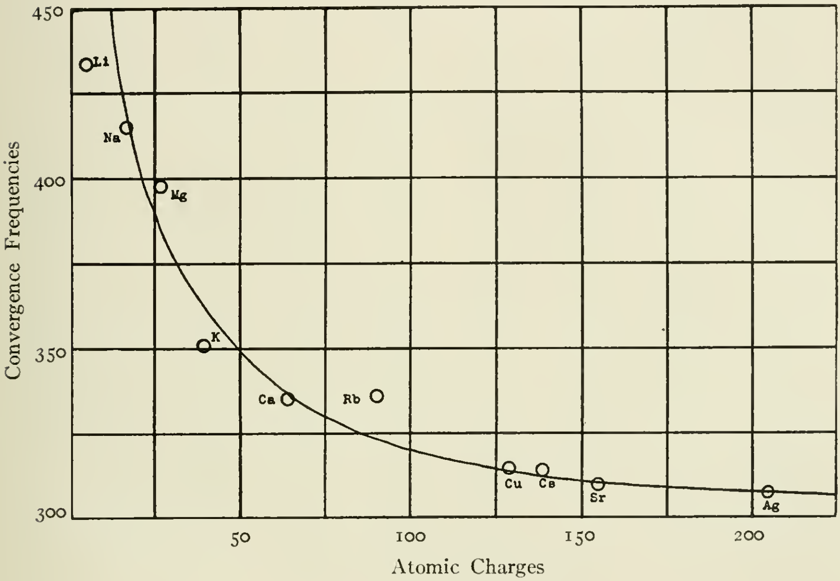
I have calculated in this way the *relative* charges of those atomic ions whose velocities in an electrolytic solution have been determined. Assuming the absolute charge of the hydrogen ion to be 4.84×10^{-10}, I have then calculated the corresponding characteristic charges of the other ions. In the papers referred to above I have shown that the numbers obtained in this way are related to all the properties of the ions which determine cohesion or affinity, and that they are likewise arranged in the order of their atoms in the voltaic series.

In the table below are given the values of these quantities for all the atoms for which I have been able to calculate them by the above method and for which the convergence frequencies are given in the table on p. 593, Vol. II, of Kayser's *Handbuch.* The convergence frequencies are in all cases the highest given in the table; that is, they are taken from the principal series in the case of the alkali metals and from the first subordinate series in the case of the other metals used.

Element	Characteristic Charge	Convergence Frequency
Li	3.96	43584
Na	15.8	41543
K	38.8	35091
Rb	89	33762
Cs	138	31509
Mg	26.4	39796
Ca	69.6	33919
Sr	155	31030
Cu	128.4	31592
Ag	205	30712

In the case of the bivalent elements the charges calculated for a single valence (for which the ionic velocities are given in the tables) are multiplied by two. The charges are all multiplied by 10^{10}.

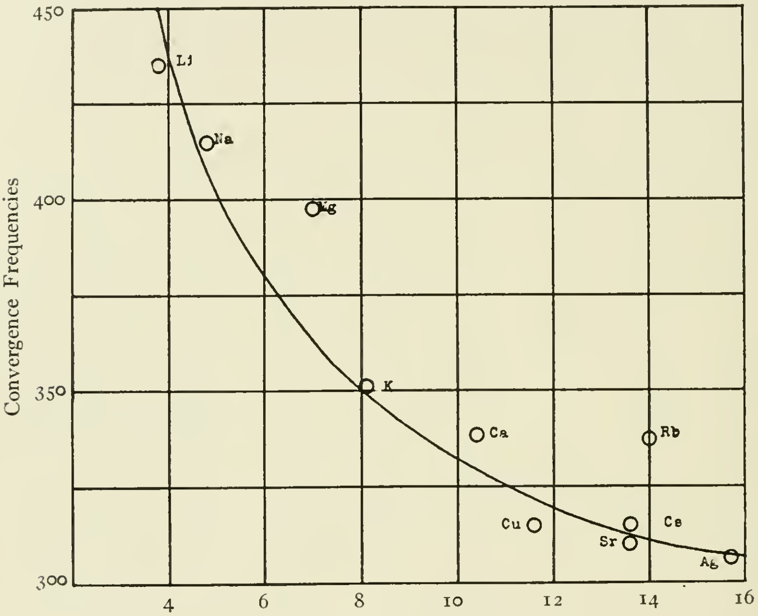
To show the relations existing between the numbers in the two columns, they are plotted against each other in Curve I, the atomic charges being used as abscissae and the convergence frequencies as ordinates. Only three decimal places of the convergence frequencies are used. The hyperbola drawn in the figure is calculated from the equation $(e+5)\ (f-295)=2840$. It is not claimed that this is the correct equation for the relation between the two sets of quantities, or even that it is the nearest possible approximation to it, but only that it does approximate to the relation under consideration.



Curve I

If a similar curve be plotted using the first subordinate series of the alkali metals, it will be found that the convergence number for hydrogen falls upon this curve.

That Curve I does show a physical relation between the convergence numbers and the atomic charges calculated in this way seems beyond a doubt. The significance of this relation seems to me to lie in the fact that when the above elements are arranged in the order of magnitude of their atomic charges they are also arranged in their order in the voltaic series. Coehn's law indicates that the voltaic series is the same as the dielectric series, and that the metals when arranged in the order of the voltaic series are also arranged in the order of their specific inductive capacities. I have elsewhere[1] given what seem to me to be valid reasons for expecting this relation between the voltaic series and the dielectric series. If this relation holds in metals, as it does in non-metals, the above table shows that the convergence frequency is greater in those elements having the lower specific inductive capacity. This seems reasonable, since the convergence frequency apparently represents the highest attainable frequency of oscillation of any electron in a given system. If these electrons vibrate under the influence of a



Gladstone's Refraction Equivalents

CURVE II

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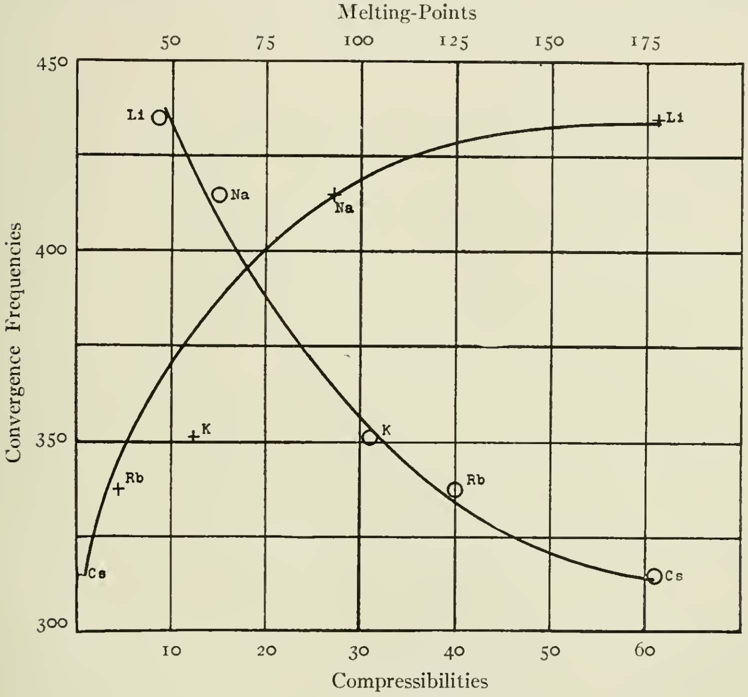
[1] "Physical Theory of Electrification," pp. 41–43.

central electric force, there is no question that, other things being equal, the higher the specific inductive capacity of the system the weaker the central force and the slower the oscillation.

In an article on "Pressure-Shift of Spectral Lines" in this *Journal*, January 1912, I have already called attention to the relation between the refraction constants of some of the elements and their atomic charges. A similar relation between the refraction constants and convergence frequencies of the elements in the above table is shown in Curve II, where Gladstone's refraction constants[1] are taken as abscissae and the convergence frequencies as ordinates. The abscissae are so chosen that the two curves shall be on nearly the same scale.



CURVE III

The curve shows that the refraction constant varies in the inverse direction as the convergence frequencies. It seems to be pretty definitely established that the inductive capacity of a medium is a principal determining factor of the light-velocity in that medium, and hence of its refraction constant.

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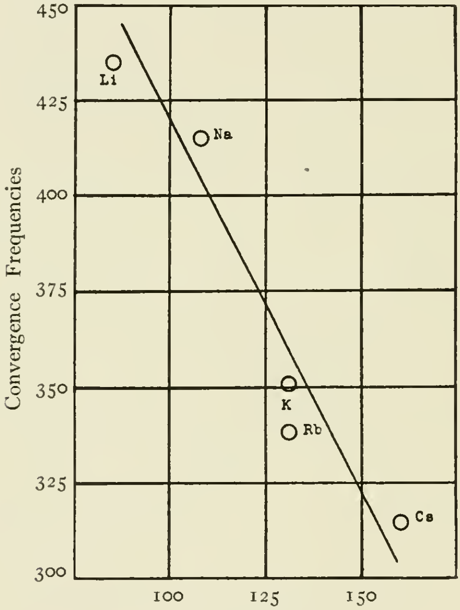
[1] *Phil. Trans.*, 1870, pp. 23–26.

In the paper on pressure-shift mentioned above, I also called attention to the relation of this property to the compressibility and the melting-points of the alkali metals. The relation between these properties and the convergence frequencies is shown in Curve III. In Curve III the points represented by crosses indicate the melting-points, the values of which are given at the top of the curve, while the circles represent the compressibilities as determined by Richards.[1]



Pressure-Shifts

Curve IV.—Principal series

If the internal forces of the atoms are electrical forces, and if the forces which hold together the atoms in a metal are of the same nature, then both the compressibilities and the melting-points must depend upon the inductive capacity of the atoms.

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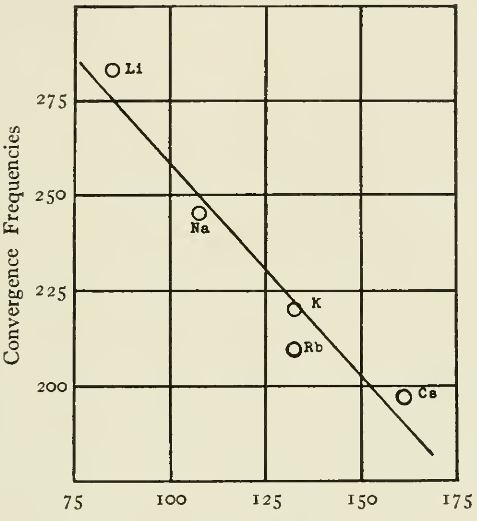
[1] *Zeit. Phys. Chem.*, 61, 192, 1910.

It follows from the relations which have been shown that the pressure-shift and the convergence frequencies of the elements of this group must bear an inverse relation to each other. In Curves IV and V these relations are shown for the convergence frequencies of the principal series and the subordinate series of the alkali metals. Whether, if the lines of these different series were considered separately, their pressure-shifts would be found to differ, I do not know.



Curve V.—Subordinate series

In his reply to my article on pressure-shift, Humphreys dismisses my arguments with the statement that pressure-shift is equally well related to all the other properties of the atom which are periodic functions of the atomic weight. This is no doubt true in a mathematical sense. It would be equally true if the pressure-shift varied in the inverse, instead of the direct, order of the specific inductive capacity. It is, however, a matter of importance to one who attempts to *explain* phenomena by mechanical laws, rather than to find rules for computing their magnitude, to know in what direction with reference to each other they vary. It seems to be plainly shown that in one group, at least, the pressure-shift varies in the inverse direction to the forces which hold together the electrical parts of the atom and the atoms of the solid. If this shift is proportional to the magnetic fields of the atoms when luminous, it follows that those atoms which have the strongest magnetic fields when luminous are those which attract each other with the weakest forces when not luminous. I see no physical reason why this should be the case. Neither do I see why those atoms whose electrons are capable of rotating at the highest speeds when luminous (if we assume the Saturnian atom) should have the weakest magnetic fields, also when luminous.

STANFORD UNIVERSITY
July 1912

REVIEWS

Die optischen Instrumente: Aus Natur und Geisteswelt, Sammlung wissenschaftlichgemeinverständlicher Darstellungen 88stes Bändchen. Von DR. MORITZ VON ROHR. Leipzig: B. G. Teubner, zweite, vermehrte und verbesserte Auflage, 1911. Pp. vi+140. Bound, M. 1.25.

Dr. von Rohr's book, which appears now in its second edition, aims to state clearly and accurately the principles underlying the construction and operation of the better known forms of optical instruments. In the second edition new material has been introduced at several points and the whole book seems to be printed in a new style of type, much easier on the eyes than was that of the first edition. The arrangement and subject-matter are largely the same in both editions—lenses, the human eye, binocular vision, the photographic camera, the stereopticon, the cinematograph, eyeglasses, reading-glasses, the microscope, and the telescope being taken up in order. The discussion of the microscope is especially interesting. The use of prisms for the purpose of total reflection is shown in the case of several styles of binoculars. The prism as a dispersing medium and its use in simple refraction, as well as its use in the treatment of strabismus, might have been mentioned.

Some care has been taken in the discussion of the different kinds of aberration in lenses, and a short account of the different types of lenses and their historical development round out the little book, of which the small size has made it necessary for the author to condense much that might have been written more at length.

Dr. von Rohr is well known through his book on the *Theory and History of the Photographic Objective* and through his frequent contributions to the *Zeitschrift für Instrumentenkunde.* A short time ago Czapski secured his help on the second edition of *Die Theorie der optischen Instrumente;* and Dr. von Rohr is publishing, in collaboration with the other scientific assistants of the great Zeiss firm of Jena, a series of volumes on the theory of optical instruments. This series, when complete, will probably surpass all other works on the subject, and will be a stimulus, as its first volume has already been, to the production of treatises and books on optics, more nearly complete and more valuable than have previously appeared in English.

The little book of Dr. von Rohr's is, on the other hand, popular in style, not theoretical but descriptive. It can be depended on as far as it goes.

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265.

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THE ATTRACTION OF SUN-SPOTS FOR PROMINENCES

By FREDERICK SLOCUM

In the autumn of 1910 a large group of sun-spots passed several times across the face of the sun. Its first passage was from August 2 to August 15. It was then given the Greenwich number 6874. The group consisted of a large regular spot, crossed by bright bridges, and accompanied by several small spots. It reappeared on August 31, as Greenwich No. 6880, and passed around the west limb on September 11. On the third apparition, from September 27 to October 8, the group was greatly extended. The main spot with its immediate attendants was numbered 6894, and a stream of spots just preceding 6893. The whole disturbed area was covered with calcium flocculi extending from latitude $-10°$ to latitude $-25°$, and from longitude 32° to longitude 65°. Its appearance on October 1, as photographed in the light of calcium with the spectroheliograph, is shown on Plate XII.

This same region returned to view on October 22, but the big spot had vanished and only a large area of bright flocculi remained to mark the place. Throughout its history this group was the scene of great activity. Bright eruptions were constantly appearing and disappearing, and, on several dates, dark flocculi were observed in the immediate vicinity. Photographs of the H and K region of the spectrum of various parts of the spot-group show distortions and displacements, corresponding to velocities up to 40 km per second.

The particular features, however, to which I wish to call attention, are the forms and movements of the prominences which were associated with this disturbed area. Active prominences were observed in this region from August 2 to November 5, but the best displays occurred on the west limb on October 8 and on the east limb on October 22. On the former date nine photographs were secured, of which six are reproduced in Plate XI.

The spot-group at this time was very near the limb, as shown in Figs. 1 and 2, Plate XII. Fig. 1 is a low-level view and Fig. 2, high-level, both taken in the light of calcium. On account of the foreshortening, comparatively little idea of the great extent of the group can be derived from pictures taken so near the limb. I have, therefore, shown in Figs. 3 and 4, Plate XII, the high- and low-level appearance of the region when it was near the central meridian of the sun's disc. On Plate XI the position of the center of the largest spot of the group is indicated by the tip of the arrow.

The most striking feature of the views on this plate is that the prominences are pouring from both sides apparently right down into the big spot. There can be no question about the direction of motion, for a sufficient number of details can be identified on two or more plates to establish this. For example, note the three bright knots on the long streamer coming in from the right at $4^{h}26^{m}.6$ and $4^{h}34^{m}.9$, Plate XI. These three knots give velocities along the apparent trajectory of 16, 20, and 60 km per second, at distances of 170,000, 130,000, and 75,000 km from the center of attraction. The values of the velocities as well as the distances are, of course, affected by an unknown component in the line of sight, but, since these three points are upon the same trajectory, an accelerated velocity is undoubtedly indicated. Other points that can be identified on two plates give apparent velocities ranging from 15 to 90 km per second.

In addition to the general attraction manifested by this spot, there are also evidences of repulsion. The plates taken at $3^{h}24^{m}.9$ and $5^{h}26^{m}.5$ show bright straight jets coming directly out of the spot. These are generally so short-lived that they do not appear on consecutive plates. One such eruption, however, can be identified on plates taken at $5^{h}43^{m}.7$ and $5^{h}50^{m}.5$, and this shows a velocity of ascent of 40 km per second, at a distance of 60,000 km from the spot.

A remarkable feature shown by this group of prominences is the great distance to which the influence of the spot is exerted. The prominences covered 45° of the sun's limb, with the spot nearly in the center, and the prominence matter, as far as 260,000 km from the spot, is evidently drawn in toward it. This is especially well shown to the right, or south, of the spot, where two centers of activity will be noticed. From the one nearer the spot, a stream of matter travels a long distance parallel to the surface of the sun and then bends down into the spot. From the prominence on the extreme right, a similar stream of matter starts off in the same direction, but, as it rises a trifle above the former stream, it apparently meets a counter current and is turned back, curving up to a height of $2\frac{1}{2}'$, or 107,000 km. This is best shown at $4^{h}34^{m}.9$.

This group of prominences of October 8 extended from latitude $+6°$ to latitude $-37°$ on the west limb. When this same region came to the east limb on October 22, a similar display of prominence activity was observed, extending from latitude $+12°$ to latitude $-36°$. The sun-spot was no longer visible, but the prominences were gathered around the extensive area of bright flocculi, which still marked the place. To the north of the flocculi were numerous jets and prominences of frail structure, one of which rose to a height of $3\frac{1}{2}'$ or 150,000 km. To the south was a very bright and massive prominence, from the top of which poured a stream of matter down into the southern edge of the flocculi, striking the surface of the sun a few degrees south of where the big spot had been two weeks before.

I was fortunate enough to catch this stream in the act of descending. See Plate XIII. Between $5^{h}50^{m}.1$ and $5^{h}52^{m}.3$ the apparent velocity of the point of the stream was 87 km per second, and between $5^{h}52^{m}.3$ and $5^{h}58^{m}.1$, 110 km per second, again indicating acceleration. The small detached mass on the extreme right of the pictures of Plate XIII was descending at the rate of 10,000 km per second.

The above observations show that sun-spots, or the regions of sun-spot activity, exert a strong attraction upon some prominences.

In 1908, Professor Hale[1] reported a somewhat similar observation upon a dark hydrogen flocculus in the vicinity of a sun-spot. The flocculus approached the spot at velocities ranging from 140 to 76 km per second.

Again Evershed,[2] in his spectroscopic study of the radial motions around sun-spots, finds that "the gases of the higher chromosphere, hydrogen and calcium, show a tendency to move inward with a diminishing velocity toward the spot center."

In one respect the results of my observations differ from both those of Hale and of Evershed. My results show an acceleration toward the center, while theirs indicate diminishing velocities.

YERKES OBSERVATORY
August 10, 1912

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[1] *Astrophysical Journal*, **28**, 109, 1908.

[2] *Memoirs of the Kodaikanal Observatory*, Vol. 1, Part 1, p. 54, 1909.

ELEMENTS OF THE ECLIPSING VARIABLES *W DELPHINI, S CANCRI, SW CYGNI, AND U CEPHEI*

By HARLOW SHAPLEY

For the reasons given in the preceding paper,[1] the most satisfactory eclipsing stars for the detection of darkening at the limb from the light-curve of the principal minimum, and most suitable for the critical application of the above methods, are those whose curves are thoroughly determined by the photometric work of an experienced observer, and which show a constant phase at minimum of sufficient length with respect to the depth to insure that the eclipse is total rather than annular. Among the best existing photometric observations of variables are those made with a polarizing photometer at the Harvard Observatory by Professor Wendell, and the four stars, *W Delphini*, *S Cancri*, *SW Cygni*, and *U Cephei*, for which he publishes observations in *Harvard Annals*, 69, Part I, satisfy the requirements.[2] No other total eclipses have been sufficiently well observed to be included; and, moreover, it would be a doubtful advantage for the present study to introduce work by another observer and another instrument into this homogeneous set of data. The following table of information concerning the four selected stars is taken from Wendell's summary.[3]

Star	R.A. 1900			Dec. 1900	Max.	Min.	Period	No. Obs.	Av.Dev.
	h	m	s		mg	mg			mg
W Delphini..	20	33	7	+17° 55′.9	9.39	12.11	$4^{d}.8061$	500	0.053
S Cancri.....	8	38	14	+19 23.6	8.00	10.10	9.4845485	407	0.040
SW Cygni...	20	3	50	+46 0.5	9.06	11.72	4.57294	192	0.029
U Cephei....	0	53	24	+81 20.3	6.82	9.18	2.4928840	695*	0.055

*Erroneously 691 in Wendell's summary.

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[1] *Astrophysical Journal*, **36**, 239, 1912.

[2] It is hardly necessary to show the impossibility of annular eclipses for these stars. The light lost at primary minimum is $2^{mg}.7$, $2^{mg}.1$, $2^{mg}.7$, and $2^{mg}.4$, respectively. Therefore, for annular eclipses by a totally obscure companion, the k's must necessarily be greater than 0.95, 0.92, 0.95, and 0.94, and must be still nearer unity if the companions are not wholly dark. The duration of constant minimum light in each case, as well as the general shape of the light-curve, immediately excludes such large values of k.

[3] *Harvard Annals*, **69**, 96.

The values of maximum and minimum light differ slightly from those used below which were determined by a more detailed consideration of the observations. The last column contains Wendell's determination of the average deviation of a single observation from the mean light-curve. The observations were made throughout the interval from October 1895 to December 1902. They represent a total of 28,704 comparisons of the light of the variables with that of their reference stars. All four stars belong to spectral class A.

1. *W Delphini.*—The 500 observations of this star have been discussed, and "uniform" elements derived, by Professor Russell, who publishes the orbit in the September number of this *Journal* as a part of his third paper on eclipsing variables. The table of normal places is reproduced here with an additional column of residuals.

NORMAL MAGNITUDES OF *W Delphini*

Phase	Mag.	No. Obs.	O.−C.$_u$	O.−C.$_d$	Phase	Mag.	No. Obs.	O.−C.$_u$	O.−C.$_d$
	mg		mg	mg		mg		mg	mg
−0^{d}2894..	9.41	6	+0.01	−0.01	+0^{d}0560..	11.76	7	−0.01	0.00
.2637..	9.49	5	+ .02	.00	.0659..	11.58	8	+ .01	+ .01
.2458..	9.58	5	+ .04	+ .03	.0753..	11.33	7	− .04	− .04
.2306..	9.59	4	− .01	− .02	.0859..	11.14	5	− .02	− .02
.2200..	9.67	5	.00	.00	.0937..	10.97	5	− .05	− .04
.2106..	9.73	8	+ .01	.00	.1036..	10.88	8	+ .02	+ .02
.2007..	9.79	10	.00	.00	.1147..	10.73	8	+ .05	+ .04
.1911..	9.88	12	+ .02	+ .02	.1246..	10.56	12	+ .03	+ .02
.1817..	9.95	10	+ .01	+ .01	.1351..	10.39	14	.00	.00
.1718..	10.02	8	.00	.00	.1445..	10.31	11	+ .04	+ .04
.1615..	10.16	17	+ .04	+ .04	.1546..	10.13	10	− .02	− .02
.1506..	10.23	14	.00	.00	.1641..	10.10	11	+ .04	+ .03
.1396..	10.37	14	+ .01	.00	.1744..	9.97	10	.00	.00
.1311..	10.44	16	− .03	− .04	.1847..	9.90	9	+ .02	+ .01
.1212..	10.59	17	− .03	− .04	.1941..	9.79	9	− .02	− .03
.1121..	10.78	14	+ .01	.00	.2050..	9.71	8	− .02	− .03
.1013..	10.91	17	− .04	− .03	.2157..	9.71	6	+ .04	+ .03
.0906..	11.12	14	− .01	.00	.2242..	9.63	8	+ .01	.00
.0809..	11.30	10	− .02	− .02	.2345..	9.57	7	.00	− .01
.0715..	11.51	12	.00	.00	.2507..	9.50	7	.00	− .01
.0617..	11.69	10	.00	.00	.2708..	9.48	7	+ .03	+ .02
.0509..	11.88	7	.00	.00	+0.2811..	9.43	4	+0.02	0.00
.0313..	12.05	5	− .04	− .01					
.0169..	12.08	4	− .02	− .02	+0.94....	9.42	5	+0.02	+0.02
−0.0082..	12.07	7	− .03	− .03	1.90....	9.35	5	− .05	− .05
+0.0060..	12.16	5	+ .06	+ .06	2.04....	9.41	7	+ .01	+ .01
.0139..	12.09	4	− .01	− .01	2.67....	9.38	5	− .02	− .02
.0261..	12.03	5	− .07	− .04	3.04....	9.42	3	+ .02	+ .02
.0356..	12.02	6	− .03	.00	4.04....	9.44	6	+ .04	+ .04
+0.0460..	11.87	6	−0.05	−0.05	+4.48....	9.36	7	−0.04	−0.04

An independent solution for the "uniform" elements yielded results identical with those obtained by Russell, which are, therefore, adopted as final for this paper. The determination of the "darkened" k followed the method of the other assumption. The mean value, derived from 10 points along the free-hand observation curve (assumed symmetrical as usual), is 0.703, with an average deviation of ±0.026. To determine the constants for the theoretical light-curve, Table IIx was entered with this value of k, and the resulting values of $\psi(k,\alpha)$, for the fractions $\alpha^1{}_1$ used in determining k were plotted against the corresponding values of $\sin^2\theta$. The intercept and the slope of the straight line that best represented the plotted points gave at once, by equation (9),[1] $A=0.0364$, $B=0.02525$. It was found that by plotting the points on a large scale the determination of A and B was remarkably definite. A least-squares solution, based on the residuals from the adopted straight line, gave negligible corrections for both uniform and darkened disks. The probable error of each quantity is about one unit in the fourth decimal place. The determination of A and B (upon which depends the theoretical light-curve and the remaining orbital elements) from the plot of equation (9) was adopted, therefore, as a suitable substitute for a least-squares adjustment for the other orbits here considered. Because of the lack of information concerning the secondary minimum, a correction to k was not included among the unknowns of the least-squares solutions. It might be noted that with a change in k of one or two hundredths an adjustment of A and B is possible that would give a light-curve nearly identical with that adopted, but greater alterations would begin to show unallowable residuals. Equation (9) and the relation $\theta=\frac{2\pi t}{P}$ were used with the constants found above to compute the theoretical curve. The deviations, O.−C.$_d$, in columns five and ten of the table of normals, refer to this computed curve. The theoretical "uniform" curve determined by Russell is in the fourth column below; and the difference between the two, U.−D., in thousandths of a day, is in the fifth.

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[1] The equations and tables referred to throughout this paper are those of the preceding discussion, *Astrophysical Journal*, **36**, 239, 1912.

The last column contains the difference in the magnitudes, computed by the two curves, at a time midway between t_d and t_u.

COMPUTED LIGHT-CURVES FOR *W Delphini*

α^1_1	Mag.	t_d	t_u	U.−D.	Δm
1.00........	12.100	$0\overset{\text{d}}{.}016$	$0\overset{\text{d}}{.}028$	+12	+0.02
0.99........	11.985	.042	.043	+ 1	+ .03
0.98........	11.884	.050	.049	− 1	− .02
0.95........	11.624	.064	.064	0	.00
0.9	11.292	.081	.081	0	.00
0.8	10.835	.106	.106	0	.00
0.7	10.514	.127	.127	0	.00
0.6	10.266	.146	.146	0	.00
0.5	10.066	.166	.166	0	.00
0.4	9.896	.185	.185	0	.00
0.3	9.749	.206	.205	− 1	.00
0.2	9.620	.229	.227	− 2	− .01
0.1	9.505	.258	.252	− 6	− .01
0.0	9.400	0.309	0.292	−17	−0.01

The curves are sensibly coincident except near the beginning and end, and at these points the residuals in the table of normals show that the difference is in favor of the "darkened" curve. The tendency of the observations to fall below the best attainable theoretical "uniform" curve near the beginning and end of eclipse has been observed by Professor Dugan in two partial eclipses,[1] and I have found it quite common among both the total and partial eclipses investigated. The theory of darkened stars takes care of at least a part of this discrepancy.

Table III and equations (10) and (11) give the other "darkened" elements of the orbit of *W Delphini:* $i=86° 7'$, $r_1=0.241$, $r_2=0.170$. The stellar radii are expressed as usual in terms of the radius of the relative orbit. Since the loss of light at principal minimum is $2\overset{\text{mg}}{.}70$, the small bright star has 0.917 of the light of the system. The ratio of the mean surface intensities, $\frac{J_1}{J_2}$, is $\frac{1}{22}$—a considerable change from the relative brightness on the hypothesis of uniform disks, which is entirely due to the increase in the relative size of the smaller star. The depth of the secondary minimum, which has not been observed, would be nearly double the theoretical $0\overset{\text{mg}}{.}027$ required by the uniform brightness theory, and its precise measurement would be of considerable value. If we assume the components equally massive, we find that the density of the brighter is 0.060, and of the fainter, 0.021, in terms of the sun's density. The detailed comparison of the two sets of elements and of the light-curves will be made later.

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[1] *Contributions from the Princeton University Observatory*, No. 1, 1911; No. 2, 1912.

2. *S Cancri.*—The observations have been collected into 51 normal places, of which eight are composed of measures made clearly outside the limits of minima, two occur near the epoch of secondary minimum, and the remainder define the light variation of the primary. Wendell's value of the period was adopted, but his epoch of zero phase was $0^{\text{d}}.02$ too early. The phases of all the observations were corrected accordingly. From 29 observations, constant minimum light is fixed at $10^{\text{mg}}.093$, with a probable error of seven thousandths of a magnitude. Maximum light, from 63 observations, is $7^{\text{mg}}.977 \pm 0^{\text{mg}}.004$. If the orbit is circular, the mid-epoch of secondary minimum is at phase $4^{\text{d}}.7423$. Two normal points in the table below are within three hours of this, hence, well within the minimum. I believe their deviations from constant light

NORMAL MAGNITUDES OF *S Cancri*

Phase	Mag.	No. Obs.	O.−C.$_u$	O.−C.$_d$	Phase	Mag.	No. Obs.	O.−C.$_u$	O.−C.$_d$
	mg		mg	mg		mg		mg	mg
$-0^{\text{d}}.4480$..	7.97	7	−0.01	−0.01	$+0^{\text{d}}.1388$..	9.58	10	+0.02	+0.01
.4260..	8.03	8	+ .05	+ .05	.1546..	9.37	8	.00	− .01
.4056..	8.04	9	+ .06	+ .06	.1644..	9.29	7	+ .02	+ .01
.3804..	8.07	9	+ .08	+ .05	.1740..	9.18	6	.00	.00
.3560..	8.06	7	+ .03	+ .02	.1888..	9.05	8	+ .02	+ .01
.3368..	8.13	7	+ .04	+ .03	.2112..	8.87	10	+ .05	+ .04
.3143..	8.19	7	+ .01	.00	.2245..	8.71	7	.00	.00
.2942..	8.28	10	+ .02	+ .02	.2385..	8.58	10	− .02	− .02
.2741..	8.40	11	+ .03	+ .03	.2550..	8.46	7	− .02	− .02
.2565..	8.49	10	+ .01	+ .02	.2711..	8.34	9	− .05	− .04
.2360..	8.62	12	.00	.00	.2850..	8.26	8	− .05	− .04
.2209..	8.74	6	.00	.00	.2990..	8.21	7	− .02	− .02
.2026..	8.90	4	.00	.00	.3194..	8.09	7	− .05	− .06
.1819..	9.08	6	− .01	− .01	.3393..	8.05	9	− .03	− .04
.1642..	9.23	7	− .03	− .03	+0.3913..	7.95	4	−0.03	−0.04
.1425..	9.50	6	− .01	− .02					
.1264..	9.66	10	− .02	− .03	+0.43 ..	7.97	9	−0.01	−0.01
.1029..	9.92	10	+ .02	+ .02	0.58 ..	7.97	5	− .01	− .01
−0.0203..	10.07	9	− .02	− .02	0.69 ..	7.98	10	.00	.00
+0.0238..	10.11	9	+ .02	+ .02	1.30 ..	8.00	9	+ .02	+ .02
.0498..	10.12	11	+ .03	+ .03	2.39 ..	7.97	9	− .01	− .01
.0690..	10.08	10	− .01	.00	3.53 ..	7.98	10	.00	.00
.0850..	10.06	7	+ .02	+ .03	4.6387..	8.01	5	(+ .03)	(+ .03)
.0941..	9.99	7	.00	.00	4.7715..	8.03	5	(+ .05)	(+ .05)
.1039..	9.91	7	.00	.00	7.05 ..	7.99	6	+ .01	+ .01
+0.1209..	9.74	11	0.00	0.00	+8.17 ..	7.97	5	−0.01	−0.01

of $+0^{\mathrm{mg}}.03$ and $+0^{\mathrm{mg}}.05$, respectively, demonstrate the measurable existence of the secondary dip. The average deviation of the normals of maximum light is not quite $\pm 0^{\mathrm{mg}}.01$. In the following table the mean magnitudes have been rounded off to the second decimal place, though in plotting the curves and determining the residuals the third place was used.

The range of variation at primary minimum is $2^{\mathrm{mg}}.116$, hence the light, L_2, of the smaller bright star is 0.858 of the total light. In the usual way, for several fractions, α^1_1, of the light lost, corresponding values were successively determined of the magnitude, and of t, θ, $\sin^2\theta$, $B\psi$, and ψ. Entering the two ψ-tables for k, adjusting the weights of the individual determinations to reflect the relative accuracy of all parts of the curve, and taking the mean, the result, with considerable definiteness, was $k_u=0.370$, $k_d=0.523$. The straight-line determination of A and B, and the further derivation of the orbital elements, gave for the "uniform" and "darkened" assumptions, respectively,

$$
\begin{array}{ll}
A=0.0208, & 0.0208, \\
B=0.01276, & 0.01267, \\
i=83^\circ\ 10', & 85^\circ\ 25', \\
r_1=0.203, & 0.184, \\
r_2=0.075, & 0.096, \\
\dfrac{J_1}{J_2}=\frac{1}{44}, & \frac{1}{22},
\end{array}
$$

and the "equal-mass" densities, $\bar\rho_1$, $\bar\rho_2$, are

$$
\begin{array}{ll}
\bar\rho_1=0.009, & 0.012, \\
\bar\rho_2=0.178, & 0.084.
\end{array}
$$

The subscript 1 refers in all cases to the larger star. For the disks uniformly bright the secondary minimum, k^2L_1, should be two hundredths of a magnitude, and on the opposing hypothesis twice as deep. Existing observations are insufficient to distinguish, though the average error of Wendell's constant light observations suggests that such a refinement is not hopeless.

The above values of A, B, and k are used in computing the theoretical curves. Their representation of the observations is shown by the two columns of residuals in the table of normals.

The persistence of sign near the beginning and end of eclipse does not indicate a discrepancy between theory and observation, but rather indicates a small, real, and unaccountable asymmetry. The fit of the theoretical curves to the mean of the branches of the observation curve is nearly everywhere within the uncertainty of the normal points. It should be noticed that the remarkable "stand-still" in the ascending branch of the curve, so long credited on the basis of the uncertain eye estimates of the early observers of this star,[1] was an illusion only.

COMPUTED LIGHT-CURVES FOR *S Cancri*

α^1_1	Mag.	t_u	t_d	U.−D.	Δm
1.00........	10.093	$0^{\mathrm{d}}.071$	$0^{\mathrm{d}}.053$	+18	+0.02
0.99........	10.029	.087	.085	+ 2	+ .02
0.98........	9.969	.096	.095	+ 1	+ .01
0.95........	9.807	.114	.115	− 1	.00
0.9.........	9.581	.136	.136	0	.00
0.8.........	9.235	.168	.169	− 1	− .01
0.7.........	8.973	.194	.195	− 1	− .01
0.6.........	8.762	.219	.219	0	.00
0.5.........	8.585	.242	241	+ 1	.00
0 4.........	8.433	.265	.263	+ 2	+ .01
0.3.........	8.300	.287	.286	+ 1	.00
0.2.........	8.181	.312	.313	− 1	.00
0.1.........	8.075	.340	.344	− 4	− .01
0.05........	8.025	.358	.366	− 8	− .02
0.00........	7.977	0.386	0.413	−27	−0.01

3. *SW Cygni.*—The observations of this star are much fewer in number than those of the other three, but the higher accuracy of the work makes it of nearly equal importance. The principal minimum is thoroughly determined, but the value of maximum light, $9^{\mathrm{mg}}.06$, must depend upon only eight observations; and, as in the case of *W Delphini*, there are no observations near the phase of the secondary minimum. Constant minimum light, depending on 29 observations, is $11^{\mathrm{mg}}.720 \pm 0^{\mathrm{mg}}.002$. There is a slight, though perhaps not real, lack of symmetry in the curve. No alteration of the period given by Wendell was necessary, but a shift of $-0^{\mathrm{d}}.0044$ of the zero phase was made. The 192 observations are collected into the 38 normal points of the following table—the last two groups falling outside the minimum.

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[1] Clerke, *Problems in Astrophysics*, p. 306, London, 1903.

NORMAL MAGNITUDES OF *SW Cygni*

Phase	Mag.	No. Obs.	$O.-C._u$	$O.-C._d$	Phase	Mag.	No. Obs.	$O.-C._u$	$O.-C._d$
	mg		mg	mg		mg		mg	mg
$-0^{d}.2265$ . .	9.28	4	+0.01	0.00	$+0^{d}.0092$ . .	11.73	5	+0.01	+0.01
.2075 . .	9.43	6	+ .04	+ .04	.0343 . .	11.68	4	− .04	− .04
.1849 . .	9.57	5	.00	+ .01	.0544 . .	11.63	4	− .09	− .07
.1681 . .	9.74	5	.00	+ .01	.0682 . .	11.53	4	.00	.00
.1538 . .	9.90	6	.00	.00	.0792 . .	11.32	5	+ .02	+ .02
.1434 . .	10.06	6	+ .02	+ .02	.0907 . .	11.05	4	+ .03	+ .03
.1352 . .	10.17	6	+ .02	+ .02	.1002 . .	10.81	5	+ .01	.00
.1276 . .	10.30	5	+ .03	+ .03	.1090 . .	10.62	8	.00	.00
.1217 . .	10.39	4	+ .02	+ .01	.1195 . .	10.40	7	.00	.00
.1141 . .	10.53	5	+ .01	.00	.1290 . .	10.23	4	− .02	− .02
.1038 . .	10.71	5	− .01	− .02	.1389 . .	10.09	6	− .01	.00
.0949 . .	10.89	3	− .02	− .02	.1537 . .	9.91	5	.00	.00
.0847 . .	11.12	3	− .04	− .04	.1750 . .	9.66	6	.00	.00
.0729 . .	11.39	3	− .04	− .04	.1965 . .	9.48	5	.00	.00
.0629 . .	11.63	3	− .01	.00	.2116 . .	9.34	5	− .02	− .02
.0537 . .	11.70	4	− .02	− .01	.2368 . .	9.22	6	.00	.00
.0410 . .	11.73	5	+ .01	+ .01	.2641 . .	9.12	8	+ .02	.00
.0254 . .	11.73	7	+ .01	+ .01	0.288 . .	9.07	3	+ .01	.00
−0.0098 . .	11.72	8	0.00	0.00	+1.76 . .	9.05	5	−0.01	−0.01

The average deviation of a single determination of *k* from the mean of ten was less than ±0.03 for each of the limiting assumptions. The computations gave the following results:

	Uniform	Darkened
k	0.494	0.666
A	0.0430	0.04285
B	0.02636	0.02621
i	83° 41′	87° 28′
r_1	0.266	0.247
r_2	0.131	0.166
$\frac{J_1}{J_2}$	$\frac{1}{18}$	$\frac{1}{24}$
ρ_1	0.017	0.021
ρ_2	0.141	0.070

The smaller star has 0.914 of the light. The secondary minimum should be about $0^{mg}.02$ and $0^{mg}.05$, respectively. The light-curves below, and the residuals from them in the table above, show general agreement throughout the middle, and the characteristic disagreement at the extreme ends.

COMPUTED LIGHT-CURVES FOR *SW Cygni*

α^1_1	Mag.	t_u	t_d	U.−D.	Δm
1.00	11.720	0.056	0.052	+ 4	+0.01
0.99	11.611	.065	.065	0	.00
0.98	11.514	.070	.070	0	.00
0.95	11.258	.081	.081	0	.00
0.90	10.936	.094	.094	0	.00
0.8	10.485	.116	.116	0	.00
0.7	10.168	.134	.134	0	.00
0.6	9.922	.152	.152	0	.00
0.5	9.722	.170	.169	+ 1	+ .01
0.4	9.554	.187	.186	+ 1	.00
0.3	9.408	.205	.205	0	.00
0.2	9.280	.225	.226	− 1	.00
0.1	9.165	.248	.253	− 5	− .01
0.05	9.111	.263	.270	− 7	− .02
0.00	9.060	0.286	0.310	−24	−0.02

4. *U Cephei.*—The inclinations of the orbits in the three cases above approach 90° when the "uniform" theory is replaced by the other limiting solution—an inevitable result of the large increase of k. When the former requires the maximum inclination of 90°, that is, a central transit, it is obvious, from equation (11), which then may be written $\phi^1_2(k)=\frac{B}{A}$, that in order to keep the solution real on the assumption of complete darkening, the quantities k, and A and B cannot be determined independently. No degree of darkening can hope to improve the solution where the "uniform" assumption demands a central eclipse, and the most satisfactory work possible in such a case is a compromising adjustment of k and the curve elements, A and B, keeping the inclination, of course, at its maximum. The "uniform" orbit of *U Cephei* shows this characteristic, and for that reason is suitable to show a limitation in the application of the "darkened" method. Although the most observed of all the eclipsing stars I have studied, a satisfactory orbit for it has been one of the hardest to derive.

The sets of measurements during the principal minimum have been collected into groups that cover approximately equal intervals of time, but are of very different weights because of the bunching of observations on the steep parts of the curve. The 54 observations during the constant minimum phase show a non-symmetrical tendency that persists throughout the entire primary minimum. A downward slant of the bottom of the light-curve with increasing time is the observed effect, which I believe may be real and due to the irregular luminosity of the dark companion. The slight asymmetry of *S Cancri* is similar, and a sloping bottom seems to be so common with stars of large range, and bears such a definite relation to the asymmetrical tendencies of the remainder of the curve, that it is hardly probable the whole effect is subjective. The mean constant minimum light is $9^{\mathrm{mg}}.177 \pm 0^{\mathrm{mg}}.007$. The observations outside primary minimum at once indicate a secondary whose depth is about $0^{\mathrm{mg}}.07$, but the 63 observations within its possible limits are not sufficient to fix the mid-epoch accurately, nor the

NORMAL MAGNITUDES OF *U Cephei*

Phase	Mag.	No. Obs.	O.−C.$_u$	O.−C.$_d$	Phase	Mag.	No. Obs.	O.−C.$_u$	O.−C.$_d$
	mg		mg	mg		mg		mg	mg
$-0^{\mathrm{d}}.2420$..	6.82	6	+0.03	+0.03	$+0^{\mathrm{d}}.0958$..	7.82	33	0.00	−0.02
.2230..	6.84	4	+ .06	+ .05	.1056..	7.62	17	− .03	− .04
.2058..	6.88	4	+ .06	+ .06	.1159..	7.46	10	− .04	− .03
.1912..	6.91	5	+ .05	+ .06	.1262..	7.33	12	− .03	− .01
.1761..	6.93	5	− .01	+ .02	.1367..	7.25	7	.00	+ .03
.1584..	7.11	7	+ .04	+ .07	.1465..	7.13	11	− .03	.00
.1391..	7.22	8	.00	+ .02	.1574..	7.08	10	+ .01	+ .04
.1210..	7.46	5	+ .03	+ .04	.1661..	7.00	6	.00	+ .03
.1074..	7.65	7	+ .02	+ .02	.1761..	6.92	7	− .02	+ .01
.0955..	7.84	10	.00	− .01	.1865..	6.92	8	+ .03	+ .06
.0861..	8.02	19*	.00	− .02	.2007..	6.82	8	− .01	.00
.0763..	8.26	33	− .02	− .03	.2222..	6.82	7	+ .03	+ .03
.0659..	8.62	36	.00	+ .02	+0.2414..	6.79	4	+0.01	+0.01
.0571..	8.91	33	+ .01	+ .05					
.0490..	9.05	10	− .11	− .02		SECONDARY MINIMUM			
.0435..	9.16	7	− .02	.00	+1.2672..	6.80	11		
.0360..	9.17	13	− .01	− .01	1.2962..	6.85	13		
.0263..	9.14	10	− .04	− .04	1.3259..	6.85	14		
.0164..	9.19	8	+ .01	+ .01	1.3547..	6.86	12		
−0.0057..	9.19	8	+ .01	+ .01	1.3859..	6.88	8		
+0.0049..	9.15	4	− .03	− .03	+1.4352..	6.83	5		
.0154..	9.20	3	+ .02	+ .02					
.0262..	9.20	4	+ .02	+ .02		MAXIMUM LIGHT			
.0366..	9.24	4	+ .06	+ .06	+0.44....	6.78	5	0.00	0.00
.0438..	9.22	4	+ .05	+ .07	0.54....	6.81	5	+ .03	+ .03
.0490..	9.14	8	− .02	+ .06	0.81....	6.79	7	.00	.00
.0569..	8.94	27	+ .03	+ .07	0.87....	6.82	5	+ .04	+ .04
.0668..	8.60	51	+ .02	+ .03	0.95....	6.81	5	+ .03	+ .03
.0766..	8.28	59	.00	.00	1.01....	6.73	5	− .05	− .05
.0863..	8.02	53	0.00	−0.02	+1.93....	6.74	5	−0.04	−0.04

* Includes one half-weight observation.

shape and depth with much definiteness.[1] Thirty-seven observations are entirely clear of both minima and fix constant maximum light at $6^{\text{mg}}.784 \pm 0^{\text{mg}}.009$. The phases published by Wendell have been corrected for an error of the zero point of $-0^{\text{d}}.0015$.

The "uniform" k, on first determination, was 0.636 ± 0.012, but with the best obtainable values of the curve elements, $A = 0.0724$, $B = 0.0434$, and the maximum inclination, the value of k, restricted by equation (11), had to be reduced to 0.631 in order to make the solution real. The agreement of this solution with the observed curve is reasonably satisfactory as is indicated by the first column of residuals in the table above. The remaining elements of the adopted "uniform" system are: $r_1 = 0.324$, $r_2 = 0.205$, $\frac{J_1}{J_2} = \frac{1}{20}$, $\bar{\rho}_1 = 0.032$, $\bar{\rho}_2 = 0.126$, and the depth of secondary minimum should be $0^{\text{mg}}.049$.

From the same values of the ψ-function used to determine the "uniform" k, we can enter the other table and obtain 0.814 as a mean "darkened" k that represents the whole curve with equivalent exactness. Then entering Table II*ax* we find $\phi^1{}_2(k) = 0.710$, which should be equal to $\frac{B}{A}$ for a central eclipse. But this is not reconcilable with the quotient $\frac{B}{A} = 0.600$, of the last paragraph, necessary for the best representation of the observations. A smaller k gives a smaller quotient, but pulls the ends of the computed curve away from the observations. However, the far heavier weight of the middle part makes this adjustment preferable to changes of A and B. After a few trials the best values obtainable, on the assumption of complete darkening, for k, A, and B, were 0.687, 0.0724, and 0.0436, respectively. The other elements are: $r_1 = 0.319$, $r_2 = 0.219$, $\frac{J_1}{J_2} = \frac{1}{17}$, $\bar{\rho}_1 = 0.034$, $\bar{\rho}_2 = 0.104$, and the secondary minimum would be $0^{\text{mg}}.077$ in depth. The range of $2^{\text{mg}}.393$ indicates that regardless of the assumption as to the distribution of the luminosity the smaller star has 89 per cent of the system's light.

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[1] Observations by Professor E. C. Pickering with the meridian photometer also indicate the existence of the secondary minimum. Shapley, *Astronomische Nachrichten*, **192**, 79.

LIGHT-CURVES FOR *U Cephei*

α^1_1	Mag.	t_u	t_d	U.−D.	Δm
1.00........	9.177	0.048	0.040	+8	+0.03
0.99........	9.092	.052	.048	+4	+.08
0.98........	9.014	.054	.052	+2	+.07
0.95........	8.809	.060	.059	+1	+.03
0.9	8.535	.068	.067	0	+.01
0.8	8.134	.082	.083	−1	−.02
0.7	7.842	.095	.096	−1	−.02
0.6	7.613	.108	.108	0	.00
0.5	7.423	.121	.120	+1	+.01
0.4	7.260	.136	.133	+3	+.03
0.3	7.121	.150	.147	+3	+.03
0.2	6.997	.167	.162	+5	+.03
0.1	6.885	.187	.182	+5	+.02
0.05........	6.833	.200	.195	+5	+.02
0.00........	6.784	0.221	0.225	−4	0.00

The apparently unsymmetrical position of the secondary minimum would indicate that the orbit of *U Cephei* is eccentric. The evidence is too uncertain, however, to warrant a solution for elliptic elements. The observations would allow the zero phase of secondary minimum to be as much as one-tenth of a day after the mid-epoch of constant light. Using equation (31) of Russell's second paper, we would find in that case $e \cos \omega = 0.06$ where e is the orbital eccentricity and ω the longitude of periastron counted from the ascending node. There is no evidence of the relative durations of the two minima, so we cannot hope to separate e and ω. André has found the values, $e = 0.113$, $\omega = 210°$, from a consideration of the supposed periodical variation in the length of the orbital period.[1] His investigation gave also, approximately, $r_1 = 0.5$ and $r_2 = 0.3$, in units of the radius of the orbit; but he states that the dark companion is the smaller[2]—an impossible condition for a range of over two magnitudes and a constant minimum phase one-fifth the duration of total light change. The elements derived by Blažko[3] are based upon the light-curve determined by Yendell, whose observations were made by the Argelander method. The depth of minimum is three-tenths of a magnitude less than that

[1] *Traité d'astronomie stellaire*, **2**, 240, Paris, 1900.

[2] *Op. cit.*, pp. 209, 215, 232.

[3] *Op. cit.*, p. 99.

measured by Wendell, and the difference between Blaˇko's "uniform" elements and mine is largely due to this difference in range of variation. From the closeness of the component stars we should expect to find a small effect of prolateness, but Wendell's observations in maximum light give no definite sign of it.

The two limiting sets of elements for each of the four stars are collected into the following table. The densities are in terms of the sun's density, and for each system the radius of the circular orbit is the unit of length.

	W Delphini		*S Cancri*		*SW Cygni*		*U Cephei*	
	Uniform	Darkened	Uniform	Darkened	Uniform	Darkened	Uniform	Darkened
k......	0.528	0.703	0.370	0.523	0.494	0.666	0.631	0.687
i......	83° 25′	86° 7′	83° 10′	85° 25′	83° 41′	87° 28′	90°	90°
r_1......	0.256	0.241	0.203	0.184	0.266	0.247	0.324	0.319
r_2......	0.135	0.170	0.075	0.096	0.131	0.166	0.205	0.219
$\frac{J_1}{J_2}$.....	$\frac{1}{40}$	$\frac{1}{22}$	$\frac{1}{44}$	$\frac{1}{22}$	$\frac{1}{43}$	$\frac{1}{24}$	$\frac{1}{20}$	$\frac{1}{17}$
$\bar{\rho}_1$......	0.017	0.021	0.009	0.012	0.017	0.021	0.032	0.034
$\bar{\rho}_2$......	0.118	0.060	0.0178	0.084	0.141	0.070	0.126	0.104
r_1+r_2..	0.391	0.411	0.278	0.280	0.397	0.413	0.529	0.538

The "darkened" solution, unrestricted, doubles the ratio of mean surface intensities, and, while not materially increasing the density of the large faint companion, reduces the density of the brighter star to about one-half that obtained on the assumption of uniform disks. This point is of importance in indicating that the average density of the A-type stars, computed on the limiting basis of uniform luminosity, may be in error by as much as 100 per cent. The average "uniform" density of the brighter component for the four stars is 0.141; the "darkened" average, 0.080; and for the fainter companion, 0.019 and 0.022, respectively. These densities are computed on the assumption that both stars have equal masses. But it is well known that the brighter component of a close double is regularly the more massive, and, as a consequence, the brighter star will generally be more dense, and the fainter less dense, than computed above. It is probable, then, that the computed "uniform" densities very nearly represent the real conditions—the unequal masses being compensated for by a considerable degree of darkening at the limb.

The sum of the "uniform" radii is increased slightly in all cases by the introduction of complete darkening, but the maximum change is only 5 per cent. The effect on k is relatively large; but while the radius of the larger star is decreased, that of the smaller is increased, and the distance separating the stars is not much lessened.

The general effects of darkening on the elements here deduced, contradict, in part, the results derived by Blažko.[1] But he simply makes use of the longer duration of light change, and has made a secondary ordinary solution with the beginning of eclipse sooner and the beginning of totality later than for the "uniform" case. This will obviously increase k, but also increases the radius of the larger star.

Finally, we will compare the relative accuracy with which the two computed curves represent the observations. Referring to the pairs of curves in the individual discussion of each star, we see that the run of differences, U.−D., is negative and large at the beginning of eclipse where $\alpha^1_1=0$, changes sign three times, becoming positive at the beginning of totality. That is, for darkened stars the eclipse begins sooner and the total phase begins later, as found by Blažko. The large values of U.−D. near the beginning and end of eclipse do not indicate a large separation of the curves, for at those points the light changes slowly with the time and the curves are nearly flat. The relative fit of the two is shown by the average deviations of the normal points in the table below. It is sufficient for this purpose to give each point equal weight, except for *U Cephei* where unit weight was given for every ten observations or fraction of ten.

AVERAGE DEVIATIONS OF NORMAL POINTS

AVERAGE DEVIATION FOR	*W Delphini*		*S Cancri*		*SW Cygni*		*U Cephei*	
	Unif.	Dark.	Unif.	Dark.	Unif.	Dark.	Unif.	Dark.
	mg	mg	mg	mg	mg	mg	mg	mg
Upper third.....	0.015	0.011	0.037	0.034	0.010	0.009	0.028	0.033
Middle third....	.022	.020	.016	.015	.015	.013	.012	.023
Lower third.....	.026	.019	.014	.015	.023	.019	.018	.029
Constant light..	0.029	0.029	0.009	0.009	0.01:	0.01:	0.027	0.027

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[1] *Op. cit.*, p. 98.

For the three stars that are free from the central transit restriction, the "darkened" curves have a slight advantage throughout the whole course of variation. But as remarked before, there is practically no difference between the two curves in the middle third, which is usually the most heavily observed portion. A more searching test is afforded by taking the *mean* observed residuals from the two computed curves for each star, grouping the normal points of the preceding tables as follows:

1. Normals for which α^1_1 is less than 0.2. If the stars are darkened at the limb, the observed light should here be fainter than that computed on the "uniform" hypothesis.
2. Normals with α^1_1 between 0.2 and 0.98, for which the two hypotheses should give practically identical results. These observations are numerous and have been grouped in three divisions.
3. Normals for which α^1_1 lies between 0.98 and 1.00. For these the "darkened" curve should be sensibly above the "uniform" curve. The interval covered by these phases is so short that the number of observations is very small.
4. Observations during constant light at minimum. We thus derive the following summaries. The weights assigned represent simply the number of observations combined into all the normals used. The weights of the individual normals have been considered in forming these groups. The mean results for the first three stars have been combined with equal weights—the column headed "weight" representing here only the total number of observations.

For the first three stars the observations unquestionably deviate from the "uniform" curve in just the manner that might be expected to result from darkening at the limb. But the minute observable differences between a zero and a unit darkening show the futility of attempting to determine the degree of darkening in this way; however, it seems probable that in these stars (which are of spectral type A), the darkening is very considerable. Before anything very definite can be said about its amount, a better determination of the observed curves at the beginning and end of eclipse will be necessary, and also sufficient information concerning the maximum light and secondary minimum to enable the elimination of "reflection" and prolateness effects.

The best "darkened" curve which can be obtained for *U Cephei*, consistent with the condition of central transit, is not nearly as good as the "uniform" curve. The observed depth of secondary minimum, however, appears to be in better accordance with the hypothesis of darkening.

Part of Curve		Beginning or End		Upper Slope		Middle Slope		Lower Slope		Just Outside Totality		Constant Minimum Phase	
		Res.	Wt.	Res.	Wt.	Res.	Wt.	Res.	Wt.	Res.	Wt.	Res.	Wt.
W Delphini													
Descending Branch	U	+0.009	43	+0.017	61	−0.018	78	−0.006	53	−0.040	5	0.000	20
	D	.000		+ .017		− .023		− .004		− .010		.000	
Ascending Branch	U	+ .010	47	+ .005	49	+ .025	53	− .024	38	− .048	11		
	D	− .001		− .001		+ .022		− .021		− .018			
Mean	U	+ .010	90	+ .012	110	− .001	131	− .013	91	− .046	16	.000	20
	D	.000		+ .009		− .005		− .011		− .016		.000	
S Cancri													
Descending Branch	U	+ .047	39	+ .020	33	− .002	28	− .007	33	....	0	+ .011	29
	D	+ .034		+ .023		− .002		− .012		....		+ .011	
Ascending Branch	U	− .037	35	− .024	33	+ .026	31	+ .005	36	+ .002	24		
	D	− .040		− .021		+ .018		.000		+ .009			
Mean	U	+ .005	74	− .002	66	+ .013	59	− .001	69	+ .002	24	+ .011	29
	D	− .003		+ .001		+ .008		− .006		+ .009		+ .011	
SW Cygni													
Descending Branch	U	....	0	+ .011	26	+ .020	26	− .022	17	− .02:	4	.000	29
	D	....		+ .013		+ .017		− .024		− .01:		.000	
Ascending Branch	U	+ .008	17	− .004	21	− .006	25	+ .015	18	− .09:	4		
	D	− .003		− .004		− .003		+ .012		− .07:			
Mean	U	+ .008	17	+ .004	47	+ .007	51	− .003	35	− .055	8	.000	29
	D	− .003		+ .005		+ .007		− .006		− .040		.000	
Mean for all three stars	U	+0.007	181	+0.006	223	+0.006	241	−0.006	195	−0.033	48	+0.004	78
	D	−0.002		+0.005		+0.003		−0.008		−0.016		+0.004	
U Cephei													
Descending Branch	U	+ .040	25	+ .010	30	− .007	88	− .019	50	....	...	− .002	53
	D	+ .052		+ .013		− .007		+ .029		....	...	− .002	
Ascending Branch	U	+ .001	64	− .017	72	+ .006	163	+ .022	39	....	...		
	D	+ .024		− .024		+ .002		+ .068		....	...		
Mean	U	+ .012	89	− .009	102	+ .002	251	.000	89	....	...	− .002	53
	D	+0.033		−0.012		−0.001		+0.046		....	...	−0.002	

SUMMARY

1. Orbital elements are obtained for the eclipsing variables, *W Delphini*, *S Cancri*, *SW Cygni*, and *U Cephei*, upon the hypotheses of uniform brightness and of complete darkening.

2. The computed light-curves on the latter hypothesis are more satisfactory, evidently indicating the certain existence of considerable darkening at the limb in stars of spectral class A, but the difference between the two sets of curves is so small that the determination of the amount of darkening seems to be far beyond the power of the very best existing photometric observations. The beginning and end of eclipse and of totality are the only times when the light-curves differ perceptibly, and at these points the greatest differences are a few hundredths of a magnitude. For intermediate degrees of darkening computed curves will in general fall between those here derived, which may be considered limiting solutions, and all the orbital elements will likewise be intermediate.

3. Elements derived on the assumption of complete darkening will differ in general from those obtained on the uniform disk hypothesis as follows: k will be increased on the average by about 25 per cent, the larger star becoming smaller, the smaller larger; the distance separating the surfaces of the component stars will, however, remain practically unchanged; the density of the brighter star, whose light determines the spectrum, will be approximately one-half so great; the ratio of surface intensities, $\frac{J_1}{J_2}$, will be nearly doubled, that is, on account of the effect on the surface areas of the stars, the difference between the intensities per unit area will be lessened; the inclination of the orbit will approach more closely to 90°, and consequently, in the cases where the uniform disk solution requires a central transit (as for *U Cephei*), the solution for darkened stars will not always be an improvement.

Princeton University Observatory
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BAND SPECTRA OF ALUMINIUM, CADMIUM, AND ZINC

By EMILY E. HOWSON

It is a well-known fact that the spectra of the arc in air and *in vacuo* differ materially in the number of the lines and in the comparative intensities of the same lines. When the arc of certain metals is under reduced pressure, bands appear in their spectra which are not present in the arc burning in air. The object of this paper was to measure, as accurately as possible, the wave-lengths of the lines in these bands; then, to test some of the formulae which have been suggested for the mathematical representation of bands and series. The spectra of aluminium and cadmium were used for this purpose and the results are recorded below.

APPARATUS AND MEASUREMENTS

To obtain the spectra *in vacuo* a vessel was used of the type described by Dr. Barnes[1] in a recent article. The spectra were photographed in the first order of a Rowland concave grating having a radius of fourteen feet and a dispersion of 3.639 Å to the millimeter.

The spectra of the metal to be examined and of iron were photographed on the same plate. To accomplish this, one electrode was iron and the other the metal or brass filled with metal. By this means both spectra were obtained at the same time without a shift of one relatively to the other. The iron and the metal lines overlapped along the center of the plate, and it was through this portion of the plate that measurements on the lines were made, by means of a Gaertner micrometer microscope reading to 0.001 mm. Three iron lines near the band under investigation were used as standards and the metal wave-lengths calculated from these. A mean was taken of several settings and two or more plates were measured to insure accuracy. The wave-lengths of the standard iron lines were taken from Kayser's[2] table.

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[1] *Astrophysical Journal*, 34, 154, 1911.

[2] *Ibid.*, 32, 217, 1910.

ALUMINIUM

The band spectrum of aluminium has been observed by a number of investigators; as yet very few have drawn particular notice to the bands with heads at $\lambda\lambda$ 4241, 4260, 4288, and 4353. Basquin[1] observed them in a rotating arc in hydrogen and Barnes[2] found that they appeared very strongly in an arc *in vacuo;* both recorded the wave-lengths and gave their conclusions as to the origin of these bands. No attempt has yet been made to connect the lines of these bands by a formula. The difference in the wave-lengths of adjoining lines increases directly as the distance from the head. It was further found that in most cases the second differences were approximately constant. In consequence it seemed but right to conclude that, knowing the head of the band and certain constants, the wave-lengths of the remaining lines could be calculated.

Fowler[3] working with the bands which occur in the spectrum of magnesium found that they could be best represented by a formula of the type:

$$\lambda_m = a + bm + cm^2,$$

where a, b, c are constants for a band and m an integer representing the number of the line in the series. Much the same result was obtained in the case of the bands of aluminium, as will be seen in what follows.

The lines were measured as has been described, using three iron lines as standards. In some cases plates were used which did not contain iron lines. It was therefore necessary to use an aluminium line as standard, its wave-length having been carefully determined from the plates containing the iron spectrum. These standard aluminium lines, one selected in each band, were calculated from three iron lines, namely, λ 4250.129, λ 4250.789, and λ 4282.404, the average wave-lengths for the standards being λ 4244.024, λ 4261.668, λ 4287.113. It is well known that pressure produces a change in wave-length which is different in different substances, and for different lines of the same substance. The errors thus introduced into the standard lines by changes in the wave-lengths of the iron lines might amount to as much as .005 Å, but the relative wave-lengths of the band lines would not be affected.

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[1] *Ibid.*, **14**, 8, 1901.

[2] *Ibid.*, **34**, 159, 1911.

[3] *Phil. Trans. Roy. Soc.*, **209** A, 447, 1909.

The following Table I gives the values of the wave-lengths of lines in the four bands which we have called α, β, γ, and δ. The line λ 4259.400 was so close to the following line that an accurate measurement of its wave-length was difficult, so the value, as obtained from the formula given below, has been recorded.

TABLE I

α Band	β Band	γ Band	δ Band
4241.193	(4259.400)	4280.545	4353.162
41.652	59.635	83.805	53.359
42.318	60.016	87.113	54.057
43.023	60.497	90.574	55.068
44.024	60.999	94.173	56.426
45.250	61.668	98.019	58.097
46.528	62.451	4301.997	60.449
48.005	63.382	06.218	61.081
49.617	64.463	10.712	62.047
51.317	65.759	15.428	63.348
53.205	67.112	20.504	65.018
55.178	68.732		67.058
57.416	70.566		69.488
	72.593		71.340
	74.871		74.961
	77.556		78.935
			81.660

The second differences are approximately constant for all the bands except for β. Here the third differences are constant as shown by Table II.

TABLE II

First Differences	Second Differences	Third Differences
0.283		
0.355	0.072	0.014
0.441	0.086	0.012
0.539	0.098	0.015
0.652	0.113	0.014
0.779	0.127	0.014
0.920	0.141	0.014
1.075	0.155	0.012
1.242	0.167	0.015
1.425	0.183	0.012
1.620	0.195	0.015
1.830	0.210	0.014
2.054	0.224	0.012
2.290	0.246	0.016
2.542	0.252	

In the δ band there are two subseries which seem to be closely related to each other on account of the agreement of their first and second differences as shown in Table III.

TABLE III

First Subseries		Second Subseries	
First Differences	Second Differences	First Differences	Second Differences
0.698		0.632	
1.011	0.313	0.966	0.334
1.358	0.347	1.301	0.335
1.671	0.313	1.670	0.369
2.352	0.681	2.040	0.370
		2.430	0.390

Several formulae such as the following were tried upon the bands given in Table I.

$$n_m = n_0 - \frac{N_0}{(m+\mu)^2},$$

$$\lambda_m = a + bm^2 .$$

The one which gave the best results was

$$\lambda_m = a + bm + cm^2 + dm^3 .$$

Four lines were thus necessary to obtain the values of the four constants. Two separate calculations were made for these constants the average of which is given in Table IV.

TABLE IV

Band	a	b	c	d
α	4241.193	0.3809	0.0900	−0.0008
β	59.400	0.2522	0.0289	0.0023
γ	80.515	3.2240	0.0314	0.0046

The following Table V gives the differences between the observed values (Table I) and the calculated values as obtained with the above formula having the constants given in Table IV. The positive sign means that the observed value is greater than the calculated one.

TABLE V

m	α	β	γ
0	±0	±0	+0.030
1	−0.011	−0.048	+0.030
2	+0.010	−0.022	−0.003
3	−0.101	+0.018	−0.019
4	−0.077	−0.011	−0.026
5	+0.002	−0.003	+0.024
6	−0.018	+0.001	+0.019
7	+0.012	+0.012	+0.040
8	+0.027	+0.018	±0
9	−0.001	+0.072	+0.009
10	+0.003	±0	
11	−0.030	±0	
12	+0.074	+0.004	
13		−0.023	
14		−0.035	
15		+0.108	

CADMIUM

The cadmium bands extending from λ 4300 to λ 4500 have been observed before this but no relationship between the lines has so far been recorded. Jones[1] working with cadmium vapor in a discharge tube observed them and describes two bands with heads at λ 4494 and λ 4299. Later Fowler and Payne[2] using an arc *in vacuo* and Morse[3] by means of a Wehnelt interrupter found traces of these bands. Gallenkamp[4] mentions four heads—the first between the two principal lines, the second at λ 4400, the third at λ 4200, and the last at λ 4000.

It was found impossible to use a rod of metallic cadmium as one electrode for it rapidly melted away at the temperature of the arc. It was therefore placed in a cavity in the lower electrode made of brass. The lines in the spectrum due to the brass were eliminated by comparing the plates obtained when both cadmium and brass were used with those obtained when brass electrodes alone were used.

The differences in the spectra of cadmium in the arc in air and *in vacuo* are shown on Plate XIV. Fig. 1 is the spectrum in air and Fig. 2 the same *in vacuo*. Four distinct bands can be readily seen in the second figure. By applying the method of second differences, it was found that each band consisted of one or more series. The measurements of the wave-lengths of these lines were carried out in the same way as described above. The iron lines used as standards in these measurements were, for the first and second bands, λ 4494.571, λ 4482.266, and λ 4476.055; for the third and fourth, λ 4307.908, λ 4299.243, and λ 4294.124. Table VI gives these values. The series are marked by the letters α, β, γ, δ, ϵ, and ζ.

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[1] *Wied. Ann.*, **62**, 30, 1897.

[2] *Proc. Roy. Soc.*, **72**, 253, 1903.

[3] *Astrophysical Journal*, **21**, 223, 1905.

[4] *Zeits. f. Wiss. Photogr.*, **5**, 299, 1907.

TABLE VI

4510.033	4486.746 δ	4454.674 γ	4373.740	4279.872 ζ
09.488	86.370 γ	53.668 δ	68.177 δ	77.145
09.048 α	85.032 δ	52.382	65.716	76.634 ζ
08.907 β	84.518 γ	49.924 γ	60.119	76.181
08.628 α	83.071 δ	49.138 δ	57.507	73.743
08.288 β	82.325 γ	47.609	13.310 ϵ	73.111 ζ
07.953 α	80.898 δ	44.733 γ	12.982 ϵ	70.059
07.498 β	79.940 γ	44.448 δ	12.414 ϵ	69.325 ζ
07.017 α	78.265 δ	39.455 δ	11.534 ϵ	66.226
06.404 β	77.977	37.762	10.338 ϵ	65.173 ζ
05.696 α	77.224 γ	36.867	08.773 ϵ	62.087
05.005 β	75.394 δ	34.284 δ	06.948 ϵ	60.864 ζ
04.121 α	74.172 γ	32.353	04.844 ϵ	56.306 ζ
03.420 β	72.327 δ	30.289	02.390 ϵ	53.068
02.265 α	70.851 γ	28.813 δ	4299.673 ϵ	51.365 ζ
01.504 β	69.022 δ	26.925	97.535 ζ	48.183
4499.972 α	68.074	24.026	96.847 ζ	46.634
99.315 β	67.289 γ	23.080 δ	95.965 ζ	46.106 ζ
97.401 α	65.588 δ	17.114 δ	94.913 ζ	43.062
96.791 β	65.253	10.849 δ	93.582 ζ	40.645 ζ
91.838	63.394 γ	04.409 δ	92.018 ζ	34.860 ζ
91.622	61.860 δ	4397.708 δ	90.137 ζ	28.737 ζ
91.209 γ	60.639	90.719 δ	88.024 ζ	22.349 ζ
90.812 γ	59.198 γ	89.185	85.598 ζ	20.169
90.272 γ	58.366	83.471 δ	82.817 ζ	15.812 ζ
89.351 γ	57.871 δ	81.603	80.882	
88.088 γ&δ	56.951	75.938 δ	80.254	

The same formula as that used for the aluminium bands, viz.:

$$\lambda_m = a + bm + cm^2 + dm^3,$$

TABLE VII

Bands	a	b	c	d
α	4509.073	−0.28234	−0.13306	−0.00176
β	08.907	−0.41268	−0.14084	+0.00039
γ	4491.199	−0.07485	−0.18919	+0.00264
δ	88.075	−1.18740	−0.16340	+0.00190

TABLE VIII

m	α	β	γ	δ
0	+0.025	±0.000	−0.010	−0.013
1	+0.028	+0.066	+0.159	−0.020
2	+0.009	+0.023	+0.041	−0.059
3	−0.005	+0.008	−0.008	+0.023
4	+0.006	+0.023	−0.046	−0.078
5	−0.006	−0.050	+0.055	+0.026
6	−0.056	−0.059	−0.019	+0.085
7	+0.001	−0.064	−0.015	+0.080
8	−0.011	±0.000	−0.090	+0.060
9			+0.013	−0.050
10			−0.001	−0.099
11			+0.146	

was found the most satisfactory for these cadmium bands. The values of the constants a, b, c, d, for the α, β, γ, and δ bands are given in Table VII and the agreement between calculated and observed wave-lengths in Table VIII.

ZINC

In eliminating the brass lines from the spectrum of the brass and cadmium combined a fluting in the neighborhood of λ 4300 was found to appear on all the plates. By the same method of elimination as employed above, this band was found to be due to the zinc in the brass. As their wave-lengths were measured in connection with the cadmium work, they are recorded in the following Table IX. Those lines which appear to belong to a series are marked with the letter η.

TABLE IX

4300.959	4281.436 η	4254.396	4222.014
4297.322	78.328 η	49.609	21.653
97.541 η	77.806	44.579	18.121
95.965 η	74.909 η	39.315	14.136
94.687 η	70.997 η	37.658	09.874
93.135 η	67.156	35.919	09.188
91.430 η	63.127	33.094	05.277
89.329 η	58.857	30.226	00.454
86.987 η	58.310	28.002	4195.288
84.333 η	57.728	24.987	89.745

In conclusion I wish to thank Dr. Barnes for his assistance and suggestions which he so kindly made during the course of the work.

Bryn Mawr College
June 1912

OBSERVATIONS OF THE SPECTRUM OF *NOVA GEMINORUM* No. 2[1]

By WALTER S. ADAMS and ARNOLD KOHLSCHUTTER

The first observations of the spectrum of *Nova Geminorum* No. 2 were made by us on March 22, nine days after its discovery by Enebo, the delay being due to unfavorable weather and a somewhat late receipt of the announcement of its position. From that time until May 27, observations were continued as the weather and the time at the disposal of the spectrograph permitted.

All of the spectrograms were obtained at the 80-foot focus of the large reflector with the Cassegrain spectrograph. At first a single prism of 63° angle with a camera lens of 102 cm focal length was used, but after April 5 this lens was replaced by one of 46 cm focal length. With the latter lens and suitable tilt of the photographic plate it is possible to obtain good definition throughout the entire spectrum from $H\epsilon$ to $H\alpha$ with the exception of a limited region near $H\beta$. For purposes of close comparison, however, we have photographed mainly the region between λ 3900 and λ 5000, only two plates having been taken of the less refrangible portion. A list of the photographs follows:

Plate No.	Date	G.M.T.	Camera	Region
γ 1076	1912 March 22	17^h 28^m	Long	H–λ 5000
1083	23	17 21	Long	H–λ 5000
1087	24	15 36	Long	H–λ 5000
1089	28	16 24	Long	$H\beta$–$H\alpha$
1091	30	16 30	Long	$H\epsilon$–λ 5000
1102	April 3	17 09	Long	$H\epsilon$–λ 5000
1103	5	17 23	Short	$H\epsilon$–λ 5000
1104	6	15 51	Short	$H\epsilon$–λ 5000
1105	6	16 39	Short	$H\epsilon$–λ 5000
1118	22	16 15	Short	$H\epsilon$–λ 5000
1149	28	16 21	Short	$H\epsilon$–λ 5000
1185	May 5	16 08	Short	$H\epsilon$–λ 5000
1188	10	16 15	Short	$H\epsilon$–$H\alpha$
1201	27	15 50	Short	$H\epsilon$–λ 5000

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 62.

In discussing the results obtained from the reduction of these spectra we shall consider the subject under several heads:

1. General characteristics of the spectrum and changes during the observations.
2. Radial velocity.
3. The hydrogen lines.
4. The helium lines.
5. Nebular lines.
6. Probable identifications.

I. GENERAL CHARACTERISTICS OF THE SPECTRUM AND CHANGES DURING THE OBSERVATIONS

The first spectrum of the *Nova*, taken on March 22, showed a strong continuous spectrum with many dark lines, some sharp and narrow and others broad and diffuse. At numerous places faint bright bands are present, in most cases accompanied by strong dark lines, usually double, on the more refrangible side. Apart from hydrogen the more prominent of these bands are due to parhelium. The hydrogen lines are represented by strong bright bands from 20 to 30 Ångströms wide with strong double absorption lines to the violet. Each of the bright hydrogen bands has strong maxima of about equal intensity at the two edges and between the two is a broad faint absorption band at least partially broken up into numerous narrow dark lines. The effect is very similar to that observed by Campbell and Wright in the hydrogen bands of *Nova Persei* and described by them in their well-known memoir.[1]

The most noticeable changes in the spectrum in the days immediately following March 22 were the decrease in intensity of the continuous spectrum, the rapid changes and disappearance of the absorption lines, the very marked alterations in the hydrogen absorption lines, and the rapid changes in the maxima of the bright hydrogen bands. On the photographs of March 23 and 24 the dark hydrogen lines are still strong and double. On March 28, however, they are much weaker and the bright band between them appears like a wing on the bright hydrogen lines. The red line

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[1] *Astrophysical Journal*, 14, 269, 1901.

Hα shows the effect most strongly. The next spectrogram, obtained on March 30, again shows the strong absorption lines, but each is itself double, thus forming a set of four. On succeeding spectrograms the more refrangible component grows fainter and finally disappears as does the other after persisting somewhat longer.

The two chief maxima of intensity at the edges of the bright hydrogen bands, after remaining approximately equal from March 22 to March 30, show a great change after that time. On March 30 the violet maximum is decidedly broader and somewhat stronger, while on the succeeding spectrogram taken on April 3 the red component is much the more prominent. This continues to hold true to the end of our series of observations on May 27. Within the bands there are numerous changes in the secondary maxima as well.

The main feature of the later photographs of the *Nova* is the gradual change of the spectrum in the direction of the nebular spectrum, which is apparently characteristic of all *Novae*. It is marked chiefly by the following changes:

1. A steady decrease in the intensity of the continuous spectrum.
2. A corresponding increase in the intensity of numerous bright bands in the spectrum, many of which are characteristic of the spectrum of nebulae. The most important of these is the chief nebular line, λ 5007, which is first seen with certainty on April 6. It increases very rapidly after that time and on the last photographs in the latter part of May it is nearly equal to *Hβ* in intensity and one of the strongest features of the spectrum.

The second nebular line, λ 4959, was first seen on a spectrogram of April 22 and from this time until the end of our series of observations it grew rapidly stronger. On April 28 it formed one of a nearly equal pair with the helium line λ 4922 but after that time it was much the stronger of the two.

Other lines which increased greatly in intensity toward the end of the series of observations were the well-known nebular lines λ 4364 and λ 4687. The former is seen as a heavy wing upon the red side of *Hγ* on photographs taken subsequent to April 22. The line λ 4687 of the principal series of hydrogen was first seen as a faint shading upon the red side of the very bright band

λ 4614–λ 4662. It rapidly grew stronger and on the last photograph is a well-marked feature of the spectrum.

The very complex region between λ 4600 and λ 4700 undergoes many changes. On the earliest photographs there is merely a trace of some faint bright bands in this region. Between March 24 and March 30, however, a strong double bright band develops extending from λ 4628 to λ 4670 with the centers of its components at λ 4643 and λ 4662. This band later grows in width and intensity, the width reaching a maximum about April 28 when the entire band extends from λ 4574 to λ 4681. Its intensity is about one-half that of the brightest part of $H\beta$. After this time the band begins to break up into components the brightest of which extends from λ 4622 to λ 4661, its center being at λ 4642. On our last spectrograms this band has an intensity nearly equal to that of $H\beta$. It evidently coincides with the line observed by Campbell in the spectra of nebulae at 464 $\mu\mu$. The changes of intensity during the interval March 22 to May 10 are shown by the curves of Plate XVII.

In giving the results of our measures we shall give the values for four groups of plates. The first consists of the three spectrograms γ 1076, γ 1083, and γ 1087 taken on March 22, 23, and 24. The second group consists of γ 1091–γ 1105, taken between March 30 and April 6. The third group includes γ 1118 and γ 1149, taken April 22 and April 28. The fourth group consists of the photographs taken after May 5.

The less refrangible region from λ 5050 to λ 6600 is considered separately. The wave-lengths given are corrected for radial velocity.

Owing to the very rapid changes in the spectrum there are some cases of apparent contradiction in the results given in a single column; for example, the center of a dark line nearly coincides with the center of a bright band. These are due to changes on the photographs the results of which are grouped, and so are only apparent. It is hardly necessary to say that it is impossible in all cases to distinguish between true absorption lines and dark spaces, as well as between bright bands and portions of the continuous spectrum. Thus complex regions such as λ 4550–λ 4700 may be analyzed in very different ways by different observers.

TABLE I

Portion of Line or Band	Observed Wave-Lengths for Different Plate Groups				Identification
	γ 1076 –γ 1087	γ 1091 –γ 1105	γ 1118 –γ 1149	γ 1185 –γ 1201	
Violet edge bright band.....	......	......	3882.7	3881.5	
Center dark band..........	......	......	3884.7	3885.2	
Center bright band.........	......	......	3891.3	......	$H\zeta$ band λ 3889
Red edge bright band.......	......	......	3899.7	3900.8	
Violet edge bright band.....	3925.0	......	......	......	K band
Narrow dark line..........	3933.9	......	......	......	*Ca* λ 3934
Red edge bright band......	3948.5	......	......	......	
Center dark band or line....	3950.5	3952.0	.....	......	
Center faint bright band....	3955.2	3954.4	......	......	
Center dark band or line....	3959.0	3958.0	......	......	
Violet edge bright band.....	3960.9	3962.6	3960.8	3960.7	$H\epsilon$ band λ 3970
Center dark line............	......	3966.0	......	......	λ 3951–λ 3981
Narrow dark line..........	3968.7	3968.6	3969.2	3967.8	*Ca* λ 3969
Center bright band.........	3970.7	3972.6	3971.2	......	
Center dark line...........	......	3973.4	......	......	
Center bright maximum....	......	3977.6	3977.6	......	
Red edge bright band......	3980.2	3981.9	3982.0	3981.8	
Center dark line...........	3984.1	3983.5	3982.6	......	
Violet edge bright band.....	......	......	3987.1	3986.0	
Center dark band..........	......	4003.6	......	......	
Red edge bright band......	......	4009.0	4006.6	4007.6	
Red edge dark band........	......	4018.2	4018.2	4018.8	
Center faint bright band....	......	4035.0	4034.4	......	
Red edge bright band......	......	......	4052.3	4052.0	
Violet edge bright band.....	......	4062.2	4061.5	4061.8	
Center dark band..........	......	4067.0	......	4066.3	
Center bright band.........	......	4067.9	......	......	Nebular band
Violet edge bright band.....	4069.7	......	......	......	λ 4069
Red edge bright band......	......	4073.6	......	......	
Center dark band..........	4072.9	4075.1	......	4074.0	
Violet edge stronger bright band..................	4077.0.	......	......	......	Probably part of $H\delta$ band
Maximum in dark band....	......	4078.6	......	......	
Violet edge strong dark line..	4080.4	4081.1	......	......	
Center strong dark line.....	4082.5	4083.4	......	4084.0	
Red edge strong dark line...	4085.0	......	......	......	
Center bright band.........	4086.6	4085.2	......	......	
Violet edge dark line.......	4088.9	4087.0	......	......	
Center dark line...........	4090.5	4089.0	......	......	
Violet edge bright band.....	4092.6	4092.8	4089.9	4090.2	$H\delta$ band λ 4102 λ 4081–λ 4113

TABLE I—*Continued*

PORTION OF LINE OR BAND	OBSERVED WAVE-LENGTHS FOR DIFFERENT PLATE GROUPS				IDENTIFICATION
	γ 1076 –γ 1087	γ 1091 –γ 1105	γ 1118 –γ 1149	γ 1185 –γ 1201	
Maximum in bright band...	4096.0	......	......	4094.4	
Violet edge dark band......	4098.4	......	......	......	
Center dark line...........	......	......	......	4098.6	
Center bright band.........	4102.7	4103.7	4100.4	4102.6	
Center dark line...........	......	4101.8	......	......	
Center dark band..........	4104.2	4105.4	4105.2	4102.0	
Violet edge bright maximum	......	4105.6	......	......	
Center bright maximum....	......	......	4109.4	4108.0	
Red edge dark band........	4109.2	......	......	......	
Center dark line...........	......	4110.0	......	4110.7	
Center bright maximum....	4111.1	4110.1	4111.2	......	
Red edge bright band......	4113.1	4114.5	4114.6	4115.1	
Center dark line...........	......	4123.6	......	......	
Center dark line...........	4133.4	......	......	......	
Violet edge faint bright band	......	......	......	4135.9	
Center dark line...........	4153.6	4154.3	......	......	*He* band λ 4144
Red edge faint bright band..	......	4156.2	......	4157.8	
Center faint dark band.....	4160.4	4158.2	4160.4	......	
Center dark line...........	......	......	......	4168.0	
Center faint bright band....	4173.1	......	......	......	*He* λ 4169
Violet edge faint bright band	......	4180.9	......	4179.9	
Center bright band.........	......	4185.1	4182.3	......	
Center dark band..........	......	4191.6	......	......	
Red edge bright band......	4192.7	4190.4	......	......	
Center dark line...........	4196.7	4197.4	......	......	
Center dark line...........	......	4200.4	......	4198.8	
Center faint bright band....	4200.6	......	......	......	(H′ λ 4201)
Center dark line...........	......	4206.7	......	......	
Center faint dark line	4210.9	4209.2	......	......	
Violet edge dark band......	......	4210.7	......	4212.7	
Center dark band..........	4213.2	4214.2	4215.6	4213.3	
Center faint bright band....	4217.0	......	......	......	
Center dark line...........	4219.1	4220.2	......	......	
Center strong dark line.....	4222.5	......	......	4224.6	
Violet edge bright band.....	4224.0	4223.0	4225.8	......	
Violet edge bright band.....	......	......	......	4231.0	
Center bright band.........	4235.2	4236.0	4238.2	......	
Center bright band.........	......	......	......	4240.6	
Center dark line...........	......	4234.7	......	......	
Center dark line...........	4236.0	......	......	......	
Red edge bright band......	4246.5	4244.8	......	......	
Center dark line...........	4248.9	4247.7	4250.0	4248.6	
Red edge bright band......	......	4252.8	4252.6	4251.3	
Center dark band..........	......	4256.3	4255.1	4255.7	

TABLE I—*Continued*

Portion of Line or Band	Observed Wave-Lengths for Different Plate Groups γ 1076 –γ 1087	γ 1091 –γ 1105	γ 1118 –γ 1149	γ 1185 –γ 1201	Identification
Violet edge bright band.....	......	4259.6	4258.4	4259.3	
Center dark line...........	......	......	4263.3	4263.5	
Center faint bright band....	4271.4	4269.7	4269.3	4269.4	Nebular band λ 4265
Violet edge faint bright band	......	4275.6	......	......	
Center dark line............	4279.5	......	......	......	
Red edge bright band......	......	4280.0	4280.2	4280.0	
Center dark line...........	4283.4	......	4282.8	4281.8	
Center faint bright band....	......	4283.7	......	4285.6	
Center dark line...........	4289.1	......	......	......	
Violet edge bright band.....	4291.4	4292.3	......	......	
Red edge bright band......	......	4292.2	......	......	
Center dark line...........	......	4293.9	......	4294.4	
Center bright band.........	4294.8	......	......	......	
Violet edge bright band.....	......	4297.2	......	......	
Center dark band..........	4302.5	......	4303.4	4304.3	
Center bright band.........	......	4304.8	......	......	
Violet edge bright maximum	4305.5	4306.0	4305.6	......	
Center dark line...........	......	......	......	4306.6	
Center bright maximum....	4310.9	4309.5	......	......	
Center dark line...........	......	4313.0	......	......	(*Ti* λ 4313.0)
Red edge bright band......	......	4313.9	......	......	
Center dark line...........	......	4316.0	4315.6	......	
Red edge bright band......	4317.3	......	......	......	
Center dark line...........	......	4318.0	4317.5	4316.2	
Center strong dark line.....	4320.0	4319.8	......	......	
Violet edge bright band.....	4322.6	4321.9	......	......	
Center bright band.........	4324.5	4323.6	......	......	
Red edge bright band......	4325.9	4325.4	......	......	
Maximum in dark line......	......	4326.2	......	......	
Center dark band..........	4328.4	4327.8	......	......	
Violet edge bright band.....	4330.8	4330.4	4328.6	4329.4	*H*γ band λ 4341 λ 4316–λ 4354
Narrow bright maximum ...	4334.2	......	4334.4	4333.8	
Dark line.................	4335.0	4334.2	......	......	
Center bright maximum....	4335.5	4335.6	......	......	
Violet edge dark band......	4336.8	......	......	......	
Narrow dark line..........	4337.4	4337.4	4337.4	4336.9	
Dark line.................	......	4339.8	......	4339.4	
Dark line.................	4342.2	4343.6	......	4343.0	
Center bright band.........	4342.6	4343.7	4341.4	4342.0	
Center broad dark band....	4343.4	4343.3	......	4341.9	
Red edge dark band.........	4349.8	4348.4	......	......	
Dark line.................	4348.7	4350.8	......	......	
Center bright maximum....	4351.1	4350.9	4350.1	4350.5	
Strong narrow maximum....	......	......	......	4351.8	
Red edge maximum........	4353.9	4353.1	......	......	
Red edge bright band......	4353.4	4354.4	4354.4	4353.9	

TABLE I—*Continued*

Portion of Line or Band	Observed Wave-Lengths for Different Plate Groups				Identification
	γ 1076 –γ 1087	γ 1091 –γ 1105	γ 1118 –γ 1149	γ 1185 –γ 1201	
Dark line	4356.3	4355.8	4354.8	4354.6	
Center broad dark band	......	......	......	4361.6	
Center bright wing	4361.4	4359.5	......	......	
Red edge bright wing	4363.3	4365.2	......	......	
Dark line	4364.8	4365.1	......	4364.2	
Center bright band	......	......	......	4364.8	Nebular band λ 4364
Dark line	......	4367.0	......	4368.2	
Violet edge bright band	......	4368.8	......	......	
Dark line	......	4372.6	......	......	
Bright maximum	......	......	......	4373.2	
Violet edge bright band	4376.0	......	......	......	
Red edge bright band	......	......	4379.0	4377.1	
Bright maximum	4379.0	......	......	......	
Center dark band	......	......	4380.0	4380.9	
Bright maximum	......	4382.2	......	......	
Dark line	4384.4	4386.6	......	......	
Center bright band	4386.0	4386.0	......	4388.1	*He* band λ 4388 λ 4376–λ 4397
Dark line	4389.2	......	......	......	
Center faint bright band	4391.3	......	......	......	
Red edge whole bright band	......	......	......	4392.2	
Dark line	4392.6	4392.6	......	......	
Red edge bright band	4395.4	4398.5	......	......	
Dark line	4397.2	......	......	......	
Dark line	4399.4	4399.7	......	4398.9	
Violet edge bright band	4401.2	......	......	......	
Dark line	4402.5	......	......	4403.7	
Violet edge bright band	......	4406.0	4405.8	4406.2	
Dark line	4406.4	......	......	......	
Maximum in bright band	......	4409.4	......	......	
Dark line	4415.7	4416.6	......	4415.9	
Center bright band	4416.5	4418.0	......	4417.8	
Dark line	......	4419.4	4418.2	......	
Dark line	4423.5	4423.0	......	......	
Bright maximum	......	4425.0	4424.7	......	
Red edge bright band	4428.2	......	4429.8	4428.8	
Dark line	4429.0	......	......	4430.0	
Center dark band	4432.3	4431.1	4432.7	4433.2	
Violet edge bright band	4435.4	......	4435.8	4435.8	
Center bright maximum	......	4438.4	......	......	
Center bright band	4442.6	4441.6	4442.1	4443.0	*He* band λ 4438 λ 4436–λ 4450
Violet edge dark band	......	4446.2	4448.1	4447.0	
Strong dark line	......	4446.9	......	......	
Red edge bright band	4450.5	......	......	4449.8	

TABLE I—*Continued*

PORTION OF LINE OR BAND	OBSERVED WAVE-LENGTHS FOR DIFFERENT PLATE GROUPS				IDENTIFICATION
	γ 1076 –γ 1087	γ 1091 –γ 1105	γ 1118 –γ 1149	γ 1185 –γ 1201	
Center dark band	4452.4	4452.2	4452.1	4451.3	
Maximum in dark band	......	......	4455.0	4455.0	
Violet edge bright band	4455.6	4457.9	......	......	
Very strong dark line	4460.8	4460.3	4461.0	......	*He* band λ 4472 λ 4452–λ 4485
Violet edge bright band	4463.0	......	4461.5	4462.5	
Bright maximum	......	4465.0	......	4465.0	
Dark line	4467.0	4467.2	......	4467.7	(*Fe* λ 4466.7)
Dark line	......	......	......	4471.2	
Center bright band	......	4473.3	4472.6	4473.8	
Dark line	4477.7	......	......	......	
Center bright maximum	......	4480.6	......	4481.0	(*Mg* λ 4481.4)
Red edge bright band	4486.5	4485.3	4484.6	4484.5	
Dark line	4487.2	......	......	......	
Dark line	4489.3	4488.8	......	4487.8	(*Ti* λ 4488.5)
Center dark band	......	4492.2	4491.0	4490.8	
Dark line	4495.5	4493.8	......	4494.3	(*Fe* λ 4494.7)
Violet edge bright band	4496.3	......	4495.6	......	
Red edge dark band	......	4499.6	......	......	
Dark line	4501.5	......	......	4501.6	(*Ti* λ 4501.4)
Dark line	4505.2	4505.6	......	......	
Dark line	4508.1	4508.0	......	4506.8	(*Fe* λ 4508.5)
Violet edge bright band	......	4508.1	4507.0	4508.1	
Dark line	......	4510.2	......	......	
Dark line	4511.9	4512.9	......	......	
Violet edge stronger bright band	4512.4	4512.1	......	......	
Bright maximum	......	4517.0	......	4516.2	
Dark line	4518.9	4517.8	......	4517.2	
Center whole bright band	......	4520.1	......	4520.5	
Dark line	4522.8	4522.1	......	......	
Center bright band	4522.8	4523.2	......	......	
Dark line	......	4525.5	......	4526.6	(*Fe* λ 4525.3)
Dark line	4529.0	4529.0	......	......	(*Fe* λ 4528.9)
Dark line	......	......	......	4532.0	
Red edge bright band	4533.7	4534.2	......	......	
Dark line	4534.7	......	......	4535.2	
Broad dark line	......	4536.7	......	......	
Dark line	4537.6	4538.3	......	4538.6	
Red edge bright band	......	......	......	4538.4	
Violet edge bright band	4538.7	4539.8	......	......	
Dark line	4543.8	4544.8	......	......	
Red edge bright band	......	......	......	4546.6	
Dark line	4547.0	4547.0	......	......	
Center bright band	4552.4	4551.3	......	......	
Dark line	......	4551.6	......	......	
Dark line	4554.6	4555.0	......	......	
Red edge bright band	......	4562.7	4564.7	......	

TABLE I—*Continued*

PORTION OF LINE OR BAND	OBSERVED WAVE-LENGTHS FOR DIFFERENT PLATE GROUPS				IDENTIFICATION
	γ 1076 –γ 1087	γ 1091 –γ 1105	γ 1118 –γ 1149	γ 1185 –γ 1201	
Narrow dark line..........	4562.5	......	......	......	
Maximum in dark line......	......	4566.2	......	......	
Red edge bright band......	4568.6	......	......	4567.0	
Center dark band..........	4569.1	4569.5	4570.2	4570.5	
Maximum in dark band	4571.9	4570.9	4573.2	4571.4	(*Ti* λ 4572.2)
Violet edge bright band.....	4573.3	4574.9	4573.7	4575.2	
Dark line.................	4575.8	4575.6	......	......	
Center bright band.........	......	4579.4	......	......	
Dark line.................	4580.9	4580.8	......	4579.9	
Center bright band.........	4585.7	4586.0	......	......	
Dark line.................	4586.7	4587.8	......	4585.5	
Center bright maximum....	......	4594.8	......	......	
Dark line.................	......	4596.4	......	......	
Red edge bright band......	4598.2	4597.9	......	......	
Dark line.................	......	4598.9	......	......	
Bright maximum..........	......	......	......	4600.1	(Nebular 459 μμ)
Dark line.................	4601.0	4600.6	......	......	
Center bright band.........	......	4601.0	......	......	
Dark line.................	4603.3	4603.2	......	......	
Strong dark line..........	......	4605.1	......	......	
Red edge bright band......	4606.3	......	......	......	
Dark line.................	4608.3	4607.8	......	......	
Center dark band..........	4610.2	4610.4	......	......	
Center bright band.........	......	4611.7	......	......	(Nebular λ 4610)
Violet edge bright band.....	4613.0	4615.5	......	......	
Dark line.................	......	4617.7	......	4616.4	
Violet edge bright band.....	......	4621.2	......	4618.4	
Center bright band........	......	4622.0	......	......	
Dark line.................	4623.0	4621.2	......	......	
Violet edge stronger bright band..................	......	4622.5	4622.4	......	
Dark line.................	4624.3	4625.7	4626.6	4626.0	
Bright maximum..........	4627.4	......	......	......	
Center bright band.........	4631.4	4632.2	......	......	
Center dark band..........	4632.3	4633.9	......	......	
Center dark line...........	......	4636.3	......	......	
Bright maximum..........	4638.4	......	4636.4	......	
Center bright band.........	......	4640.7	......	4640.8	Nebular 464 μμ
Center dark band..........	4643.0	......	......	......	
Center bright maximum....	......	4643.4	4646.0	......	
Dark line.................	4646.2	4645.3	......	......	
Red edge bright band......	4649.5	......	......	......	
Dark line.................	......	4651.6	......	4650.7	
Center bright band.........	......	4654.3	......	......	
Center dark band..........	4655.9	......	......	4657.0	
Violet edge bright band.....	4659.2	......	......	......	

TABLE I—*Continued*

Portion of Line or Band	Observed Wave-Lengths for Different Plate Groups				Identification
	γ 1076 –γ 1087	γ 1091 –γ 1105	γ 1118 –γ 1149	γ 1185 –γ 1201	
Center bright maximum	......	4661.9	......	......	(Nebular λ 4663)
Red edge bright band	......	4664.0	......	4662.4	
Dark line	......	4667.8	......	4667.0	
Center bright band	4667.4	......	......	......	
Center bright band	4672.8	......	......	......	
Dark line	4674.8	4675.2	......	......	
Violet edge bright band	......	......	......	4674.2	
Red edge bright band	4675.3	4675.0	......	......	
Center bright maximum	......	......	......	4678.4	
Dark line	4678.4	4679.2	......	4679.7	
Center bright maximum	......	4682.3	......	......	Nebular λ 4687
Red edge bright band	4683.4	4682.4	4680.8	......	λ 4674– λ 4700
Dark line maximum	......	4686.1	......	4685.8	
Center bright band	......	......	......	4687.1	
Violet edge bright band	......	4689.0	......	4685.8	
Center bright maximum	......	......	......	4694.8	
Dark line	4694.0	4693.0	4690.7	4692.6	
Bright maximum	......	4696.8	......	.....	
Red edge bright band	......	......	......	4700.3	
Red edge bright band	......	4706.4	4708.6	......	
Center bright maximum	4709.9	4709.1	4707.5	......	
Dark line	......	4720.3	......	......	
Red edge faint bright band	......	4726.6	4725.1	4723.8	
Center faint bright band	4738.5	......	......	......	
Violet edge dark band	......	4744.0	......	......	
Maximum in dark band	4751.6	4752.1	4753.1	4751.6	
Red edge dark band	......	4756.2	......	......	
Dark line	4756.0	4756.2	......	......	
Dark line	4762.0	4762.4	......	......	
Red edge dark band	......	4765.2	......	......	
Violet edge faint bright band	......	......	4772.7	4771.9	
Dark line	4781.4	......	......	4782.2	
Dark line	......	......	......	4789.6	
Center faint bright band	......	4792.6	4793.8	......	
Dark line	......	4794.4	4794.6	......	
Center faint bright band	......	......	......	4804.1	
Violet edge dark band	......	4818.2	4817.6	4816.3	
Dark line	4824.7	4825.3	......	4826.0	
Center dark band	......	4828.9	......	4830.5	
Strong dark line	......	4833.2	......	......	$H\beta$ band λ 4862
Violet edge strong dark band	4835.8	......	......	......	λ 4833–λ 4877
Dark line	......	4835.8	......	......	
Center narrow bright band	4837.2	4836.3	......	......	
Center strong dark line	4837.8	4837.9	......	......	
Violet edge bright band	......	4837.8	......	......	

TABLE I—*Continued*

Portion of Line or Band	Observed Wave-Lengths for Different Plate Groups				Identification
	γ 1076 –γ 1087	γ 1091 –γ 1105	γ 1118 –γ 1149	γ 1185 –γ 1201	
Dark line	......	4841.1	......	......	
Center bright band	......	4841.7	......	......	
Red edge dark band	4841.4	4841.5	......	......	
Dark line	......	4842.3	......	4842.8	
Center bright line	4843.6	4842.9	......	......	
Red edge bright band	4845.9	4845.8	......	......	
Center strong dark line	4847.6	4847.6	......	......	
Violet edge bright band	4849.9	4849.8	4847.0	4847.8	
Narrow bright maximum	......	4852.0	......	......	
Bright maximum	4854.1	4854.4	4853.4	4854.0	
Narrow dark line	......	4853.2	......	4852.2	
Bright maximum	......	4855.7	4856.4	......	
Dark line	......	4856.2	......	......	
Violet edge dark band	4856.9	......	......	......	
Center dark line	4858.1	4857.8	4858.5	4859.0	
Center whole bright band	4863.0	4864.0	4862.6	4863.0	
Center dark band	4863.7	4864.5	......	4862.1	
Maximum in dark band	......	4863.9	......	......	
Maximum in dark band	......	4866.6	......	4866.5	
Red edge dark band	4870.7	......	......	......	
Bright maximum	4871.6	4872.3	4870.9	4871.2	
Center strong bright maximum	4872.9	4873.5	......	......	
Red edge bright band	4875.9	4877.6	4877.7	4877.7	
Center bright wing	......	4883.0	......	......	Part of *Hβ* band
Dark line	4885.1	......	......	......	
Red edge bright wing	......	4888.6	......	......	
Dark line	4889.9	4890.1	......	......	
Center strong dark line	......	4896.9	......	4898.2	
Dark line	4900.5	......	......	4901.6	
Center bright band	4905.4	......	......	......	
Center dark band	4909.9	4909.1	4909.0	......	
Maximum in dark band	4911.2	......	......	......	
Violet edge bright band	4912.1	4911.2	......	4911.4	
Violet edge stronger bright band	4913.4	4914.1	4913.7	4912.7	*He* band λ 4922 λ 4901–λ 4938
Dark line	......	4915.7	......	......	
Center bright maximum	......	4917.5	......	......	
Dark line	4920.7	4919.8	4920.3	4921.2	
Center dark band	4925.9	4925.1	......	4923.7	
Center bright band	4925.8	4926.2	4926.2	4924.4	
Center stronger bright band	......	4927.4	......	......	
Red edge bright band	4938.1	4938.0	4938.3	4936.1	
Center strong dark line	......	......	4940.5	4940.7	
Violet edge bright band	......	......	4943.1	4945.8	
Center bright band	......	......	4959.0	4960.1	Nebular λ 4959
Red edge bright band	......	......	4976.3	4973.6	

TABLE I—*Continued*

Portion of Line or Band	Observed Wave-Lengths for Different Plate Groups				Identification
	γ 1076 −γ 1087	γ 1091 −γ 1105	γ 1118 −γ 1149	γ 1185 −γ 1201	
Strong dark line	......	......	......	4978.0	
Violet edge faint bright band	4980.3	......	......	......	
Center faint dark band	......	......	4980.0	4983.3	
Strong dark line	......	......	......	4985.3	
Violet edge bright band	......	4990.8	4991.2	4989.5	
Center bright band	......	4992.8	......	......	
Violet edge stronger bright band	......	4993.5	......	4992.5	
Center strong dark line	4994.7	......	......	......	
Center bright maximum	......	4996.1	......	4997.1	*He* band λ 5016
Red edge bright band	5001.9	5003.7	......	......	λ 4995–λ 5032
Dark line	5004.5	5003.8	......	5004.0	
Dark line	......	5007.2	......	......	
Violet edge bright band	5006.9	5010.3	......	......	
Center bright band	......	5007.0	5007.8	5007.1	Nebular λ 5007
Center stronger part of bright band	5012.5	5011.9	......	......	
Dark line	5014.3	5015.2	......	5011.7	
Red edge bright band	......	5020.2	5023.6	5022.5	
Center bright band	5019.3	5020.1	......	......	
Dark line	5023.0	5024.2	......	......	
Center bright maximum	......	5027.2	......	......	
Red edge bright band	5032.1	5032.8	......	......	
Violet edge bright band	......	......	......	5040.6	
Center faint bright band	......	......	......	5048.7	*He* band λ 5048
Red edge bright band	......	......	......	5060.1	λ 5041–λ 5060

Portion of Line or Band	Observed Wave-Lengths		Identification
	γ 1089	γ 1188	
Violet edge faint bright band	5160.4	......	
Center bright band	5172.5	5176.7	*Mg b* group
Red edge bright band	5183.5	......	
Center very faint bright band	5239.0	......	
Center faint bright band	5279.0	......	
Violet edge bright band	5308.3	......	
Center bright band	5319.4	......	
Red edge bright band	5332.5	......	
Center faint bright band	5379.1	......	
Violet edge bright band	5566.9	......	
Dark line	5577.2	......	

TABLE I—*Continued*

Portion of Line or Band	Observed Wave-Lengths		Identification
	γ 1089	γ 1188	
Center bright band	5580.4	......	
Red edge bright band	5591.5	......	
Violet edge bright band	5649.2	......	
Violet edge bright band	......	5666.7	
Center bright maximum	......	5676.2	
Center bright band	5680.8	5680.2	
Red edge bright band	......	5693.9	
Red edge bright band	5710.6	......	
Violet edge bright band	5743.1	5742.6	
Center faint dark band	......	5754.5	
Center bright band	5757.3	5755.9	Nebular λ 5752
Bright maximum	......	5765.7	λ 5743–λ 5770
Red edge bright band	5770.0	5770.0	
Violet edge bright band	5847.3	......	
Violet edge bright band	......	5864.7	*He* band λ 5876
Maximum in bright band	5870.1	......	λ 5847–λ 5909
Center bright band	5877.9	5878.1	
Dark line	5880.9	......	
Center stronger part of band	5885.7	......	
Red edge bright band	......	5891.2	
Dark line	5890.3	......	D_2 *Na* λ 5890
Dark line	5896.6	......	D_1 *Na* λ 5896
Red edge bright band	5909.4	......	
Center faint bright band	......	5942.3	
Violet edge faint bright band	5989.0	......	
Center faint bright band	6004.8	......	
Red edge bright band	6017.1	......	
Center faint bright band	6158.8	......	
Center faint bright band	6248.8	......	
Violet edge bright band	6287.7	6289.3	
Center bright band	6302.4	6304.7	Nebular λ 6301
Red edge bright band	6317.8	6320.3	
Violet edge bright band	6352.0	......	
Center bright band	6367.6	6370.0	
Red edge bright band	6382.0	......	
Violet edge bright band	6449.5	......	
Center bright band	6474.1	......	
Center bright maximum	6480.1	......	
Center bright band	......	6486.5	
Dark line	6502.0	......	
Red edge bright band	6509.0	......	
Dark line	6519.6	......	

TABLE I—*Continued*

Portion of Line or Band	Observed Wave-Lengths		Identification
	γ 1089	γ 1188	
Violet edge bright band	6529.7	......	
Center bright band	6536.9	......	Wing on *Hα* band
Red edge bright band	6543.0	......	
Dark line	6545.1	......	
Violet edge bright band	6548.3	6545.4	
Maximum in bright band	6555.3	......	*Hα* band λ 6563
Center bright band	6567.0	6569.4	λ 6545–λ 6585
Center dark band	6567.2	......	
Maximum in bright band	6578.6	......	
Red edge bright band	6585.2	......	
Center bright wing	6593.7	......	
Red edge bright band	......	6595.2	
Red edge bright wing	6600.5	......	Part of *Hα* band
Bright maximum	......	6672	
Center faint bright band	......	6677	*He* λ 6678

2. RADIAL VELOCITY

The principal lines available for the determination of the radial velocity of the *Nova* are the narrow absorption lines H and K of calcium, certain faint lines, for the most part due to iron, which appear on some of the plates, and the narrow lines D_1 and D_2, which are present upon one of the photographs taken in the less refrangible region. Owing to the strong absorption of the glass of the prism employed, the spectrograms are rather weak in the H and K region and the results obtained from these lines are not very accordant. The following is a summary of the values from the various lines:

Plate	Lines	Velocity	Reduction to Sun	Radial Velocity
		km	km	km
γ 1076	H	+40.2	−29.3	+10.9
1083	H, K	+47.0	−29.4	+17.3
1087	H, K	+33.1	−29.3	+ 3.8
1104	H	+35.0	−29.4	+ 5.6
1118	H	+28.5	−27.5	+ 1.0
1089	D_1, D_2	+47.4	−29.5	+17.9
1076	4222, 4528	+47.3	−29.3	+18.0
1083	4222, 4236, 4528	+39.6	−29.4	+10.2
1087	4222, 4250, 4494, 4501, 4528	+35.2	−29.3	+ 5.9

Although the range shown by these observations is rather large, in view of the character of the lines we do not consider any certain variation of velocity as established. The mean of the results as given in the table is

$$+10 \text{ km}$$

and the mean of the final results of the six plates is

$$+10 \text{ km.}$$

Determinations of the radial velocity by other observers are as follows:

Curtiss	+ 9 km
Plaskett	+12 km with a probable variation of 15 km
Küstner	+ 7.0 km

3. THE HYDROGEN LINES

The bright hydrogen bands are characterized by definite edges upon the red side which can be measured with a very fair degree of accuracy. On the violet side the dark absorption lines upon the earlier photographs form the boundary which seems to persist as the absorption lines grow fainter. These two limits, accordingly, represent the principal part of the emission band, while a broad fainter band extends toward the violet. The strong band is the

CENTERS OF BRIGHT HYDROGEN BANDS

Plate	$H\epsilon$	$H\delta$	$H\gamma$	$H\beta$	$H\alpha$
γ 1076	3971.0	4103.2	4342.4	4863.0	......
1083	3970.1	4102.5	4343.3	4862.8	......
1087	3970.7	4102.3	4342.2	4862.9	......
1089	......	......	......	4862.9	6566.8
1091	3972.5	4102.7	4342.2	4863.1	......
1102	3972.5	4102.3	4342.8	4863.5	......
1103	......	4103.0	4343.1	4863.9	......
1104	3972.7	4104.7	4343.1	4867.9	......
1105	......	4105.8	4343.9	4864.6	......
1118	3972.0	4103.0	4342.2	4862.7	......
1149	3970.6	......	4340.9	4862.5	......
1185	3971.2	4103.2	4341.3	4862.4	......
1188	......	4101.9	4341.9	4863.0	......
1201	......	4103.4	4342.4	4863.3	......
Mean	3971.5	4103.2	4342.4	4863.3	6566.8
Δ	+1.3	+1.3	+1.8	+1.8	+3.8

portion upon which our measures of width and displacement have been made. In the accompanying tables we have collected the results of our measures upon both the dark and the bright hydrogen lines.

An examination of the measures of the bright hydrogen bands shows that as in the case of *Nova Persei* the centers are displaced toward the red. On our photographs we have measured the centers directly, as well as the violet and the red edges of these bands. Accordingly, two independent determinations are available. Collecting all of the results we obtain the table on p. 308.

The displacements from the normal positions of the lines are given opposite Δ in the last row of the table. It is a peculiar fact that these values are better satisfied by a law in which the displacement is directly proportional to the square of the wave-length than they are by one in which it is proportional to the first power such as is found for the dark hydrogen lines and the widths of the bright hydrogen bands. The difference is due almost entirely to *Hα* for which but one determination is available. A least-squares solution gives the following residuals in the two cases:

	λ	λ^2
Hε	−0.2	0.0
Hδ	−0.2	−0.1
Hγ	+0.2	+0.2
Hβ	0.0	−0.1
Hα	+1.3	+0.2

The dark hydrogen lines could be measured satisfactorily upon but a few of the earlier spectrograms. The results follow:

DARK HYDROGEN LINES

Plate	*Hε*		*Hδ*		*Hγ*		*Hβ*		*Hα*
	I	II	I	II	I	II	I	II	I
γ 1076	......	......	4090.2	4082.2	4328.3	4319.5	4847.6	4837.0	......
1083	3959.0	3950.5	4090.6	4082.4	4328.4	4320.2	4847.5	4837.5	......
1087	3958.9	3950.5	4090.8	4082.8	4328.5	4320.4	4847.7	4839.0	......
1089	......	......	......	......	......	......	4848.0	......	6545.2
Mean	3959.0	3950.5	4090.5	4082.5	4328.4	4320.0	4847.7	4837.8	6545.2
Δ	−11.2	−19.7	−11.4	−19.4	−12.2	−20.6	−13.8	−23.7	−17.8

If we take the values of the displacements of the dark lines, given as Δ in the table, and determine their variation with wave-length we find the relationship obtained by Campbell and Wright for the dark hydrogen lines in *Nova Persei*, and by Plaskett in the case of the present *Nova*—that is, the displacement is directly proportional to the first power of the wave-length. The residuals from a least-squares solution assuming the values of equal weight are as follows:

	Component I O.−C.	Component II O.−C.
$H\epsilon$	+0.2	+0.5
$H\delta$	0.0	−0.4
$H\gamma$	+0.1	−0.4
$H\beta$	+0.4	+0.3
$H\alpha$	−0.4	. . . . .

These residuals are within the limits of error of the measurements.

We have already referred to the fact that on plate γ 1091 taken March 30 each component of the hydrogen lines is itself double, although in the case of $H\delta$ and $H\beta$ we have been unable to measure the separate portions of the less refrangible component. The values for this plate are as follows:

	$H\delta$			$H\gamma$				$H\beta$		
	I	II		I		II		I	II	
		a	*b*	*a*	*b*	*a*	*b*		*a*	*b*
γ 1091	4088.9	4078.6	4082.2	4326.2	4328.5	4315.9	4319.8	4846.5	4833.2	4837.9
Separation		3.6		2.3		3.9			4.7	

The values of the separation of the two lines forming component II appear to be closely proportional to the wave-length. It is a noteworthy fact that the position of the *b* line of component II in each case occupies the same position as the center of the whole component upon the plates γ 1076–γ 1089. In other words, the doubling of the lines upon γ 1091 is due to a new component formed about 4 Ångströms to the violet of the original lines. The change, accordingly, can hardly be ascribed to anything in the nature of a variation of pressure or to a possible magnetic field, but must rather be due to the action of an additional mass of absorbing gas, and it is probably allied to some of the phenomena of multiple reversal in laboratory spectra.

The widths of the bright hydrogen bands are, of course, influenced by the density of the spectrograms, and similarly the relative widths are affected by the curve of sensitiveness of the photographic plates employed. A comparison of the widths of the lines for the whole series of photographs is of considerable interest, however. In the accompanying table the values are given in Ångström units. Owing to a defect in the photograph at the position of the $H\alpha$ line on γ 1188, its value is omitted.

WIDTHS OF BRIGHT HYDROGEN BANDS

Plate	$H\epsilon$	$H\delta$	$H\gamma$	$H\beta$	$H\alpha$
γ 1076.......	18.7	20.8	22.5	26.1	....
1083.......	19.3	20.8	....	26.9	....
1087.......	20.2	19.9	22.4	25.7	....
1089.......	....	....	....	26.3	38.8
1091.......	22.7	23.0	22.0	27.2	....
1102.......	....	24.5	25.6	29.0	....
1103.......	....	24.1	24.0	28.5	....
1104.......	17.4	20.1	23.2	28.5	....
1105.......	....	17.2	22.1	28.2	....
1118.......	20.5	22.0	25.8	29.6	....
1149.......	22.1	27.1	25.7	31.9	....
1185.......	21.0	25.6	25.4	32.1	....
1188.......	....	25.0	24.6	30.1	....
1201.......	....	23.1	22.8	24.8	....
Mean......	20.2	22.3	23.8	28.2	38.8

An investigation of these values shows that, like the displacements of the hydrogen absorption lines, they vary in direct proportion to the wave-length. The residuals given by a least-squares solution assigning weights according to the number of measures are as follows:

	O.–C.
$H\epsilon$..............	-1.8
$H\delta$..............	-0.5
$H\gamma$..............	-0.3
$H\beta$..............	$+1.2$
$H\alpha$..............	$+2.4$

A similar relationship has been found by Professor Plaskett for the widths of the bright hydrogen bands.

4. THE HELIUM LINES

The helium and parhelium lines which can be identified with reasonable certainty in the *Nova* spectrum are given in the following list. The absorption lines accompanying the bright bands are also included, the more doubtful identifications being placed in parentheses.

CENTERS OF HELIUM LINES

He	Bright Band	Absorption Lines	
		I	II
4143.92	4146.4	(4133.4)	(4123.6)
4169.13	4173.1	......	......
4388.10	4387.4	......	......
4437.72	4438.4	(4429.5)	(4418.6)
4471.65	4473.2	4460.6	(4452.2)
4922.10	4925.9	4911.1	4900.6
5015.73	5019.5	5004.2	4994.7
5047.82	5048.7	......	......
5875.87	5878.0	......	......
6678.37	6677	......	......

In addition to these lines λ 4713.25 is probably present, at least upon the later photographs, but blended with a portion of the great band which extends from λ 4600 to beyond λ 4700. The average displacement of the bright bands toward the red is 2.1 Ångströms, or about the same as that of the hydrogen lines, but the measurements are much less accurate except in the case of λ 4926 and λ 5020, the bands being fainter and probably complicated with other bands in some cases. It seems probable that the displacements of both dark and bright lines are proportional to the wave-length.

Campbell and Wright in their discussion of the spectrum of *Nova Persei* have called attention to the interesting fact that on the earlier spectrograms of the star the parhelium lines are the ones observed, while in the later observations these have become very faint or have disappeared and the helium lines have become prominent. They refer to the observations of Runge and Paschen which indicate that with increase of pressure in the radiating gas the helium lines become stronger relatively to the parhelium lines. Of the lines listed in the table above as observed in *Nova Geminorum*, λ 4471, λ 4713, and λ 5875 belong to the helium series and the remainder to parhelium. The line λ 4471 was probably present upon the earliest photographs, but very faint. It was first measured by us on a spectrogram taken on April 6, and continuously throughout the series after that time. The line λ 4713 we have already referred to as probably being present upon the later photographs merged in another bright band. The D_3 line, λ 5876, is present on the spectrogram of March 28, but is considerably stronger upon the photograph of May 10. The parhelium lines, with the possible exception of λ 5048 and λ 6678, were much stronger upon the earlier photographs, although most of them could still be seen throughout the whole series. The line at λ 5016 was last measured on March 30. The two parhelium lines, λ 5048 and λ 6678, however, appear to be present only upon the later photographs, the former being first measured on the spectrogram of May 5, and the latter on May 10. In general the relative behavior of helium and parhelium lines seems to resemble their behavior in *Nova Persei*, but not to be identical with it.

5. NEBULAR LINES

The bands which are almost certainly identical with lines observed in the spectra of nebulae are indicated in Table I. In addition to hydrogen and helium they are as follows:

Nova		Nebulae
Limits	Center or Maximum	
4062–4074	4068	4069
4259–4280	4270	4265
	4365	4363
	4579	4574
	4599	459
	4612	461
	4641	464
4674–4700	4687	4686
4954–4974	4960	4959
4993–5022	5007	5007
5743–5770	5757	5752
6288–6319	6303	6301

The line λ 4686 is the first line in the principal series of hydrogen. The three lines of the second subordinate series may perhaps be present in the *Nova* spectrum; they are known to be present in the spectra of Wolf-Rayet stars, and their wave-lengths as determined by us in the star *BD*+30° 3639 are added for comparison.

H	*Nova*	*BD*+30° 3639
4026	Band 4018–4052	4028
4201	Center faint band 4201	4201
4542	Violet edge bright band 4540	4542

The spectrum of the *Nova* in its later stages and the spectra of Wolf-Rayet stars have numerous features in common. The greatest point of difference is, of course, the absence, or at least the comparative faintness, of some of the principal nebular lines in the Wolf-Rayet spectrum. Apart from this feature there is probably fully as much difference between spectra of individual stars of the Wolf-Rayet type as there is between the *Nova* and certain of these stars. For purposes of comparison the spectrum of the Wolf-Rayet star *DM*+37° 3821 is reproduced with the *Nova* spectra in Plate XV. The most striking feature of the spectrum of this star is the great intensity of λ 4686 and the prominence of the three lines already referred to which belong to the second subordinate series of hydrogen, λ 4026, λ 4201, and λ 4542.

A comparison of the widths of the bright hydrogen bands in the spectrum of *DM*+37° 3821 and of the *Nova* gives the following results:

	Nova	*DM*+37° 3821
$H\delta$	λ 4092–4114	4081–4117
$H\gamma$	4330–4354	4325–4354
$H\beta$	4849–4876	4843–4877

Within the limits of error of measurement the red edges of the bands have the same position. The violet edges in the case of the *Nova* are affected by the presence of absorption which is absent in the case of the Wolf-Rayet stars. The violet edges, accordingly, differ by roughly the same amount in the two cases. It is clear, therefore, that in a type of spectrum which we are accustomed to consider as relatively permanent, emission bands of the same order of width are present as in the spectra of *Novae*. On the other hand, the hydrogen lines in the spectrum of the Wolf-Rayet star $BD+30^\circ$ 3639 are very narrow while other bands in the spectrum are as much as 20 Ångströms wide. This seems to us an objection to the hypothesis of Kayser that such bands may be due to the Doppler effect in gases under the bombardment of *Kanalstrahlen*.

In 1907 Hartmann made an important investigation of the spectrum of *Nova Persei* nearly six years after its discovery[1] and found its spectrum to resemble very closely that of the Wolf-Rayet star $BD+35^\circ$ 4001. *Nova Persei* at that time was of about the eleventh magnitude.

6. PROBABLE IDENTIFICATIONS

The sodium lines D_1 and D_2 appear as narrow absorption lines in a bright band whose red edge falls at λ 5910. On the violet side this band blends with the bright D_3 band of helium.

A faint bright band extending from λ 5160 to λ 5184 is probably due to the *b* group of magnesium. A maximum at λ 4480 in the bright helium band λ 4463–λ 4485 is probably due to magnesium λ 4481.

The H and K lines of calcium like the sodium lines are narrow absorption lines in broad bright bands. The bright band due to the K line extends from λ 3925 to λ 3949, while the H band is lost in that of the $H\epsilon$ line. The K band is not seen after March 30. It seems very doubtful to us whether λ 4227 of calcium is represented in the *Nova* spectrum. This region is extremely complex and the bright band falling nearest the position of the line extends from λ 4224 to λ 4246 with a center at about λ 4235 on the earlier plates. On later photographs it seems to have even a longer wave-length, due probably to an extension of the band toward the red. The two absorption lines λ 4223.1 and λ 4213.9 probably belong to this band, and the wave-lengths of all three make it improbable that this can be due to calcium λ 4227.

A number of faint absorption lines present on the earliest plates may perhaps be identified with calcium. A number of these are given by Professor Küstner. Our own values are as follows:

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[1] *Astronomische Nachrichten*, **177**, 113, 1908.

Nova	Sun (Rowland)
λ 4282.1	4283.17
4302.8	4302.69
4307.3	4307.91
4459.0	4454.95
4580.7	4581.58
4587.0	4586.05
4685.9	4685.45

In view of the large number of dark lines present in the *Nova* spectrum these identifications seem to us of rather doubtful value.

A considerable number of the absorption lines may be identified with lines of iron, and several others with lines of titanium, particularly with enhanced lines of this element. The possible identifications are indicated in Table I.

In his discussion of the spectrum of *Nova Lacertae*[1] Mr. Wright has instituted an interesting comparison between the bright stellar bands and the lines of the spark spectrum of nitrogen. Other observers have noted a similarity between the spectra of Wolf-Rayet stars and the spectra of oxygen and nitrogen. If we compare the lines of nitrogen given by Wright with our measures of the bright bands of *Nova Geminorum* we obtain the list in Table II.

There is practically a continuous band in the *Nova* spectrum between λ 4600 and λ 4700, and the values given are the centers of subordinate bands or of maxima. The fainter lines of the nitrogen spectrum show about the same degree of agreement with the stellar bands. In view of the evidence we are inclined to consider the presence of nitrogen in *Nova Geminorum* as probable, although we would agree with Mr. Wright in considering it as hardly proven.

The announcement by Professor Küstner and Dr. Giebeler of the discovery of dark lines in the *Nova* spectrum due to radium and the radium emanation arrived while we were engaged in preparing our results for publication. Owing to the fact that our series of observations did not begin until March 22, or several days after the first observations at Bonn, the results are not fully comparable, especially in view of the very rapid changes in the spectrum at this time. As bearing on the question of the presence of radium absorption lines after March 22, however, the results may be of interest.

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[1] *Lick Observatory Bulletin*, No. 194, 1911.

TABLE II

Nitrogen Exner and Haschek and Neovius	Intensity	*Nova* Bright Bands
3995	50	Band 3986–4008
4026	3	Band 4018–4050
4035	4	Center 4035
4042	5	
4097	3	Conflict with $H\delta$
4103	3	
4146	4	Center band 4146 (*He?*)
4176	3	Center band 4173 (*He?*)
4207	2	
4229	3	Band 4224–4246
4237	5	Center 4237
4242	5	
4348	2	Conflict with $H\gamma$
4426	2	Maximum 4425
4433	2	Band 4435–4450
4447	20	Center 4442
4508	2	
4515	1	Band 4508–4534
4530	2	
4601	5	Maximum 4600
4607	4	Center band 4605
4614	3	
4622	4	Center band 4622
4631	15	Center band 4632
4643	5	Center band 4641
4649	5	Center band 4649
5003	10	Band 4991–5032
5007	10	*He* 5016 and on later plate N_1 5007
5045	7	Conflict with *He* 5048
5497	6	
5535	6	
5667	9	Band 5667–5694
5680	12	Center 5681
5712	6	Red edge band 5711

In the region of spectrum considered by Giebeler there are 12 lines in the spark spectrum of radium for which he has found corresponding lines in the *Nova*. Of these we have been able to measure seven upon our photographs. Two of these, however, fall within the $H\gamma$ band and their agreement with radium lines is entitled to little significance on account of the presence of a great number of narrow dark lines, so many, in fact, that accidental coincidence would be almost certain. On two of the remaining five lines, λ 4153 and λ 4532, our measures show very close agreement with those of Giebeler and indicate discrepancies of over an Ångström unit from the radium lines.

In the region of wave-lengths longer than λ 4533 we find possible coincidences with λ 4600, λ 4642, λ 4693, and λ 4826. The first of these seems to be double in the *Nova* spectrum, and the discrepancy both for it and for λ 4826 is large. The extremely bright radium line λ 4683 does not appear to be represented in the *Nova*.

The degree of correspondence in the case of the emanation spectrum is about the same as for the radium spark, about 50 per cent of the lines having possible coincidences in the *Nova* spectrum. There are dark lines near the positions of the very intense lines λ 4167, λ 4350, λ 4609, and λ 4626, but none near λ 3982, λ 4203, and λ 4681. In general there seems to be very little agreement between the relative intensities of the emanation and the radium spark lines and the dark lines in the *Nova*, and the same fact seems to be true of Giebeler's results. Thus the four strongest radium lines (omitting λ 4341, which falls on the $H\gamma$ band) in the part of the spectrum investigated, λ 4178, λ 4305, λ 4436, and λ 4533, have been measured by him upon but one photograph, while several of the fainter lines have been measured more frequently.

In view of these results we do not consider the presence of the radium or the emanation spectrum in the *Nova* as established, at least for the period covered by our observations.

We have not been able to detect with any certainty evidence of a periodic variation in the structure of the bright hydrogen lines such as was suspected by Wolf,[1] but the dates of the observations were not particularly favorable for such an investigation. It is perhaps worthy of note that there appears to be some evidence of a progressive change in the wave-lengths of the bright hydrogen bands during the period of rapid decrease in the light of the star, March 25–April 6. As the brightness grew less and the star more red in color the bands appeared to shift somewhat toward the red. It is possible that there may be a shift in the maximum of the individual bands with decrease of temperature similar to that found in a continuous spectrum.

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[1] *Astronomische Nachrichten*, **191**, 167, 1912.

The complex structure of the hydrogen bands led us to make an attempt to detect possible polarization in the spectrum of the star. Two spectrograms were obtained on April 7, using a Nicol prism and a quarter-wave plate in front of the slit. No displacement of the bright maxima was found when the quarter-wave plate was rotated. The dark hydrogen lines had become very faint at this stage of the development of the spectrum and are not visible on the photographs.

Since the above was written we have succeeded in obtaining two spectrograms of the *Nova* in the eastern sky. The first of these, γ 1472, was secured on August 19 and includes the portion of the spectrum between λ 4000 and λ 5000. The second, γ 1591, obtained on September 4, includes the entire spectrum from λ 4000 to $H\alpha$. On account of the low altitude of the star and the impossibility of giving a very long exposure because of the approach of daylight the second spectrogram was somewhat underexposed. Enlargements of the spectra are shown in Plates XV and XVI. The star appeared to be not far from the ninth magnitude at the time of these observations.

A comparison of these spectrograms with those obtained in the latter part of May indicates a further development of the spectrum of the star toward the nebular stage. The chief nebular line, λ 5007, is now considerably more intense than $H\beta$, and similarly the nebular line at λ 4365 is stronger than $H\gamma$. The second nebular line, λ 4959, and the line at λ 5757 have also increased greatly in intensity. On the other hand, the band at λ 4641 has become fainter and the principal series line of hydrogen at λ 4687 can hardly be distinguished at all. The helium lines λ 4472 and λ 5876 are of about the same intensity as on the earlier photographs.

In the accompanying table are given the results of our measures of some of the more important bands and lines on these spectrograms, together with the results obtained from the photographs taken in the latter part of May.

	γ 1472 $-\gamma$ 1591	Width Ångströms	γ 1185 $-\gamma$ 1201	Width Ångströms
He......................	3970.1	15.3	......	....
$H\delta$......................	4102.5	22.9	4102.6	24.9
Nebular..................	4269.6	19.2	4269.4	20.7
$H\gamma$ and nebular...........	4351.1	45.6	(4353.3)	(47.7)
	4442.5	15.8	4443.0	14.0
Helium..................	4473.3	20.3	4473.8	22.0
Nebular..................	4639.5	38.9	4640.8	....
$H\beta$......................	4862.6	25.7	4863.0	29.9
Nebular..................	4960.1	25.8	4960.1	27.8
Nebular..................	5007.9	28.5	5007.1	25.4
Nebular..................	5680.4	....	5680.2	27.2
Nebular..................	5757.4	26.3	5755.9	27.4
Helium..................	5878.2	....	5878.1	26.5
$H\alpha$......................	6566.5	35.8	6569.4	39.8

It is clear from this comparison that no very marked changes either in the wave-lengths or the widths of the bright bands have taken place in the interval of three months between the two series of observations. The widths of the bands are somewhat less on the later photographs, except in the case of one or two lines; but the difference is probably due in main to the fact that the density of the later negatives is less than that of the earlier series. In this respect, as has previously been stated, the bands resemble similar bright bands found in the spectra of Wolf-Rayet stars, and are widely different from the narrow sharp lines characteristic of the spectra of gaseous nebulae.

There appears to be a slight tendency toward a reversion to their normal wave-lengths of the centers of the bright bands, but the effect is too slight to be recognized with certainty. The marked change in the wave-length of $H\alpha$, and in a less degree of λ 5757, is due in all probability to the uncertainties of measurement in this part of the spectrum. A general displacement toward longer wave-lengths of the centers of the bands is still a characteristic of the spectrum, as it was in the earlier observations.

An important feature of the bright bands is the persistence of the faint broad absorption bands crossing them, which have been observed throughout the entire series of photographs. They appear to be symmetrically placed on the bright bands and are present on the nebular bands as well as on those of hydrogen and helium. In some cases they seem to be broken up into narrow lines such as were found upon some of the earliest photographs. The nature of the absorption is such as is characteristic of a thin layer of absorbing gas of considerable density.

We are indebted to Miss Lasby and Miss Ensign for many of the measures upon the spectrograms, and to Miss Burwell for the intensity-curves drawn from the original negatives. The enlargements of the spectra are due to Mr. Ellerman.

MOUNT WILSON SOLAR OBSERVATORY
September 23, 1912

THE PRIMARY STANDARD OF LIGHT

BY HERBERT E. IVES

A primary standard of light, properly to deserve the title, should be specified, as nearly as is possible, in terms of the fundamental units of length, mass, and time. No existing, so-called primary standards—the candle, the Hefner, the Violle, the pentane lamp—conform to this idea of a primary standard. In these latter the specification is in terms of physical and chemical quantities such as burner dimensions and fuel composition, which, while assuring the essential characteristic of *reproducibility*, can lay no claim to fundamental character.

The primary standard which should be defined is that of the quantity with which we are actually concerned in the use of light, namely, *luminous flux*, of which the present unit is the *lumen.* Luminous flux is flux of radiant energy of a certain quality, in virtue of which it is useful for illumination; the energy possessing, in short, the capacity to arouse through the eye the sensation of light or brightness. An immediate analysis resolves the specification of luminous flux into two parts, one physical, the other physiological. The physical part is rate of flow of radiant energy, directly expressible in fundamental physical units; the physiological part is the coefficient which evaluates or determines the efficiency of the radiant energy as a producer of the sensation of light.

The nearest approach, therefore, to a complete specification of light flux in fundamental units is attained in the specification of an energy flux, weighted according to its value as a producer of the sensation of light. Several suggestions for such a standard have been made. I proposed some time ago[1] *one watt of radiation of maximum luminous efficiency* as the unit of luminous flux, and suggested that it be determined by measuring both as radiation and as light a selected monochromatic radiation of known luminous efficiency. The mercury green line (.546 μ) lying close to or at the maximum of visual sensibility appeared particularly suitable.[1] Strache[2] has proposed that the radiation from a source be resolved into a spectrum which shall fall upon a diaphragm cut to the shape of the luminosity-curve of the normal eye. After recombination the radiation is to be measured in absolute units and the values obtained from various sources will necessarily be proportional to their luminous intensities. Houstoun[3] along similar lines suggests the use of a special absorption screen, whose spectral transmission shall be as the visual luminosity-curve.

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[1] "Energy Standards of Luminous Intensity," *Transactions Illuminating Engineering Society*, p. 258, April 1911; "Luminous Efficiency," *Electrical World*, June 15, 1911.

In order that any of these suggestions may bear fruit it is necessary to establish the relative efficiencies of various radiations, in other words, to determine the luminosity-curve of the average eye for a normal (equal-energy) spectrum. In order to determine this it is in turn necessary to possess a method of photometry which shall make possible the measurement of lights of different color.

Until recently the problem of heterochromatic photometry was far from solution. Consequently the establishment of a rational primary standard of light could not proceed beyond the stage of suggestion. (In passing it may be observed that the connection between the problem of heterochromatic photometry and the rational standard of light has not been widely noted.) Work which I have recently completed on the photometry of lights of different color makes it possible to put the proposal for a radiation primary standard of light flux in more definite form.

The complete discussion of the proposed radiation standard may best be divided into three parts: (1) the photometry of lights of different color; (2) the measurement of luminous efficiency; (3) specification of the radiation standard.

I. THE PHOTOMETRY OF LIGHTS OF DIFFERENT COLOR

The determination of the relative brightness of different colored lights has always been a difficult and unsatisfactory process. Various methods—visual acuity, persistence of vision, equality of brightness appearance, and lately the flicker method—all have been employed in the attempt to evaluate the different qualities of radiation on the basis of their common attribute of brightness, difficult as it is to separate the latter from the attribute of hue. Various physiological factors such as the Purkinje effect make the problem vastly complex, and until recently we had no definite information as to the relationship between the renderings of the different photometric methods, nor did we have sufficient information to justify adopting any one method as "right."

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[1] Fabry and Buisson have lately made such a measurement. *Comptes Rendus*, **153**, 254.

[2] Abstract in *Proc. Amer. Gas Inst.*, **2**, 401, 1911.

[3] *Proc. Roy. Soc.*, A, **85**, 275, 1911.

Recent work by the writer, now appearing in detail elsewhere,[1] has made possible a decision as to the best photometric method and conditions of measurement. It is found that the *flicker photometer* possesses the maximum number of attributes of a good system of brightness measurement where color differences exist. Its qualifications are as follows:

1. It possesses the greatest sensitiveness of any photometric method.
2. The results are more closely reproducible than those of other methods.
3. Things measured equal to the same thing measure equal to each other.
4. The sum of the measured values of the parts is equal to the measured value of the whole.
5. The results given by it agree, at high illuminations, with the estimates of equal brightness by the commonest photometric method (equality of brightness) when the disturbing subjective elements in the latter are eliminated either by the mean of many observations or by measurements performed with small hue differences.

The expression "the relative brightness of two colored areas" has no meaning unless the conditions of illumination and field-size are specified. Certain conditions must be taken as standard and values for other conditions derived therefrom. As a result of the work quoted the standard photometric conditions suggested are:

1. The flicker photometer should be used.

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[1] "Studies in the Photometry of Lights of Different Color, I to V," *Phil. Mag.* (abstracts in *Physical Review*, 1911 and 1912).

2. An illumination of 25 meter candles on a magnesium oxide surface. (In accordance with the results of Section 3 below, this is to be expressed as a brightness given by $\frac{10^{-6}}{\text{luminous effic.}} \frac{\text{watts}}{\text{cm}^2}$ normal radiation per unit solid angle.)

3. A photometric field of 2° diameter, surrounded by a bright area of 25° diameter.

4. The measurements should be made by a normal eye.

In connection with condition 4 it is to be noted that different eyes vary in their color-sensitiveness. It is necessary, therefore, to determine the color-sensitiveness of a normal or average eye, and to use only such eyes in heterochromatic photometry, or correct the results of abnormal observers to the value holding for an average eye. I have recently determined for this purpose the luminosity-curve of a normal equal-energy spectrum for a sufficient number of eyes to justify calling the mean curve that of an average eye. The standard photometric conditions above outlined were used and eighteen observers of normal vision were measured. The resulting mean normal luminosity-curve is given below. The values are expressed in terms of the maximum = unity, and are as well a scale of luminous efficiencies of the spectral radiations in terms of the maximum.

TABLE I

Luminosity-Curve of the Average Eye under Standard Conditions for Heterochromatic Photometry

(Relative Luminous Efficiencies of the Different Spectral Radiations)

λ	Relative Efficiency	λ	Relative Efficiency
.44 μ	.029*	.57	.948
.45	.047*	.58	.875
.46	.073*	.59	.763
.47	.107*	.60	.635
.48	.154	.61	.509
.49	.235	.62	.387
.50	.363	.63	.272
.51	.596	.64	.175
.52	.794	.65	.104
.53	.912	.66	.068*
.54	.977	.67	.044*
.55	1.000	.68	.026*
.56	.990		

* Extrapolated.

It is to be noted that condition 4 may be fulfilled by correcting to normal the results of an observer whose luminosity-curve is known, or it may be closely approximated by taking the results of a large number of observers. Better still, there may be substituted for the eye some radiation meter arranged to weight the radiation in accordance with data such as that of Table I. Perhaps the photo-electric cell will serve. Table I is offered as giving the data to effect a working approximation to condition 4 above given in case either a method of correction or a substitute for the eye is used.

2. THE MEASUREMENT OF LUMINOUS EFFICIENCY[1]

The term "luminous efficiency" has long been used for an almost purely physical quantity—the ratio of a certain portion of the energy radiated from a source to the total radiated energy. True, this selected portion is radiation which can produce the sensation of light and is hence called "visible"; but from deep red to dark violet the brightness of the spectrum varies so enormously that any method of estimating efficiency that calls it all "light" of equal value is foredoomed to have scant connection with real luminous efficiency. On the other hand, the ratio used in photometry and engineering, namely, lumens per watt, is a true measure of luminous efficiency, although there is in the term no indication of the ideal maximum such as is suggested by expressing the efficiency value as a percentage.

Now all that is significant and desirable in the purely physical so-called "luminous efficiency" may be retained and brought into exact parallelism with the true efficiency expressed by lumens per watt. Let the radiations from a source be weighted according to their value as light-producers, in terms of the radiation of maximum light-producing power. The ratio of the sum of these weighted radiations to the whole radiation gives the efficiency of the radiation expressed as a percentage of what it would be were all the radiation of maximum light-producing power. To this ratio Drysdale has given the name "reduced luminous efficiency," although it might better supersede the older "luminous efficiency" entirely and appropriate its name.

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[1] See Drysdale, "Luminous Efficiency," *Lond. Ill. Eng.*, p. 164, 1908; H. E. Ives, "Luminous Efficiency," *Trans. Ill. Eng. Soc.*, p. 113, 1910; P. G. Nutting, "The Luminous Equivalent of Radiation," *Bul. Bureau of Standards*, **5**, 261.

This efficiency is exactly parallel with lumens per watt. It is, in fact, the ratio of the lumens per watt of the radiation from the light-source compared to the maximum possible lumens per watt. Knowing the lumens per watt of a source, it is necessary to know only the maximum possible specific luminous output in order to determine its luminous efficiency, expressed in a percentage value of real significance.

The table which has been given for the brightness values of the normal spectrum for the average eye is at the same time a table of relative luminous efficiencies of the spectral colors. Expressed in terms of the maximum as unity, it is directly available for determining the ("reduced") luminous efficiency of a known energy distribution.

The value of luminous efficiency for any radiation may, then, in accordance with the subject-matter of this section, be obtained in either of the two ways:

1. The ratio of the total radiation weighted according to the luminosity-curve of the average eye (maximum value unity), to the total radiation.

2. The ratio of the specific luminous output of the radiation being measured (e.g., the lumens per watt) to the maximum possible specific luminous output (i.e., monochromatic radiation of wave-length approximately 0.550 μ), or

$$\mu=\frac{K}{K_{\max}}$$

μ = luminous efficiency

K = specific luminous output of the radiation in question

$K_{\max}$ = maximum possible specific luminous output (that of the ideal source)

The *total* efficiency of a light-source is obtained, *upon the same scale*, by substituting "source" for "radiation" in (2), i.e., by taking the lumens per applied watt instead of radiated watt.

3. SPECIFICATION OF THE RADIATION STANDARD

Using the symbols and the definitions given above, we arrive at the following defining equations:

Luminous flux $=KE$

$E=$ rate of flow of energy

$K=$ specific luminous output of the radiation

$=\mu K_{\max}E$

Now, E is expressed in C.G.S. units.

μ is a pure number determined by the physiological characteristics of the normal eye.

$K_{\max}$ is a quantity whose value is entirely dependent on the dimensions of the particular "primary" standard used in the system—Hefner, candle, or piece of hot platinum—and on the unit of energy flux.

The present proposal is:

1. *Make* $K_{\max}=$ *unity*
2. *Express E in watts*

whence *the unit of luminous flux is the flux from a source radiating energy of maximum luminous efficiency at the rate of one watt,* or, *it is the flux from a source of radiant luminous efficiency* μ, *radiating energy at the rate of* $\frac{1}{\mu}$ *watts.*[1] The quantity μ is of course to be determined by the general methods given above.

The absolute value of the unit of flux may in general be determined by measuring both in light and energy units a radiation of known luminous efficiency. As has been mentioned early in this paper, this may be done by several equivalent methods. For instance, a monochromatic radiation of known luminous efficiency may be measured in both ways. Or a "visual luminosity screen" may be used for the radiation measurement. The effect of the latter, when corrections are made for its minimum absorption, is to reduce all the radiation to the value it would have if of maximum luminous efficiency. This second method may be applied to known energy-distributions in the form of graphical calculation. Calculations of this sort made by applying the normal luminosity-curve to energy-distributions of known total value[2] indicate a value of $K_{\max}$ of about 800 lumens per watt.

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[1] Or, the flux from a source of *total* luminous efficiency μ, *consuming* energy at the rate of $\frac{1}{\mu}$ watts.

[2] Ives, "Luminous Efficiency," *op. cit.*

Attention might be called to the fact that the problem of determining the so-called mechanical equivalent of light becomes identified with the establishment of the primary standard, since the latter is specified by the *least mechanical equivalent.*

The definitions of the other common photometric units follow directly from that of flux. For instance, the unit of intensity is the intensity in a certain direction of a source radiating in that direction unit flux per unit solid angle, that is, $\frac{1}{\mu}$ *watts per unit solid angle.*

Using the value $E_{\max}=800$ lumens per watt and an albedo of 0.95 for $Mg\,O$, 25 meter candles becomes very nearly $\frac{10^{-6}}{\mu}\,\frac{\text{watts}}{\text{cm}^2}$ per unit solid angle, normal radiation; the value given under photometric condition 2.

SUMMARY

It is proposed that the unit of luminous flux be defined as the flux from a source radiating energy of maximum luminous efficiency at the rate of one watt. The method of estimating luminous efficiency, and the method of colored-light photometry which it is necessary to adopt preliminary to establishing the radiation standard, are described.

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THE INFLUENCE OF TEMPERATURE ON THE PHENOMENA OF PHOSPHORESCENCE IN THE ALKALINE EARTH SULPHIDES

By HERBERT E. IVES and M. LUCKIESH

In a previous paper[1] the present authors described experiments on the effect of infra-red radiation upon the phosphorescence of zinc sulphide. It was found that the effect of long-wave radiation is a function of the time which has elapsed since the termination of excitation. Shortly after excitation an exposure to red or infra-red causes a rapid drop in phosphorescent intensity, while later exposure causes a flash of light followed by a drop in brightness. The present investigation has grown out of attempts to obtain further information on this phenomenon by studying it under various changed conditions. Among these conditions temperature has proved so productive of results bearing not only upon the flashing-up phenomenon but upon other phases of phosphorescence as to warrant considering this paper as chiefly having to do with temperature and phosphorescent phenomena in general. The methods and procedure of the previous paper are closely followed, and reference should be made thereto for experimental details. The only essential difference in the apparatus consists in the uniform use of a quartz mercury arc and blue glasses in place of the carbon arc used before.

THE DECAY OF PHOSPHORESCENCE AT DIFFERENT TEMPERATURES

Various expressions have been proposed to represent the brightness of phosphorescence as a function of time since excitation. That most generally met is

$$I=\frac{1}{(a+bt)^2} \qquad I=\text{brightness} \quad t=\text{time}$$

which has a simple theoretical derivation, and has led to the common practice of plotting decay-curves in terms of $\frac{1}{\sqrt{I}}$ against time.

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[1] "The Effect of Red and Infra-Red on the Phosphorescence of Zinc Sulphide," *Astrophysical Journal*, **34**, No. 3, 1911.

Becquerel found cases in which the empirical formula

$$I=\frac{1}{(a+bt)^n}$$

held for values of n other than 2.

In our previous work on zinc sulphide we found the latter expression with a value of $n=1.03$ ($I^{-0.97}=a+bt$), to accurately represent the decay-curve found.

Several observers have studied the effect of temperature on the rate of decay of phosphorescence. In general the rate of decay is greatly increased by rise in temperature. Micheli[1] gives data on several sulphides over a temperature range of several hundred degrees, but does not derive any expression connecting rate of decay with temperature. Pierce[2] in experiments on zinc sulphide and Balmain's paint finds a change of curvature of the $\frac{1}{\sqrt{I}}$ plots with change of temperature, and seeks for an explanation in the possible multiple character of the emission bands.

In the present work the samples of phosphorescent sulphide were placed on a shallow trough of nichrome strip, arranged to be heated electrically. A separate trough was arranged on a solid brass column around which melting ice could be packed for securing zero temperature. In some of the work the phosphorescent material was laid on the outside of a square brass tube into which carbon dioxide snow was rammed. The temperatures corresponding to various currents through the nichrome strip were determined by melting various low-melting metals. The values given are not to be taken as more than rough approximations, since the details of the apparatus were somewhat changed after the temperature calibration.

Several different sulphides were experimented with, but the majority of the experiments were made with the zinc sulphide previously used. Tests on other sulphides were largely for confirmatory purposes or were made incidental to other than purely temperature investigations. The sulphides besides zinc were Balmain's paint, *BaBiK* (from Leppin & Masche) and *SrZnFl* (our own preparation).

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[1] *Arch. sc. phys. et nat.* (4), **12**, 5, 1901.

[2] *Phys. Rev.*, **26**, 314, 1908.

The results obtained have been very definite and clear cut. In every case we have found the decay-curves to be represented by the Becquerel equation

$$J^{-x}=a+bt$$

in which x is a function of the temperature, becoming smaller the higher the temperature.

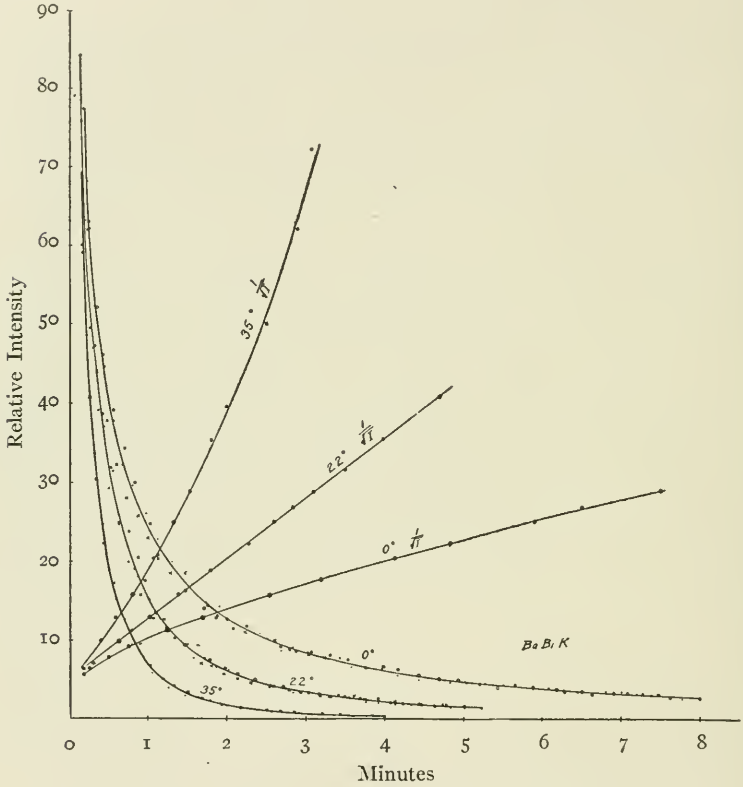

FIG. 1.—Decay of phosphorescence in *BaBiK* at different temperatures

Fig. 1 shows decay-curves at 0°, 22°, and 35° for the *BaBiK* sulphide, a compound which proved very sensitive to temperature change. When plotted in terms of $\frac{1}{\sqrt{I}}$ against time, the curve at 0° is concave to the time axis, that at room temperature is straight (taken alone this might easily have been interpreted as confirming the value $\frac{1}{2}$ for x), while that at 35° is *convex* toward the time axis. Fig. 2 shows the straight lines which result on plotting the data with the value of $x=0.8$ for 0°, $x=0.5$ for 22°, $x=0.3$ for 35°. In Fig. 3 are given the straight lines obtained in a similar manner from our more extensive data on zinc sulphide, which, however, is less sensitive to changes of temperature. It will be noted that the value $x=0.5$ is in this case not at room temperature but at over 80° C.

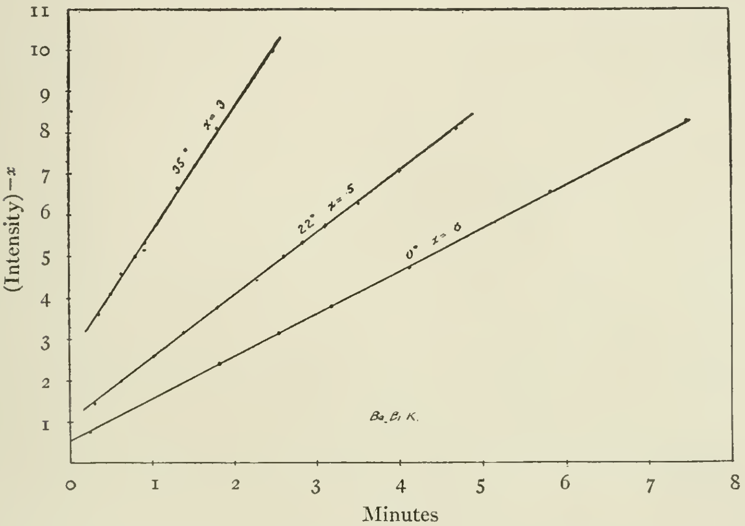

FIG. 2.—Decay of phosphorescence in *BaBiK* at various temperatures

The observation of decay-curves which plot convex to the time axis when drawn in terms of $\frac{1}{\sqrt{I}}$ (that is, with an exponent under 0.5) is not entirely new. Micheli's results for high temperature show this, Pierce found indications of such a curvature, and C. C. Trowbridge has obtained slightly convex curves in the case of gas phosphorescence. There has, however, been a tendency to look with suspicion upon such results, partly because they have been shown by data very near the limit of experimental accuracy, and doubtless partly because they are not compatible with the $\frac{1}{\sqrt{I}}$

formula. Curves concave to the time axis can be explained as summations of decays following the $\frac{1}{\sqrt{I}}$ formula, but *no summation of this sort will give a convex curve.* As the figures clearly show, our results are entirely unambiguous. We present the experimental formula, with the new significance of the exponent, as a function of temperature, merely for what it is worth as a means of representing the phenomena. It was tried in the first place without any guiding theory, and its only claim to notice is that it does represent, without exception, all the decay-curves we have obtained.

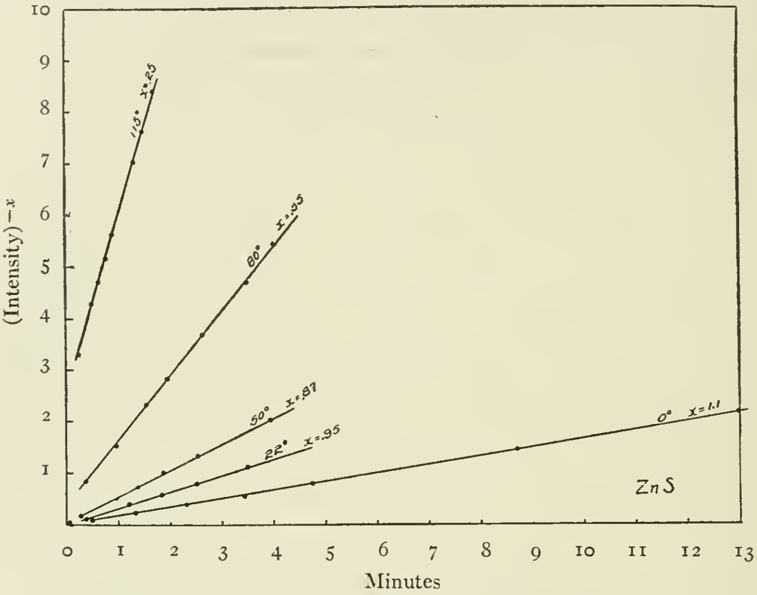

FIG. 3.—Decay of phosphorescence in zinc sulphide at different temperatures

THE FLUORESCENCE AND INITIAL VALUE OF PHOSPHORESCENCE

From the formula $I^{-x}=a+bt$ the initial value of the phosphorescence may be calculated upon solving for the constants. We have done this for zinc sulphide, with the results shown in the dashed curve of Fig. 4. The full line shows observed values of the "fluorescence" made through a yellow glass which completely absorbs the exciting blue mercury lines, and which has no effect on the measurements of phosphorescence. It is evident that the total fluorescence, and that part of it which persists as phosphorescence, are not one and the same thing. This idea is confirmed in other ways. The change from fluorescence to phosphorescence obeying the phosphorescence law takes place certainly within two or three seconds, for points on the decay-curve, obtained three seconds after excitation, fall exactly in their calculated place. As Lenard discovered, the light emitted during excitation is of two kinds, constituting what he calls the momentary and the permanent processes, the latter being predominant in the excitation caused by certain spectral regions. The momentary process is present practically only during excitation. By projecting a quartz mercury-arc spectrum on a zinc sulphide surface for a minute, and then obstructing the light, the two processes are clearly separated. The lines exciting the permanent process remain visible as phosphorescent strips, while strips which were equally bright during excitation but belong to the momentary process are almost instantly extinguished. The data of Fig. 4 are therefore capable of straightforward explanation in terms of other knowledge. It affords confirmation of Lenard's observation that at high and low temperatures the emitted light becomes more and more of the momentary variety. The excitation used throughout the work, as later study showed, consisted chiefly of three blue and near ultra-violet lines of the *Hg* arc located in the permanent process excitation region. The fluores-

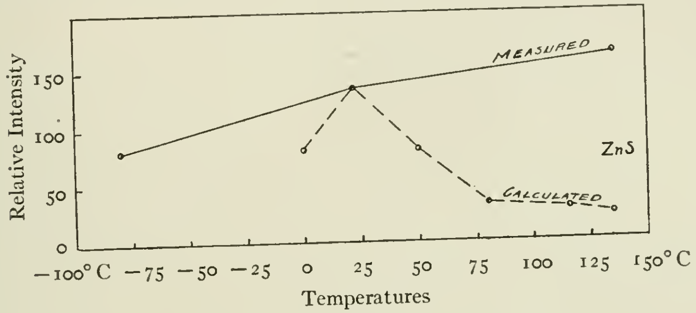

FIG. 4.—Relative initial intensities at various temperatures

cence intensity would have been relatively much higher than is shown in Fig. 3 had other "momentary" excitation regions been active.

VARIATIONS DUE TO COMPOSITION AND TREATMENT

Werner[1] has investigated the decay-curves of samples of the *SrZnFl* phosphor, in which the quantity of zinc was varied over a range of several thousand times. He found differences which he explained as due to various relative amounts of momentary and permanent process, the latter following the relationship $\frac{1}{\sqrt{I}}=a+bt$. His work was all done at room temperature. No "convex" curves were obtained. Had these been found they would have been difficult to reconcile with the suggested explanation.

In the expectation that these differences of composition might be equivalent to different temperatures and therefore capable of representation by different values of x in the above equation, we have made up a series of samples of this sulphide, following closely the procedure as given by Werner. Our samples ranged in relative composition from 1 to 100,000 parts of zinc, covering very nearly the same range in quantity as the ones used by Werner. These were measured for decay-curves at zero and at room temperature, with interesting results. By inspection it was found that all our samples had substantially the same brightness during excitation. But immediately upon extinction of the exciting light, the specimens of small zinc content decayed with great rapidity, the richer specimens displayed enduring phosphorescence, measurable for as much as five to ten minutes in the most favorable cases. The behavior of the different specimens appeared therefore to be parallel to that occasioned by different temperatures. The decay measurements, however, told a very different story. In every case where reliable measurements could be obtained an exponent of $x=1.0$ was found for zero temperature, of $x=0.8$ for room temperature. The difference in behavior of the samples of different quantitative composition appears to be represented merely by different values of the constants a and b of the equation here used. The exponent is a characteristic of the phosphor and is varied by temperature alone.

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[1] *Ann. der Physik*, **24**, 164, 1907.

The series of samples above described were all subjected to the same treatment in heating (in an electric furnace). Some experiments were made in variations of treatment. These resulted in brighter or duller phosphorescence but no certain effect on the value of the exponent was found.

INFRA-RED EFFECTS AND TEMPERATURE

The effect of infra-red on phosphorescence, which was the original problem before us, was studied at different temperatures, the experiments being confined to zinc sulphide. We were led to expect a dependence of the flash-up and extinction effects upon temperature by Lenard's observation that at very low temperatures zinc sulphide showed a strong, long-continued flash-up, in contrast to its fleeting or unnoticeable flash at room temperatures.

Fig. 5 gives a set of curves selected from a number which all show the same characteristics. At the temperature of melting carbon dioxide, −80° C., the flash-up is very pronounced; at room temperature, 22° C., it is much less pronounced; at 130° C., entirely absent. It is therefore a low-temperature effect. In this connection it is to be remembered that different phosphors are in their permanent phase at different absolute temperatures, so that the same temperature may be high for one phosphor and low for another, each having in fact its own temperature scale. In this fact may lie

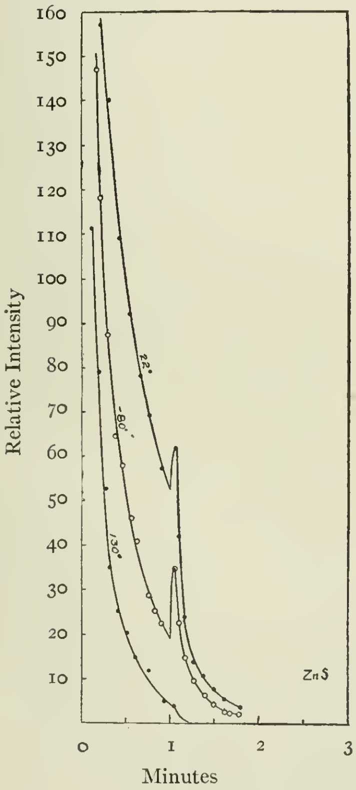

FIG. 5.—The flashing-up phenomenon under infra-red in zinc sulphide at different temperatures.

a partial explanation of the peculiar behavior of zinc sulphide. We find with other phosphors which show the flash-up—namely, Balmain's paint and *SrZnFl*—that the flash-up percentage increases in magnitude the greater the interval after excitation, and that it decreases in magnitude with any increase in temperature. With zinc sulphide at room temperature the behavior is very much as if the phosphor consisted of a mixture of components at very different effective temperatures. The "high"-temperature components which are immediately extinguished by infra-red at first predominate, leaving, later in decay, the low-temperature constituents to give the flash-up. In the other phosphors there is (on this hypothesis) immediately after excitation merely a somewhat larger proportion of the high-temperature components or the different components are only at slightly different effective temperatures, the flash-up percentage is less than it becomes later but is not masked by the effect of the high-temperature components. That these phosphors are very irregular in composition is a well-known fact, easily learned upon close inspection of the individual particles of a phosphorescent surface. Some particles are almost entirely of the momentary character; others are permanent. Differences in color are frequently visible. Of course this mixture hypothesis is far short of an explanation of the infra-red effect, but it suggests how the somewhat different behavior of the different sulphides may be differences merely of degree.

INFRA-RED EFFECTS AND COMPOSITION

The effect of different quantitative compositions on the flash-up effect in *SrZnFl* was studied, but no definite effects were found—a flash-up was always produced by infra-red, it increased with the time interval after excitation. We did not find the relationship suspected in the previous paper to which we were led by Werner's work and theory, namely that the flashing-up effect would be more marked in certain quantitative compositions than in others. We have, however, found that certain accidental differences of treatment which we were not able to control certainly do affect the magnitude of the flash. One sample, in fact, behaved almost like the *ZnS* in showing no flash at all in the early part of the decay.

INFRA-RED EFFECT DURING EXCITATION

It was noted by Nichols and Merritt that the effect of infra-red upon zinc sulphide is quite marked even during excitation, its effect being to decrease the brightness of the emitted light. According to Lenard's view the effect of infra-red is considered equivalent to a very great local increase of temperature in the phosphorescent centers. According to our results shown in Fig. 4 the effect of heat is in general to increase the brightness of the fluorescent light. According to Lenard's view, then, heat and infra-red together should give an even greater depression than infra-red alone. To test this point observations were made, through the yellow glass, as before, upon the emitted light during excitation, with and without infra-red, at room temperature and at −80° C. Fig. 6 shows the results. It is at once evident that heat and infra-red act oppositely, since the intensity at 22° under infra-red is greater than the intensity at −80° without infra-red. Had the phosphor when exposed to infra-red at −80° been raised to 22° its brightness would have increased.

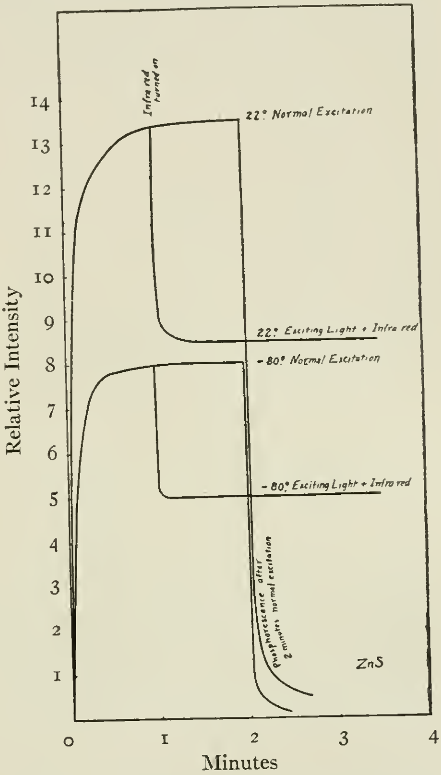

Fig. 6.—Effect of infra-red upon the intensity of the emitted light during excitation at 22° C and −80° C.

EFFECT OF SIMULTANEOUS HEAT AND INFRA-RED UPON PHOSPHORESCENCE

Lenard's view that infra-red acts as a very high temperature localized in the phosphorescent centers was subjected to a test in the case of phosphorescence, as well as in the case of fluorescence just described. The reasoning was as before. Experiment showed that a large increase of brightness or flash-up invariably occurred when the temperature of the decaying zinc sulphide was raised. There was no indication of an immediate extinction as in the case of infra-red action. In a recent paper Lenard[1] has shown that the total light given out by a phosphor is the same whether it decays naturally or is stimulated by heat; but that this is not true when infra-red acts. This peculiarity he ascribed to a radical disturbance of the light storing and emitting mechanism due to the intense heating by infra-red. If the effect of infra-red is really identical to

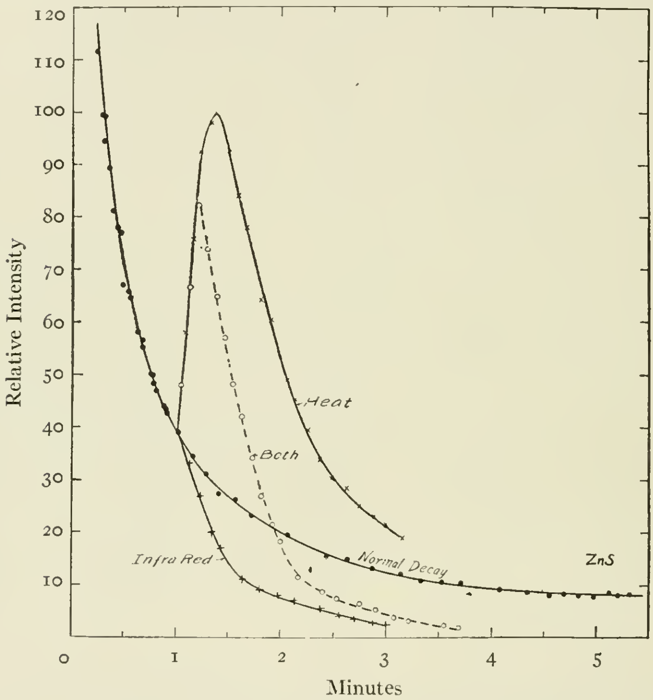

FIG. 7.—Effect of heat and infra-red upon the phosphorescence of zinc sulphide shortly after excitation.

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[1] "Lichtsummen bei Phosphore," *Sitzung. der Heidelberger Akademie der Wissenschaft*, 5, 1912.

a greatly increased temperature localized in the emission center, it would seem reasonable to suppose that the two heating effects would add if applied simultaneously. Since the increase in temperature necessary to cause a flash-up is a small fraction of that ascribed to the local action of infra-red the effect of gently heating the whole phosphor may be expected if anything merely to increase the effect of infra-red; that is, if the two are applied shortly after excitation, to cause a greater drop in brightness in our zinc sulphide than infra-red would alone.

The apparatus was so arranged that either infra-red or heat could be applied to the phosphor, or both together. Fig. 7 shows the normal decay-curve, the flash given by gentle heating, the extinction caused by infra-red, and finally the effect of heating and infra-red exposure together. Instead of the heat supplementing the infra-red it is found that they oppose each other, the result being the summation of the separate effects. This appears to us to call for a modification of Lenard's idea of the infra-red effect.

EFFECT OF INFRA-RED ON THE DIFFERENT SPECTRAL EXCITATION REGIONS

By projecting a mercury arc spectrum by a quartz prism train upon our zinc sulphide it has been possible to determine to which excitation process the extinction and flashing effects belong. Taking advantage of the fact that with this phosphor the effect of infra-red is visible even in the emission during excitation, an infra-red image of a Nernst glower was projected lengthwise through the mercury spectrum. The result was definite information that infra-red affects only the permanent process, or phosphorescence. The dark valley cut through the mercury line images scarcely touched the lines which belong to the momentary process, i.e., those that are extinguished almost instantly by the removal of the exciting light.

This experiment disposes of a hypothesis put forward in our previous paper, namely, that the early extinction effect of the infra-red is to be ascribed to a large proportion of momentary process in this part of the decay, the momentary process being assumed to be extinguished at once by infra-red. All the peculiarities noted as due to long-wave radiation are apparently to be ascribed to the permanent process.

SUMMARY AND DISCUSSION

The matter presented in this paper is almost wholly experimental. Where it has touched upon current theories of phosphorescence, as in the case of the relation between heat action and infra-red action, it has been destructive rather than constructive. The experimental results may be summarized as follows:

1. The decay of phosphorescence in the alkaline earth sulphides is accurately represented by the formula

$$I^{-x}=a+bt$$

where x is a function of the temperature, independent of the quantitative composition of the phosphor, and where a and b are functions of the composition and heat treatment of the sulphide.

2. The flashing-up phenomenon, under infra-red, is greatest at low temperatures, and decreases at high temperatures. It always shows an increase in magnitude with lapse of time after excitation.

3. In the case of zinc sulphide, infra-red and heat act, during excitation, and during the early part of decay, oppositely.

4. The effect of infra-red is upon the permanent process alone.

The suggestion made above, that these phenomena may be partially linked together by taking into account the complex character of the phosphors, is as far as we care to venture in the way of hypothesis at present. It is significant that decay-curves of the type obtained may be built up of terms of the form

$$i=ae^{-kt}$$

the law of mass action with one substance changing. Variations of a and k with temperature could produce the observed changes in the decay. Ascribing to each component its own temperature scale, the ordinarily prepared phosphor would consist of a mixture of different effective temperatures. The early part of the decay would be given chiefly by the high-temperature components, the later part by the low. Simply noting that the flash-up effect is a low-temperature effect enables us to picture why it becomes more marked as decay proceeds, without, however, giving any insight into the actual process which is thus different at high and low temperatures. The fact that the infra-red effect is localized in the spectrum, usually in two bands, would suggest, in view of the connection recently shown between phosphorescence and the selective photo-electric effect in the violet, that a study of the photo-electric effect in this zinc sulphide when exposed to infra-red both during excitation and during decay would be of interest.

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345

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ON THE DEVIATION FROM LAMBERT'S COSINE LAW OF THE EMISSION FROM TUNGSTEN AND CARBON AT GLOWING TEMPERATURES

By A. G. WORTHING

INTRODUCTION

Lambert's cosine law as applied to the radiation from a self-luminous body is expressed by the following equation

$$b \equiv \frac{dI}{dS \cos \theta} = b_0$$

where b, dI, and dS refer respectively to the brightness, the luminous intensity, and the area of an element of the body when viewed by light emitted at an angle θ, and where b_0 is the brightness viewed normally. Where the cosine law does strictly apply, the brightness does not vary with the angle of emission.

For a study of the theoretical foundations and a survey of the literature relating to Lambert's law, reference is made to Uljanin,[1] Helmholtz,[2] Liebenthal,[3] and Gross.[4] It is sufficient to say here that theoretical considerations seem to show that Lambert's cosine law is rigorously applicable only to a black body.

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[1] *Wied. Ann.*, N.F., **62**, 528, 1897.

[2] *Vorlesungen*, **6**, 150.

[3] *Praktische Photometrie* (Vieweg & Sohn), pp. 80–88.

[4] *Jahresbericht der Schlesischen Gesellschaft für vaterländische Kultur* (Naturw. Sektion), March 6, 1911.

Relative to the application of the cosine law to the emission of light by a self-luminous body, the earliest data seem to be those by Möller.[1] He investigated the light emitted by a glowing strip of platinum at various angles, making use of a polarization photometer. He concluded from his work that the cosine law of emission was fulfilled. It is to be noted, however, that his actual results point to a consistent and slight deviation therefrom, the deviation being in the direction to make the brightness of the strip appear greater at large angles of emission than at small angles. This variation was ascribed to experimental errors.

Later Uljanin,[2] making use of the optical constants obtained by Drude, computed, for various angles of emission for platinum, silver, and copper, the intensities of the beams of light polarized respectively in the plane and perpendicular to the plane of emission. From these he computed further the polarization of the emitted light and the brightness of the glowing surface as functions of the angle of emission. His computations as to the polarization of the light from platinum are closely verified by his own direct observations, in which, however, he made use of wave-lengths in the neighborhood of $3\,\mu$, whereas the Drude constants were obtained for light in the visible spectrum. Polarization measurements made by Provostaye and Desains[3] and by Millikan[4] agree closely with his results. Observations by Violle[5] and by Millikan[6] verify the polarization computations on silver. These verifications of the polarization computations lend credence to the computed brightness variations. Uljanin computed that the brightness of glowing platinum should increase from the normal value for zero angle of emergence to 1.17 times the normal brightness for an angle of 70°, decreasing at somewhat larger angles. It was suggested that Möller's observations were undoubtedly in error due to some dust which may have collected on the surface or due to the roughening of the surface of the platinum consequent on heating it up to glowing temperatures. This latter suggestion was rendered plausible by further experimentation in which the strip originally used by Uljanin to verify his polarization computations was heated to incandescence and then cooled somewhat. The further polarization measurements showed a considerable decrease in the polarization for any given angle of emission.

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[1] *Ann. d. phys.*, N.F., **24**, 266, 1885.

[2] *Loc. cit.*

[3] *Ann. de chim. et de phys.* (3), **32**, 112, 1851.

[4] *Physical Review*, **3**, 81, 177, 1895.

[5] *Comptes Rendus*, **105**, 111, 1886.

[6] *Loc. cit.*

Recently in a study of the energy losses in electric incandescent lamps, in collaboration with Messrs. Hyde and Cady,[1] the writer noted certain variations which have led to the investigation described in the present paper.

APPARATUS AND METHOD

The arrangement of the apparatus as shown in Fig. 1 is the same as that described in the energy-loss paper just referred to. The materials used in this investigation were large lamp filaments mounted in spherical bulbs, the portions used being those near the centers of the bulbs. *A* represents such a lamp; *B*, a lens so placed as to produce an enlarged image of the filament of *A* in the plane of a thin filament mounted in another bulb at *C*. *D* represents a telescope; *E*, a piece of red or blue glass. The transmissions of the glasses used are such that the light transmitted may for computational purposes be considered monochromatic, the wave-lengths being respectively 0.63 μ and 0.46 μ. *F* represents an absorbing glass screen which, though not regularly used, was used in the special case when the effect of changes in temperature was investigated.

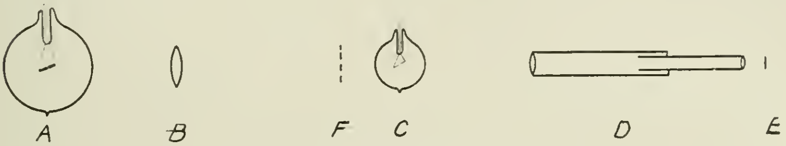

FIG. 1.—Diagrammatic sketch of apparatus

There are two ways of manipulating lamp *A* in order that the variations of brightness with the angle of emission may be investigated. The one used for the most part is that of rotating lamp *A* about a vertical axis passing through the point investigated. The other way consists in raising or lowering lamp *A* so that different portions of a cross-section of *A* are seen in the line with the filament of lamp *C*. The method is purely an optical pyrometric one. It consists in finding for various angles of emission for the light from lamp *A* the various voltages of lamp *C* at which it matches in intensity the portions of the image of lamp *A* which are in close juxtaposition. From such data one may obtain the variations in brightness of lamp *A* with angle of emission by determining the relative intensities of lamp *C* for the red or blue light of the desired wave-lengths for the different voltages obtained.

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[1] *Trans. Ill. Eng. Soc.*, 6, 238, 1911; *Ill. Eng.* (Lond.), 4, 389, 1911.

This has been accomplished by means of ordinary photometric measurements in which first the red glass and later the blue glass used at *E* (Fig. 1) were placed at the ocular opening of the Lummer-Brodhun contrast photometer-head which was used. No appreciable color differences were noticed. From a plat of the logarithm of the voltage as a function of the logarithm of the candle-power of lamp *C*, there was obtained, by means of a straight-edge, a quantity k_λ, the limiting value of the ratio of a small relative change in luminous intensity to the corresponding relative change in voltage, as a function of the lamp voltage. The results of such measurements have been plotted in Fig. 2. It is to be noticed that for any given voltage the ratio of the k_λ for the red light to the k_λ for the blue light is very closely $\frac{46}{63}$, the inverse of the ratio of the effective wave-lengths of transmission. This condition is rigorously fulfilled for black-body radiation when the two colored lights are each monochromatic. Since at any one temperature, as shown by Hyde,[1] the light from a tungsten lamp very closely matches spectrophotometrically that from a black body at some certain temperature, it follows that this condition should very closely apply to a tungsten lamp, as it does. The application of the curves of Fig. 2 to determine the brightness of the filament of lamp *A* for various angles of emission is evident.

The question of a change in the variation from the cosine law with temperature was especially suggested by the paper already referred to by Dr. Hyde. In order to make more certain whether such a change existed or not, a slight modification of the above-described method was undertaken. It consisted in taking voltage readings of lamp *C* in the ordinary way for angles of emission from lamp *A* of 0° and of 75° for a low operating temperature and then with an absorbing glass screen at *F* for a high operating temperature such that in the second instance the voltages for lamp *C* were approximately the same as those for the first instance. The two readings at 75° were made successively without any other change than the introduction or the removal of glass *F*.

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[1] *Astrophysical Journal*, 36, 89, 1912.

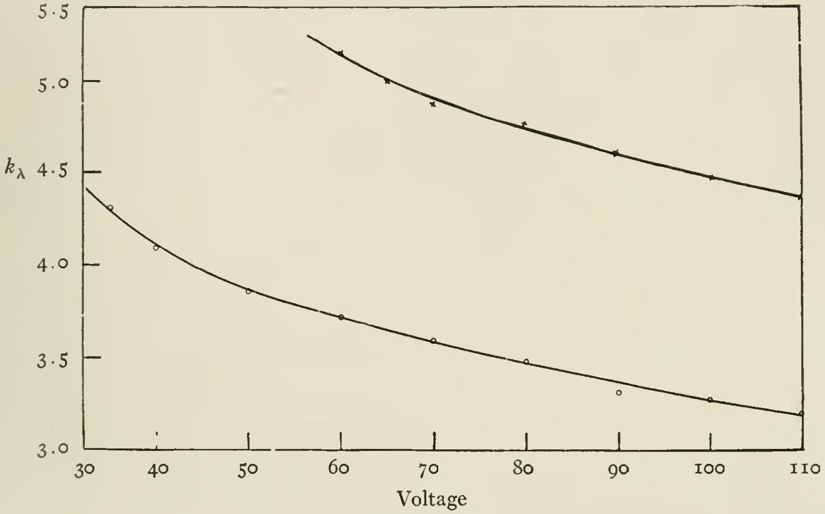

FIG. 2.—k_λ as a function of the voltage for lamp *C*, where k_λ is defined as the limiting value of the ratio of a small relative change in luminous intensity for the wave-length λ to the corresponding relative change in voltage.

× $\lambda = 0.46\ \mu$ o $\lambda = 0.63\ \mu$

RESULTS ON BRIGHTNESS MEASUREMENTS

These have been determined, using light of wave-length 0.63 μ, the brightness as a function of the angle of emission for tungsten at two different temperatures, one being that corresponding to the normal operating temperature with an efficiency of 1.25 watts per candle-power and the other to a temperature at which the wattage of the lamp is about 18 per cent of the normal, and for carbon at one temperature. The individual determinations together with the smoothed curves are shown in Figs. 3, 4, and 5.

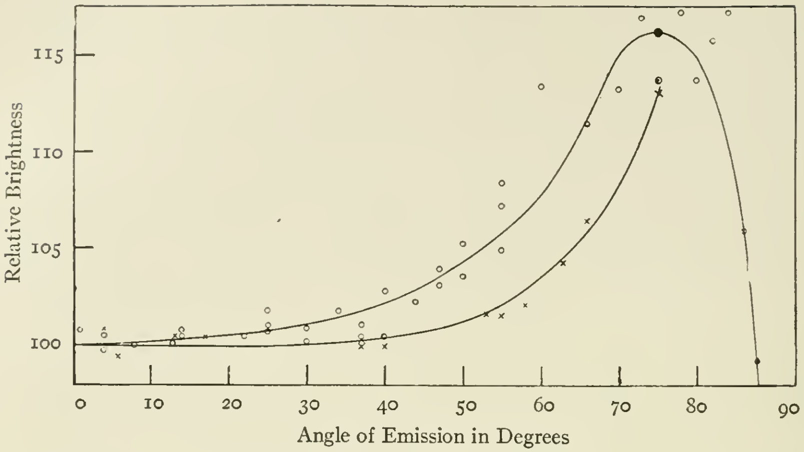

FIG. 3.—The deviation from Lambert's law of the emission from tungsten at normal operating temperature.

o $\lambda=0.63\ \mu$ × $\lambda=0.46\ \mu$

● $\lambda=0.63\ \mu$ average of several values

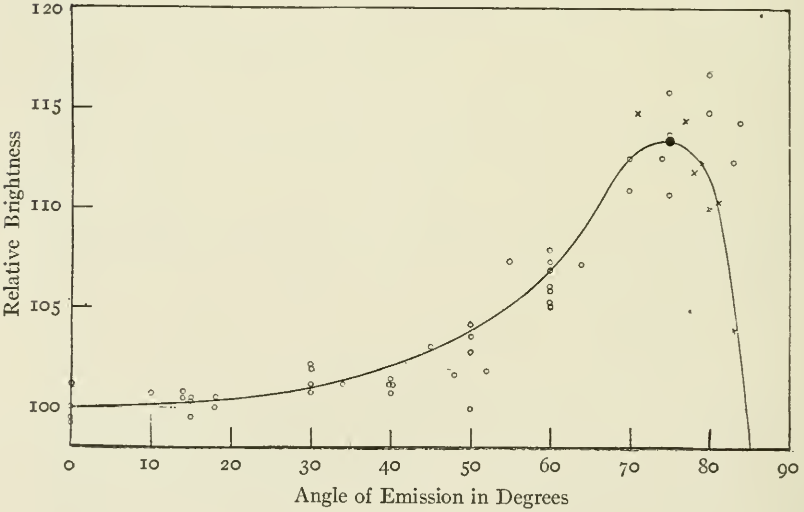

FIG. 4.—The deviation from Lambert's law of the emission from tungsten at a low operating temperature for light of wave-length $0.63\ \mu$.

● Average of several values

× Values obtained when special care was taken to eliminate errors creeping in at large angles.

A few values for the variation of brightness with angle of emission have been determined for tungsten at the higher temperature, using light of wave-length 0.46 μ, and for carbon at one temperature. In Fig. 5 there are plotted data obtained on two different lamps, both untreated carbon. The data for the red light, indicated by crosses (×), were obtained on the same lamp filament as were the data for the blue light. Because of a desire to compare the blue- and the red-light variations, the data plotted as crosses have been given preference to those plotted as circles. The large variation in the individual results in connection with tungsten is due largely to three facts: (1) The axis of rotation of lamp *A* (Fig. 1) was not tangent to the filament. With each new angle of emission there was a consequent slight apparatus adjustment. (2) The glass of the bulb was not uniform. (3) The laboratory work was

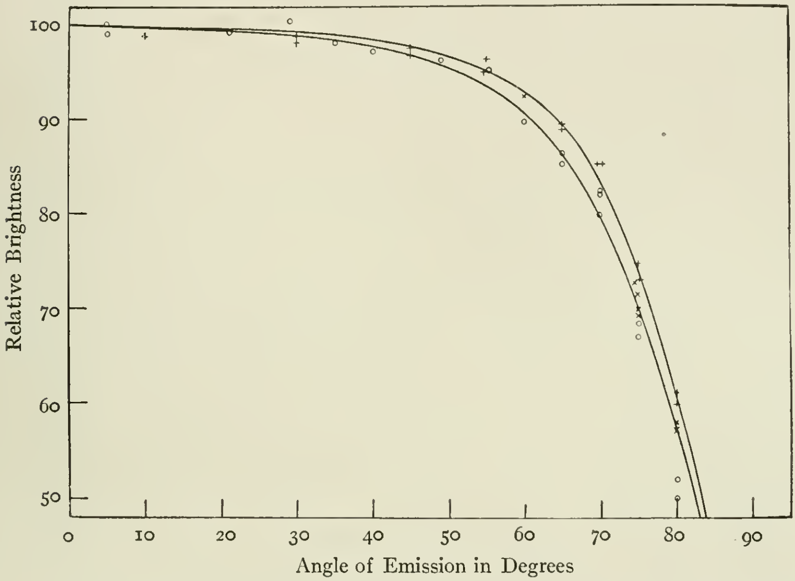

FIG. 5.—The deviation from Lambert's law of the emission from incandescent carbon.

o $\lambda = 0.63\ \mu$ first lamp studied + $\lambda = 0.46\ \mu$ second lamp studied
× $\lambda = 0.63\ \mu$ second lamp studied

Many individual values in the neighborhood of 0° have been omitted from the plat rendered difficult, due to the close location to the city streets. The difference, however, between the variations for the blue light and the red light is real for both tungsten and carbon.

In the investigation of the effect of temperature on the variations, seven sets of determinations showed that for red light tungsten does change with temperature. The average of these gave for 75° an increase in the deviation with the increased temperature amounting to 22 per cent, with an average variation from it of 5 per cent. The two operating temperatures were approximately those which were used in the regular determination of the variation of brightness with angle of emission. The results have been plotted on the curves given as filled circles at 75°. In finding the variation with temperature for blue light the various brightnesses for the blue light were directly compared with the corresponding brightnesses for the red light. Some of the experimental uncertainties which have been stated elsewhere were thus eliminated. The change with temperature for the blue light was found to be very closely the same as for the red light, the exactness of agreement falling well within the range of experimental error. The single value plotted in Fig. 3 for the blue light at 75° was obtained from these measurements, and is dependent on the correctness of the value plotted for the red light. No such changes with temperature were found for carbon.

The making of brightness determinations is difficult for large angles of emission. In Fig. 3 there are plotted as crosses values obtained for large angles of emission for which special care was taken to eliminate as many errors as possible. Two values for the relative brightness 61 and 78 for angles of emission greater than 86° were obtained but have not been included in the plat. By making use of a U-shaped filament, this decrease in brightness for large angles may be readily observed if the filament is so rotated about a central vertical axis that the two limbs are nearly in the line of sight. A fairly large portion of the lower curved portion emitting light at a large angle will then be found in close juxtaposition to fairly large areas emitting light at smaller angles of emission. In Fig. 6 there has been plotted the brightness of cylindrical glowing filaments of tungsten and carbon at normal operating temperatures as functions of the apparent distances from the central axes when viewed at a distance.

The following conclusions may be drawn from these plats:

1. The brightness of tungsten beginning with normal emission increases with the angle of emission, reaches a maximum at about 75°, and for somewhat larger angles decreases rapidly. The brightness of carbon, beginning with normal emission, decreases with increasing angle of emission, the rate of decrease increasing with the angle.

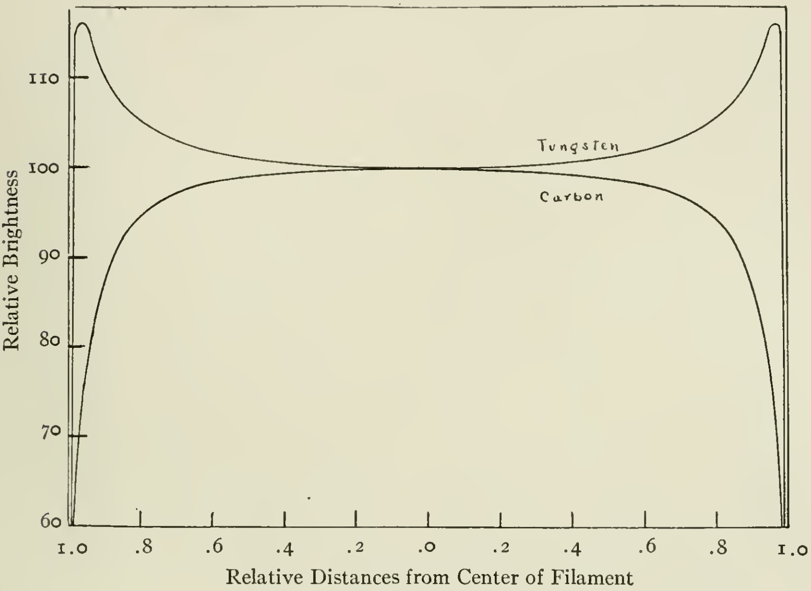

Fig. 6.—The variations in brightness across cylindrical filaments of carbon and tungsten as viewed from a distance.

2. The relative brightness variations for light of short wavelength 0.46 μ for tungsten are noticeably less than those for the light of the longer wave-length 0.63 μ. The same effect is true for carbon, though the difference is less noticeable.

3. The relative brightness variations for tungsten at the higher of the two temperatures chosen is about 20 per cent or 25 per cent greater than the corresponding variations for the

lower temperature. No definite change with temperature was found for carbon.

4. The average brightness of a tungsten cylindrical filament, viewed normally to its axis from a distance, is found to be about 3 per cent greater than that of the central part, that for a carbon filament about 5 per cent less.

It is interesting to compare the experimental results with what may be obtained by computation, making use of the optical constants obtained by H. v. Wartenberg.[1] The formulae used in these computations are the approximation formulae given by Drude.[2] A few test applications of the more rigorous formulae[3] made no very apparent changes in these results. The writer is indebted to Dr. I. Langmuir of the Research Laboratory of the General Electric company for such computations relative to the emission from tungsten. Column 1, Table I, gives the angle of emission, columns 2, 3, and 4 the reflectivities for light polarized in the plane of emission, for light polarized normal to the plane of emission, and for natural light. Column 5 contains the relative brightness expressed in terms of the normal brightness, obtained by making use of Kirchhoff's law. Column 6 contains the experimental results obtained by the writer for tungsten at the higher of the two operating temperatures. It is to be noticed that the variations shown in the computed values, in both cases, though generally in the same direction as the observed values, are of a different order of magnitude, excepting perhaps for the large angles of emission. It is not to be forgotten in this connection that the Drude constants were obtained with light of wave-length 0.579 μ, while the results used by the writer in the comparisons were obtained with light of wave-length 0.63 μ.

POLARIZATION TESTS

The arrangement of apparatus for these tests is quite simple. By means of a lens an enlarged image of the filament investigated was formed on a piece of ground glass just in front of a Marten's polarization photometer.[1] Sometimes a piece of colored glass, either red or blue, was placed just back of the ocular opening. The photometer as used had the disk with a single opening at the end next to the ground glass.

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[1] *Verh. Deutsch. phys. Ges.*, **12**, 105, 1910.

[2] Winkelmann, **6**, 1300.

[3] Chwolson (German ed.), **2**, 880. (In equation [60] the first sign in the numerator should be interchanged with the first in the denominator.)

TABLE I

Computed and Observed Brightnesses and Polarizations of Tungsten ($n=2.76$, $nx=2.71$) and of Carbon ($n=2.98$, $nx=1.74$) as Functions of the Angles of Emission

Angle	Reflectivity			Relative Brightness		Polarization	
	R_p	R_s	R	Comp.	Obs.	Comp.	Obs.
			Tungsten				
0°	0.486	0.486	0.486	1.000	1.000	0.000	.....
15	.497	.474	.486	1.000	1.003	.022	.....
30	.533	.437	.485	1.002	1.011	.093	.....
45	.597	.370	.484	1.004	1.03	.220	0.156
60	.693	.264	.478	1.015	1.08	.411	.285
75	.826	.167	.497	.978	1.16	.655	.458
80	.880	.205	.542	.891	1.15	.737	.520
85	.938	.391	.664	.654	1.08	.813	.585
90	1.000	1.000	1.000	.000	.....	.875	.....
			Carbon				
0°	0.367	0.367	0.367	1.000	1.000	0.000	.....
15	.379	.354	.366	1.001	.996	.020	.....
30	.418	.314	.366	1.002	.989	.082	0.022
45	.492	.245	.368	.998	.968	.196	.054
60	.605	.142	.374	.989	.906	.370	.096
70	.702	.079	.390	.964	.80	.511	.127
75	.772	.078	.425	.908	.70	.603	.144
80	.840	.133	.486	.812	.58	.689	.160
85	.917	.354	.636	.522	.....	.774	.....

The computed values are for $\lambda=0.579\,\mu$, the observed for $\lambda=0.63\,\mu$.

The first thing to be tested was the depolarizing action of the ground glass. This was found to be very small, of the order of 3 per cent for white light and for red light, and a trifle larger perhaps for blue light. There has been investigated for carbon and for tungsten the polarization of the emitted light as a function of the angle of emission. These results are included in Fig. 7. The change in polarization with change in temperature has also been investigated. Without any other changes the temperature of the filament of lamp *A* was varied over a very large range and polarization measurements were taken for some large angle of emission. No change was found either for carbon or for tungsten. The value plotted in Fig. 7 for tungsten at 76°.5 as a filled circle is the average of 13 measurements made in this connection. Similar tests using light of different wave-lengths revealed no certain change. Because of this fact and because of the greater ease with which measurements could be made using the natural light from lamp *A*, such measurements have been included in Fig. 7.

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[1] *Phys. Zeit.*, 1, 299, 1900.

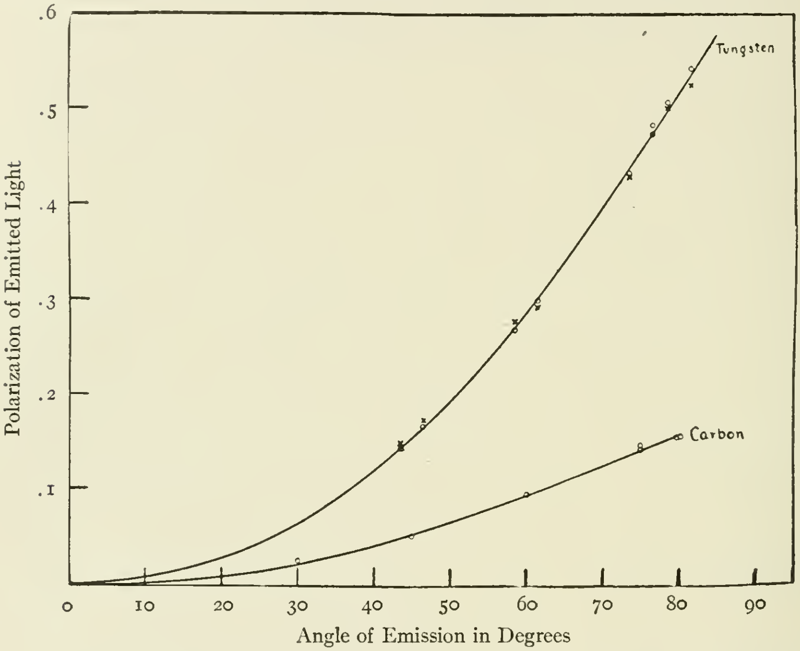

FIG. 7.—The polarization of the light from tungsten and carbon as a function of the angle of emission.

× Natural light o Light of wave-length 0.63 μ

It is interesting to compare these results for tungsten and carbon with those which may be computed from the optical constants. In column 7, Table I, will be found such computed values; in column 8 the observed values. It will be noticed for tungsten that throughout the observed values are very closely 0.7 of the computed values. For carbon the differences between the observed and the computed values are very great.

DISCUSSION OF RESULTS

As sources for the observed discrepancies between the computed and the observed results, there appear at least five facts to be considered.

1. The computed results are based on determinations made with light of wave-length 0.579 μ, while the observed data are for light of wave-lengths approximately 0.63 μ and 0.46 μ.
2. The results of Wartenburg were obtained for materials at ordinary temperatures, while the writer's were obtained with the materials at incandescent temperatures.
3. The existence of a temperature gradient in the incandescent filament.
4. The roughness of the filament surface.
5. The existence of an anisotropic surface layer.[1]

The reflectivity data on both carbon and tungsten by Coblentz[2] would indicate that, whatever variations in the optical properties there might occur with a change in wave-length in passing from the red to the blue end of the spectrum, the optical properties for an intermediate wave-length would possess corresponding intermediate values. If such is the case the discrepancies cannot be ascribed to the first of the above-named sources, for the computed deviations are not intermediate between the red and the blue observed deviations. It should be stated here that the reflectivity to be computed from Wartenberg's data[3] on graphite, viz., 36.7 per cent, differs from that obtained by Coblentz, 22.3 per cent, and from the values obtained by Aschkinass[1] for gas coke and anthracite, 3.7 per cent and 4.4 per cent.

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[1] Perhaps a sixth source of discrepancy, brought to the author's attention by Dr. C. E. Mendenhall, should be added. Irregularities in the glass of the bulb of ordinary lamps were found by him to be the source of great uncertainties, and in the present work, might be thought to produce by chance some of the variations noted. However, investigations on different filaments and at different points on the same filament by the two different methods always gave consistent results. Plate-glass windows would undoubtedly have been a great help in eliminating some uncertainties, but would certainly not have changed the order of the observed effects.

[2] *Bulletin of the Bureau of Standards*, 7, 197.

[3] *Loc. cit.*

In view of the fact that a change in the optical properties of tungsten with temperature has been found, it would seem possible that considerable of the variation between the computed and the observed results on this substance might be due to the second cause stated above. As to carbon, the experimental data do not seem to support any such explanation.

The existence of a temperature gradient in the incandescent filament needs to be considered, since the average distance of the source of radiation from the surface of the radiating body changes with the angle of emission. Such a source of error would naturally make the observed values for the cosine law variation too small for the larger angles of emission. It is interesting to see what changes might be expected due to this temperature gradient. An expression given by Angell[2] serves for computation purposes. Some work by the writer not yet published gives for the thermal conductivity of incandescent carbon a quantity of the order of $0.1 \frac{\text{watt}}{\text{deg.} \times \text{cm}}$. This combined with an input of $7.2 \frac{\text{watts}}{\text{cm of length of filament}}$ and a filament radius of 0.013 cm actual data for one of the carbon lamps investigated, gives as the temperature difference between the axis of the filament and the surface 0°003. This would certainly not account for any of the discrepancies. The formula given by Angell, however, assumes that the radiation originates at the surface of the filament. But if, as is undoubtedly the case, the radiation originates in finite depths, there might still exist near the surface a considerable temperature gradient, due to the fact that the elements of volume near the surface receive less radiation from the side toward the surface than do those elements which are farther from the surface. Such an effect, while it might account for the discrepancy as to the cosine law variation for carbon, would act in the wrong direction in the case of tungsten. Since the

[1] *Ann. d. phys.* (4), **18**, 380, 1905.

[2] *Physical Review*, **32**, 421, 1911.

polarization effects are but little if at all dependent on the temperature, such measurements would not be noticeably affected by this supposed cause.

The fact that the filaments were roughened, due either to being heated up to high temperatures or to other causes, does not explain these differences so far as the deviation from Lambert's law is concerned. Any such roughening would tend to make the glowing filament in question behave more like a black body, for which Lambert's law should hold rigorously. It therefore might be expected that the true deviations, if a polished surface could be used, would be greater than those found. That the filaments, especially the carbon filaments, were roughened, was readily visible when viewed by light emitted at large angles. In the making of the optical pyrometric measurements it was fortunately possible to select places which were apparently smooth. Because of this the writer is of the opinion that the observed variations from Lambert's law are of the right order of magnitude. The effect of this roughening might well explain, however, some of the differences between the observed and the computed polarizations. Here the observed values are noticeably less than the computed values. This is to be expected if the roughening tends to make the glowing filaments behave more like a black body. How much may be accounted for in this way is very difficult to say. If it might be assumed that the portions of the filament which appeared through the telescope to be smooth were in fact polished surfaces, the writer believes that the differences between the observed and the computed polarizations could certainly not be accounted for on the supposition that the roughening tends to make the body black.

The existence of an anisotropic surface layer is to be expected as a consequence of surface tension forces. Uljanin[1] has mentioned this in his paper and speaks as though the anisotropism were an established experimental fact. How much this might affect the observed results is a matter of conjecture without further data.

It is of course possible that the various sources of discrepancy additatively combine to explain the difference between the observed and the computed results.

[1] *Loc. cit.*

The present work seems to have a direct bearing on certain conclusions which have been reached by others. Hyde[1] has found that a tungsten-carbon combination does not satisfy what he has for convenience called "Criterion I," and that therefore, taking one as a standard, the other changes in its optical properties on being heated. Though good reasons were advanced for thinking that the change occurred in the tungsten, they were not conclusive. The present work indicates definitely a change with temperature in the optical properties of tungsten. No change has been found for carbon. The greater deviation from Lambert's law for the light of the longer wave-length for tungsten is in perfect accord with the known pronounced emission selectivity for the shorter wave-lengths and the measured reflectivities of Coblentz.[2] The greater emissivity for the blue light would suggest that for the shorter wave-length the approach to black-body radiation is more nearly complete, that consequently there would be a closer approach to Lambert's law, as is the case. Starting with the close agreement of the blue and the red deviations of carbon from Lambert's law, one is led to the conclusion that it behaves like a gray or a black body. The small variation which has been found agrees in direction with the small variation which Coblentz found in the reflectivities. That it approximates closely a gray body has been well known. It is, perhaps, surprising that so great a deviation from the cosine law should be found for carbon, but this does not, so far as the writer can see, in any way theoretically conflict with the supposed grayness. The variations found for tungsten are exactly similar to those computed by Uljanin for *Pt.*, *Ag.*, and *Cu.*

SUMMARY

1. There has been measured the brightness variation as a function of the angle of emission for carbon at one temperature and for tungsten at two temperatures, making use of light of wave-length 0.63 μ; and for tungsten and carbon at one temperature each, making use of light of wave-length 0.46 μ. The conclusions drawn from these are summarized near the end of the subdivision giving the results on brightness measurements.

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[1] *Loc. cit.*

[2] *Loc. cit.*

2. Measurements of the polarization of the light emitted at various angles of emission have been made for both carbon and tungsten. No certain changes have been found in the polarization due to changes in temperature or changes in the wave-length of the light used.

3. The comparison of the observed with the computed values for both tungsten and carbon show the observed deviation from Lambert's law to be greater than and of a different order of magnitude from the deviations computed; while for the polarization of the emitted light, it has been found that the observed values are considerably less than the computed values.

4. In certain respects the present results have been found to agree qualitatively with the experimental results of Hyde and of Coblentz.

In conclusion the writer desires to express his indebtedness to Mr. F. E. Cady and to Mr. G. Cadisch for aid given in the experimental work.

Physical Laboratory
National Electric Lamp Association
Cleveland, Ohio
July 1912

THE INTEGRATED SPECTRUM OF THE MILKY WAY[1]

By E. A. Fath

In *Harvard Annals*, **56**, No. 1, is given a study of the distribution of the spectra of over 32,000 stars. Among other results are found: (1) That over 52 per cent of the stars investigated have A-type spectra; (2) The ratio of A-type stars to all other types increases as the brightness decreases; (3) In the Milky Way two-thirds of the stars investigated are of the A-type.

In view of these observations we might seem justified in drawing the following conclusions: (1) The integrated spectrum of our stellar system would be approximately of the A-type; (2) The integrated spectrum of the Milky Way as seen from our point of view would be of the A-type.

For some years the writer had planned to make a study of the integrated spectrum of some of the brighter regions of the Milky Way in order to test the validity of the second conclusion. An opportunity presented itself from March to June of last year, when Director Campbell of the Lick Observatory very kindly loaned the spectrograph which the writer had used for work on the Zodiacal Light. This instrument is described in *Lick Observatory Bulletin*, No. 165.

The region selected for the first trial was the very bright portion of the Milky Way partially bounded by the stars γ, δ, and λ *Sagittarii*, which is the brightest region of the galaxy seen at this latitude.

The method of observation was the simplest possible. The spectrograph alone was used, as the angular diameter of the surface investigated was greater than the angular aperture of the collimator lens. The instrument was mounted in such a way as to permit motion in azimuth and altitude. It was then pointed at the region in question and moved every 10 or 15 minutes to follow the diurnal motion.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 63.

The first exposure was begun on the night of March 30 and continued, whenever possible, until May 4. The total exposure on this first plate was 30^h20^m. As the spectrum obtained was not quite as strong as desired a second plate was taken in the interval from May 27 to June 29. The second exposure totaled 65^h13^m. It was of satisfactory intensity.

The general appearance of this spectrum is like that of the sun in that it shows the F, G, H, and K lines and three broad absorption bands to the violet of K. On closer examination, however, certain differences can be noted. There appears to be a bright line at 416 $\mu\mu$ and faint additional absorption lines at 411, 421, and 448 $\mu\mu$. The bright line is just halfway between the first two absorption lines. This raises the question as to whether it may not be merely a portion of the general continuous spectrum which appears especially strong by contrast. The line appears on both plates, while the three faint absorption lines appear only on the stronger of the two. The origin of these additional lines is uncertain. There is neither a bright auroral nor a bright nebular line in the neighborhood of the bright line observed. The absorption line at 411 $\mu\mu$ may possibly be $H\delta$; those at 421 and 448 $\mu\mu$ cannot be even approximately identified. These wave-lengths are probably uncertain by 1 $\mu\mu$ although for two entirely distinct sets of measures made over a week apart the greatest difference is 0.5 $\mu\mu$. This uncertainty is due in a large measure to the very small scale of the spectrum, for the distance on the plate between λ 5000 and λ 3900 is only 2 mm. The line at G is very broad and it is not impossible that $H\gamma$ may help in its formation.

It might be thought that the three stars, γ, δ, and λ *Sagittarii*, which are south and east of the star cloud under consideration and are of the K-type, have an injurious effect, but that this is not the case is shown by measures of the brightness of this region with a photometer of the type used by Yntema and described in *Groningen Publications*, No. 22. A provisional reduction of these measures gives the brightness of a square degree of the Milky Way in this very bright region equal approximately to 0.4 the light of a standard first magnitude star. The total intensity of the light of the three stars is equal to half that of a star of the first magnitude. The angular aperture of the collimator lens may be taken as 3°. Accordingly the area of sky affecting the plate at any time was about seven square degrees. Then, assuming the most unfavorable condition possible, namely, concentrating the light from the three stars into one at the center of the region, this would amount to less than one-fifth of the total light falling on the plate at any time. As a matter of fact, these stars could send no light whatever to the plate unless the spectrograph was not moved at sufficiently short intervals of time. This occurred a few times, as it was necessary to carry on this exposure at the same time the writer was observing with the 60-inch reflector, and time could not always be taken to make the required changes. Had the exposure been made, either by having the spectrograph moved by a driving-clock or by continuous motion of the alt-azimuth mounting by the observer, the light of the three stars would never have reached the plate.

This result was of sufficient interest to continue the work, the same spectrograph again being loaned by the Lick Observatory. The next plate was exposed a total of 67^h52^m during May and June 1912, on the bright region at $\alpha=18^h42^m$, $\delta=-8°$. This time the spectrograph was attached to the coelostat of the Snow telescope, a new set of gears providing the required speed of rotation. The plate is very much like the previous one, except that it is not quite so dense and the absorption line at 421 $\mu\mu$ is absent.

A third exposure was made during July and August 1912, on the region between β and η *Cygni*. This exposure amounted to 74^h11^m. The spectrograph was again attached to the coelostat of the Snow telescope.

The intensity of the last plate is about the same as that of the stronger plate of the *Sagittarius* region and the spectrum is similar in its general appearance, but the absorption line at 411 $\mu\mu$ is relatively stronger and the possible bright line at 416 $\mu\mu$ is relatively weaker.

The four plates taken of three bright portions of the Milky Way are therefore in agreement in indicating that *the integrated spectrum of the Milky Way is approximately of solar type.*

This result receives an independent check from a series of plates taken to test the possible selective absorption, or loss, of light in space. On the strength of the Harvard result that the greater portion of the Milky Way stars are of the A-type, and assuming that this result could be extended to the fainter stars, it appeared possible to test the hypothesis that the more distant, and therefore fainter, stars are redder than the nearer brighter ones.

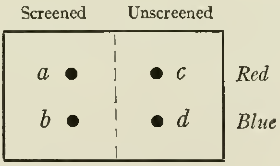
The regions selected for trial were centered at

$18^h30^m+10^\circ$	$18^h30^m+5^\circ$
18 40 +10	18 40 +5
18 50 +10	

and were photographed with the 60-inch reflector on red-sensitive plates.

On each plate two exposures of a region were made, one through a red and the other through a blue ray filter, the plate being moved a little in declination to separate the images of the two exposures. In addition one-half of the plate was kept covered by the slide of the plate-holder except during 30^s at the close of each 6^m of the exposure. Thus on the covered or "screened" half of the plate only the brighter stars are found. Accordingly it is possible to compare faint stars from the uncovered portion of the plate with bright stars from the screened portion.

The theory of the method, which was suggested by Professor Kapteyn is as follows: Let us consider two stars, one from each half of the plate, at the same distance from the optical axis so that there will be no differential aberration effect, and with the red images approximately equal. Though these images are of the same size, that for the screened portion of the plate will be of a star approximately two magnitudes brighter, and hence, on the average, a less distant star, than that whose image lies within the unscreened portion.



On the assumption that there is a greater absorption and scattering of blue than of red light in interstellar space, the screened blue image b will be larger than the blue image d. Since all of the images compared are approximately of the same size the result will be free from photographic effects that might otherwise enter.

Let us put

$$a-c=m \tag{1}$$

$$b-d=n \tag{2}$$

On the basis of our assumption the value $n-m$ will be positive, that is

$$b-d>a-c$$

Subtracting (1) from (2)

$$(b-d)-(a-c)=n-m=\text{redness}=r$$
$$r=(b-a)+(c-d)$$

If r is positive the more distant stars are redder than those nearer at hand. Should r prove to be negative then the more distant stars are bluer.

The plates taken were measured in the following manner. A circle of 38 mm radius was drawn with its center at the center of the plate. No stars outside the circle were used on account of the large aberration beyond this limit. A star pair from the screened portion was then compared with any pair in the unscreened region that was approximately at the same distance from the center, the red images being as nearly as possible of the same size.

It will be unnecessary to go farther into the details of this matter. The result of the comparison of 76 pairs of stars on the five plates showed that, in the mean, the fainter stars are distinctly redder than the brighter ones. Had we been certain that the stars were all of approximately the same spectral type we should have strong evidence that the light from the fainter stars lost more of its blue than of its red component in traversing interstellar space, as compared with the brighter stars. The spectrograms, however, offer another explanation, namely, the ratio of A-type stars to other types decreases as the stars become fainter, and, from a certain undetermined magnitude on, the stars of approximately solar type predominate, so that the fainter stars, which give us the most of the Milky Way light, are in reality redder than the brighter ones. These two investigations are therefore in agreement, although the original purpose of the second is not served.

It is unfortunate that the regions selected for the second investigation were not identical with those of which spectrograms were obtained, but the former plates were taken first. It was not advisable to try a spectrographic exposure on these regions as they are too faint, the exposure required being probably in the neighborhood of 200 hours. The first spectrogram was obtained at about the same time as the other series of plates. The results of the latter were therefore not published in the form originally intended.

The Harvard investigation probably contains very few, if any, stars fainter than the 8th magnitude. Both the investigations discussed here deal almost exclusively with stars fainter than this. It would be a matter of interest to know at what magnitude the ratio of A-type stars to all other types reaches a maximum, as well as the magnitude at which the other types begin to predominate. From the material at hand, however, there seems to be no satisfactory method of obtaining these values.

In regard to the spectrographic exposures on the Milky Way it should be stated that every precaution was taken to prevent sunlight from any source reaching the plate. The exposures were not begun until the sky was thoroughly dark and were closed before dawn. Furthermore, no exposure was made while the moon was above the horizon.

In conclusion, the writer desires to express his great appreciation to Director Campbell of the Lick Observatory for the use of the spectrograph, and to Mr. P. J. Van Rhyn, who assisted in the exposure of the last plate.

Note added October 22.—The reproductions of the spectra are not very satisfactory owing to the enlargement required and the very coarse grain of the original negatives. In *a* the G-group is faintly indicated, H and K form a broad band, and the shadings to the left are the ultra-violet bands referred to above.

MOUNT WILSON SOLAR OBSERVATORY
August 27, 1912

THE *ALGOL* VARIABLE *RR DRACONIS*[1]

BY FREDERICK H. SEARES

Among the variable stars included in the program of observations carried on at the Laws Observatory was the object discovered by Madame Ceraski in 1904 which has since received the designation *RR Draconis.* When first placed on the observing-list the character of the variation was unknown, but the measures of the first two nights showed the star to be an *Algol* of unusual interest on account of the range and rapidity of its variation. It was found normally to be a little brighter than the tenth magnitude, but during the eclipses, which occur at intervals of two days and twenty hours, it became so faint as to disappear from view in the $7\frac{1}{2}$-inch equatorial with which the observations were made. The brightness at the time of disappearance was apparently a little less than the thirteenth magnitude, so that the range covered by the visual observations was in excess of three magnitudes, the average change being at the rate of nearly a magnitude per hour.

Though the nature of the variation during the two hours that the star remained invisible in the Laws Observatory equatorial could not be determined, it was possible from the steepness of the curve to derive the epochs of minima, and consequently the period, with precision. The photometer used was of the equalizing wedge type designed by Pickering,[2] and as the wedge itself had been carefully calibrated and tested[3] it was possible to reduce the measures to an absolute scale of magnitudes, which were then combined to form a mean light-curve. The details of the observations, which extended from June 29, 1905, to September 25, 1906, and their discussion were published in *Laws Observatory Bulletin*, No. 9. The elements found were

$$\text{Min.}=\text{J.D. } 2417026.682+2^{\mathrm{d}}.831079\,E \quad \text{G.M.T.}$$

with an estimated uncertainty in the period of 1^{s}.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 64.

[2] *Astrophysical Journal*, **13**, 249, 1901.

[3] *Laws Observatory Bulletin*, No. 7, 1905.

The light-curve derived from the visual observations, reduced to the zero point of the Harvard Polar Sequence in a manner to be described later, is given in Table I. The phase values in the first and third columns are counted from minimum. With the exception of a remark by Hartwig,[1] indicating a correction of $+20^m$ to the ephemeris derived from the above elements for 1911, no further results seem to have been published.[2]

TABLE I

VISUAL LIGHT-CURVE—LAWS OBSERVATORY

Phase	Mag.	Phase	Mag.
0^d035	12.96	0^d100	10.51
0.040	12.58	0.110	10.35
0.045	12.29	0.120	10.23
0.050	12.02	0.130	10.13
0.055	11.79	0.140	10.03
0.060	11.59	0.150	9.95
0.065	11.40	0.160	9.88
0.070	11.23	0.170	9.82
0.075	11.08	0.180	9.77
0.080	10.94	0.190	9.74
0.085	10.81	0.200	9.72
0.090	10.70	0.210	9.71
0.095	10.60	0.220	9.70
0.100	10.51	0.230	9.70

In order to determine the total range and the character of the variation for the part of the curve not included in Table I, the star was followed photographically throughout the greater part of an eclipse on August 7, 1912, with the 60-inch reflector of the Mount Wilson Solar Observatory.

Eight plates including 61 exposures of 2^m each and 4 of 1^m, covering a period of six hours, were obtained; and on the following night three additional plates for the determination of the normal brightness of the variable and the zero point of the magnitude scale were made. The observational details are contained in Table II. On four of the plates exposures were made with various combinations of apertures for the purpose of establishing the magnitude scale of the comparison stars. For Nos. 784 and 785 the exposures with the full aperture alternated with those made with 32- and 14-inch diaphragms; for 790 the order of apertures was 60, 32, 14, 14, 32, and 60 inches; and for 791, 32, 9, 9, and 32 inches. Plate 792 was exposed both to the field of the variable and to the Pole, the order being, variable, Pole, Pole, variable.

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[1] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **46**, 230, 1911. This correction and the ephemeris given by Hartwig are based on the provisional elements in *Laws Observatory Bulletin*, No. 9, p. 135. The final elements on p. 138 are more precise.

[2] The observations by Lehnert, *Astronomische Nachrichten*, No. 4596, arrived just as this paper was sent to the printer.

TABLE II

LIST OF PLATES

Date	G.M.T.	Plate No.	Exp. Time	No. Exp.	Aperture	Seeing
1912 August 7....	$16^h 55^m$–$17^h 30^m$	778	2^m	9	60	5
	17 37 –18 16	779	2	8	60	4
	18 23 –19 1	780	2	8	60	3
	19 13 –19 47	781	2	9	60	3
	19 54 –20 29	782	2	9	60	2–3
	20 35 –21 11	783	2	9	60	2–3
	22 3 –22 37	784	2	9	60, 32	3
	22 42 –23 1	785	1	4	60, 14	3
August 8....	18 59 –19 11	790	1	6	60, 32, 14	5
	19 17 –19 30	791	2	4	32, 9	5
	19 34 –19 55	792	2	4	32	6

Fourteen comparison stars, all near the variable, were selected for use in the determination of the magnitude scale. The brightest of these, which is No. 1 of the list, is *B.D.*+62°1642, 9.3 mag. The others, all of which are fainter than the *Durchmusterung* limit, may be identified with the aid of the chart of the field shown in Fig. 1. On the diaphragm plates all of the comparison stars were measured. The resulting mean scale readings, corrected for distance error, are given in Table III. The subscripts in the headings of the table refer to the aperture. The mean scale readings of the stars of the Polar Sequence shown on plate 792 are also given in Table III, in the last column. The corresponding Polar Sequence numbers and magnitudes taken from *Harvard Circular*, No. 170, are in the two preceding columns. The scale readings for such stars as were used for the reduction of the remaining plates are given in Table V.

The data for plates 784–791 were reduced in the usual manner[1] for the determination of the magnitudes of the comparison stars. The various scales were brought to an arbitrarily chosen common zero point, and the separate values for the brightness of each star combined to form the provisional magnitude given in the second column of Table IV. The five columns immediately following contain the deviations of the separate determinations from the mean provisional magnitudes. There is no evidence of appreciable systematic differences between the different determinations of the scale, and in general the consistency of the results is satisfactory. The average deviation of a single magnitude, including the effect of scale error over a range of four magnitudes is 0.096 mag., which corresponds to a probable error of ±0.081 mag.

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[1] A general account of the methods of observation and reduction followed in photometric work with the 60-inch reflector is in preparation and will be published shortly.

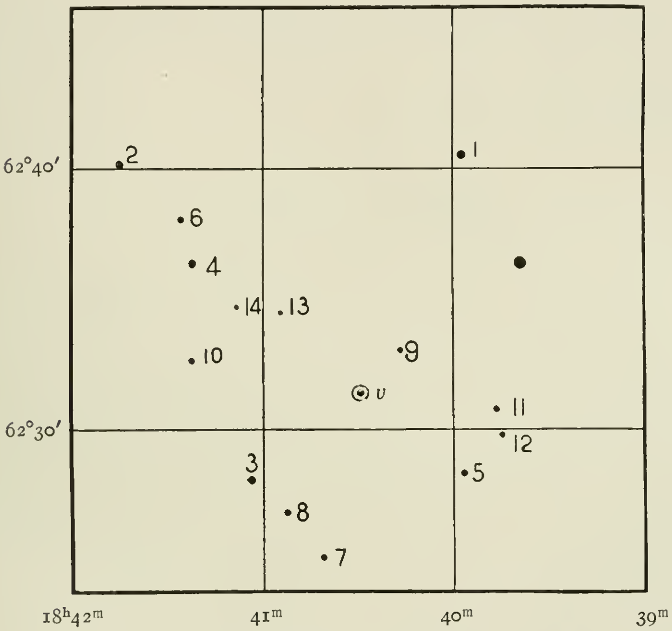

FIG. 1.—The comparison stars used with *RR Draconis*

To reduce the provisional magnitudes to the zero point of the Harvard Polar Sequence the values of the scale readings of the Polar Sequence stars given in Table III for plate 792 were plotted against the corresponding Polar Sequence magnitudes. From the curve thus established were read the values of the magnitudes corresponding to the scale readings of the comparison stars photographed on the same plate. These are entered in the eighth column of Table IV. The differences between these values and the corresponding provisional magnitudes in the second column appear in the ninth column and, disregarding extinction, represent separate determinations of the constant for the reduction of the provisional magnitudes to the Polar Sequence zero point. The interval of brightness covered by the polar stars is too short to afford a reliable comparison of the scale derived for the comparison stars with that of the Polar Sequence.

TABLE III

MEAN SCALE READINGS FOR MAGNITUDE SCALE PLATES

Star	784		785		790			791		792			
	s_{60}	s_{32}	s_{60}	s_{14}	s_{60}	s_{32}	s_{14}	s_{32}	s_9	s_{32}	Polar Sequence		
											No.	Mag.	s_{32}
v ..	3.2	8.4	5.0	12.6	2.9	8.8	11.8	6.8	12.8	6.1	14	10.52	9.8
1 ..	3.3	8.5	5.0	12.6	3.6	9.0	12.2	7.3	13.3	6.6	7*r*	10.57	10.0
2 ..	.....	13.8	10.9	.....	.....	15.5	.....	14.2	.....	13.7	5*s*	10.68	10.4
3 ..	9.3	14.3	11.4	.....	9.4	15.6	.....	14.1	.....	14.3	18	11.92	14.4
4 ..	9.2	14.3	11.8	.....	9.9	15.9	.....	14.5	.....	14.0	19	12.28	14.7
5 ..	9.3	14.6	12.2	.....	10.9	17.8	.....	14.6	.....	14.7	7*s*	12.31	14.8
6 ..	10.4	15.8	12.2	.....	11.8	.....	.....	15.5	.....	16.3	20	12.59	16.6
7 ..	10.0	15.2	12.5	.....	11.4	17.9	.....	16.7	.....	15.5	11*r*	12.87	18.5
8 ..	11.2	16.3	14.1	.....	11.9	.....	.....	17.4	.....	17.4			
9 ..	11.1	17.2	13.6	.....	12.0	.....	.....	17.6	.....	17.8			
10 ..	11.6	17.6	13.6	.....	12.1	.....	.....	17.4	.....	17.8			
11 ..	11.5	.....	13.9	.....	13.6	.....	.....	.....	.....	18.0			
12 ..	12.5	.....	14.8	.....	14.3	.....	.....	.....	.....	.....			
13 ..	13.6	.....	15.5	.....	14.7	.....	.....	.....	.....	.....			
14 ..	13.9	.....	15.4	.....	15.1	.....	.....	.....	.....	.....			

The zenith distance of the variable for the mean time of exposure of plate 792 was 34°9. That of the Pole is 55°8. The difference is greater than desirable, but could not be avoided without a serious disarrangement of the observing program. In the absence of special extinction tables for Mount Wilson an approximate value of 0.20 mag. has been derived from those of Müller.[1]

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[1] *Photometrie der Gestirne*, p. 515, 1897.

The visual extinction has been doubled, and a rough interpolation has been made for elevation above sea-level. The application of the extinction correction to the mean of the quantities in the ninth column of Table IV gives for the reduction constant of the provisional magnitudes 8.95 mags. The resulting magnitudes adopted for the comparison stars are printed in the last column of Table IV.

TABLE IV

Magnitude of Comparison Stars

Star No.	Provisional Mag.	Residuals					Mag. from Polar Plate 792	Red. to Polar Sequence	Adopted Mag.
		784 60–32	785 60–14	790		791 32–9			
				60–32	60–14				
1.....	0.90	+ 1	+17	−11	−13	+ 6	....	....	9.85
2.....	3.03	+ 5	+ 1	+ 2	....	− 7	11.87	8.84	11.98
3.....	3.12	−12	−10	+ 8	+11	+ 5	12.07	8.95	12.07
4.....	3.23	+ 1	−14	+ 5	+ 5	+ 3	11.98	8.75	12.18
5.....	3.40	+10	+ 7	−23	−12	+17	12.20	8.80	12.35
6.....	3.68	− 3	+15	−16	−14	+16	12.53	8.85	12.63
7.....	3.70	+15	+ 5	− 2	+ 2	−21	12.38	8.68	12.65
8.....	4.02	+ 8	−25	+15	+17	−12	12.70	8.68	12.97
9.....	4.03	− 5	− 5	+13	+14	−18	12.76	8.73	12.98
10.....	4.06	−18	− 2	+13	+14	− 8	12.76	8.70	13.01
11.....	4.28	+16	+ 9	−11	−15	....	12.80	8.52	13.23
12.....	4.58	+10	+ 3	− 2	− 9	....	....	....	13.53
13.....	4.80	− 7	− 2	+ 7	0	....	....	....	13.75
14.....	4.89	− 9	+11	+ 4	− 5	....	....	....	13.84
Means	....	0.086	0.090	0.094	0.101	0.113	....	8.75	
Extinction........								0.20	
								8.95	

With the magnitudes of the comparison stars once established, the determination of the brightness of the variable becomes a simple matter. It is only necessary to plot the scale readings of the comparison stars for any plate against the adopted magnitudes and then read from the resulting curve the magnitude corresponding to the scale reading of the variable.

The scale readings for the comparison stars derived from plates 778–783 and given in Table V are the means of the eight or nine exposures on each of these plates, a single estimate with the photometric scale being made for each image. The results for the variable from these plates are given separately for each image in Table VI. These are the means of three estimates with the scale.

Since the mean image was used for the comparison stars and separate images for the variable, it was necessary to correct the scale readings of the variable for the systematic deviations of the various exposures from the mean of them all. The corrections were derived by subtracting for each star the scale reading of each image from the mean scale reading for all the images of the star. The average of the differences for the same exposure for all the stars gives the correction which will reduce the scale readings for that exposure to the mean exposure for the plate. The results for the various exposures on the different plates are given in Table VII, the unit being 0.1 of a scale interval.

TABLE V

MEAN SCALE READINGS OF COMPARISON STARS

Star No.	Mag.	778	779	780	781	782	783
1......	9.85	4.1	.....	.....	.....	3.9	3.7
2......	11.98	9.2	.....	.....	.....	8.8	8.0
3......	12.07	10.6	.....	.....	.....	9.7	9.3
4......	12.18	10.4	.....	.....	10.7	11.0	10.7
5......	12.35	10.3	.....	.....	10.3	10.0	9.4
6......	12.63	11.0	.....	.....	11.6	.....	.....
7......	12.65	11.3	.....	.....	11.1	.....	.....
8......	12.97	12.0	12.1	11.8	12.0	.....	.....
9......	12.98	11.7	12.0	11.7	12.0	.....	.....
10......	13.01	12.3	12.7	12.3	12.3	.....	.....
11......	13.23	12.0	12.1	11.8	11.9	.....	.....
12......	13.53	.....	13.0	12.7	12.9	.....	.....
13......	13.75	.....	14.2	14.2	14.2	.....	.....
14......	13.84	.....	14.1	14.1	14.1	.....	.....

The corrections are in all cases small and show that the atmospheric conditions, both as to transparency and steadiness, were satisfactorily constant. The mean values in the last column give a pretty clear indication that the plates are free from the photographic effect which sometimes causes the first of a series of exposures on a plate to be systematically brighter than the last.[1]

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[1] Professor Pickering kindly allows me to quote from a personal letter as follows: "When several exposures are made on a plate, the first and last being short and equal, the images formed by the first exposure are almost always brighter than those formed by the last. The average difference is about a quarter of a magnitude."

The scale readings of the variable in Table VI are the corrected values. In the case of plates 784 and 785 the rate of change in the variable is so small that the mean scale reading of the various exposures is used, the only distinction being a separation of the results for the different apertures. The seventh column of Table VI contains the values of the brightness of the variable read from the magnitude curves for the various plates. The results are also shown graphically in Fig. 2.

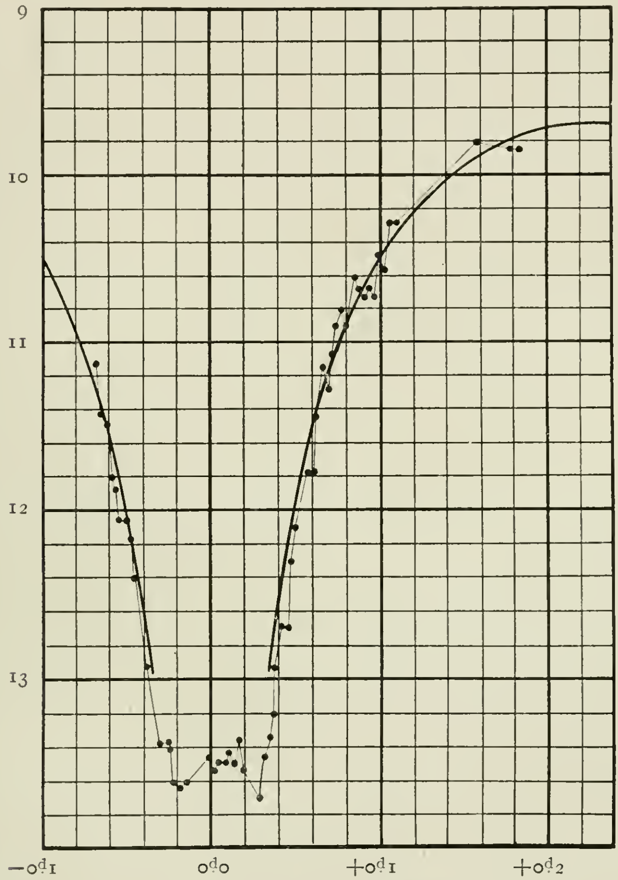

Fig. 2.—Light-curve of *RR Draconis* at eclipse

Broken line—Photographic observations
Full line—Laws Observatory visual light-curve

TABLE VI

Mean Scale Readings and Magnitudes of *RR Draconis*

1912, August 7 and 8

Plate	Exposure	G.M.T.		Phase	*s*	Mag.	Phot.—Vis.
778.....	1	$16^{\mathrm{h}}\,56^{\mathrm{m}}$	$0^{\mathrm{d}}.705$	$-0^{\mathrm{d}}.068$	7.3	11.12	−18
	2	17 1	0.709	−0.064	8.1	11.43	− 1
	3	5	0.712	−0.061	8.2	11.49	− 6
	4	9	0.714	−0.059	9.0	11.80	+16
	5	13	0.717	−0.056	9.2	11.88	+14
	6	17	0.720	−0.053	9.6	12.05	+17
	7	21	0.723	−0.050	9.6	12.05	+ 3
	8	25	0.726	−0.047	9.9	12.16	− 2
	9	29	0.728	−0.045	10.5	12.40	+11
779.....	1	17 38	0.735	−0.038	11.8	12.91	+18
	2	51	0.744	−0.029	13.0	13.37	−13*
	3	55	0.746	−0.027	13.0	13.37	−13
	4	59	0.749	−0.024	13.1	13.41	− 9
	5	18 3	0.752	−0.021	13.6	13.60	+10
	6	7	0.755	−0.018	13.7	13.63	+13
	7	11	0.758	−0.015	13.6	13.60	+10
	8	15	0.760	−0.013	13.6	13.60	+10
780.....	1	18 32	0.772	−0.001	13.1	13.46	− 4
	2	36	0.775	+0.002	13.3	13.53	+ 3
	3	40	0.778	+0.005	13.2	13.49	− 1
	4	44	0.781	+0.008	13.2	13.49	− 1
	5	48	0.784	+0.011	13.0	13.43	− 7
	6	52	0.786	+0.013	13.2	12.49	− 1
	7	56	0.789	+0.016	12.8	13.35	−15
	8	19 0	0.792	+0.019	13.3	13.53	+ 3
781.....	1	19 14	0.802	+0.029	13.8	13.70	+20
	2	18	0.805	+0.032	13.1	13.45	....
	3	22	0.808	+0.035	12.8	13.33	+37
	4	26	0.810	+0.037	12.5	13.20	+39
	5	30	0.812	+0.039	11.9	12.93	+27
	6	34	0.815	+0.042	11.4	12.69	+23
	7	38	0.818	+0.045	11.4	12.69	+40
	8	42	0.820	+0.047	10.5	12.30	+12
	9	46	0.823	+0.050	10.1	12.10	+ 8
782.....	1	19 55	0.830	+0.057	8.9	11.78	+ 7
	2	59	0.832	+0.059	8.9	11.78	+15
	3	20 4	0.836	+0.063	8.0	11.44	− 4
	4	8	0.839	+0.066	7.2	11.14	−23
	5	12	0.842	+0.069	7.6	11.29	+ 3
	6	16	0.844	+0.071	7.0	11.06	−14
	7	20	0.847	+0.074	6.6	10.90	−21
	8	24	0.850	+0.077	6.4	10.81	−21
	9	28	0.852	+0.079	6.6	10.90	− 7
783.....	1	20 36	0.858	+0.085	6.6	11.00	+19
	2	40	0.861	+0.088	5.8	10.68	− 6

* The residuals for the phase values $-0^{\mathrm{d}}.03$ to $+0^{\mathrm{d}}.03$ refer to the minimum *photographic* brightness of 13.50 mags.

TABLE VI—*Continued*

Plate	Exposure	G.M.T.		Phase	*s*	Mag.	Phot.—Vis.
783.	3	20^h44^m	0^d864	$+0^d091$	5.9	10.72	+ 4
	4	48	0.867	+0.094	5.8	10.68	+ 6
	5	52	0.870	+0.097	5.9	10.72	+16
	6	56	0.872	+0.099	5.3	10.48	− 5
	7	21 0	0.875	+0.102	5.5	10.56	+ 8
	8	6	0.879	+0.106	4.8	10.28	−13
	9	10	0.882	+0.109	4.8	10.28	− 9
784.	60	22 20	0.931	+0.158	3.2	9.81	− 8
	32	20	0.931	+0.158	8.4	9.81	− 8
785.	60	22 51	0.952	+0.179	5.0	9.85	+ 7
	14	54	0.954	+0.181	12.6	9.85	+ 8
790.	60	19 5	1.796	+1.023	2.9	9.61	− 9
	32	5	1.796	+1.023	8.8	9.78	+ 8
	14	5	1.796	+1.023	11.8	9.71	+ 1
791.	32	19 23	1.808	+1.035	6.8	9.69	− 1
	9	23	1.808	+1.035	12.8	9.69	− 1
792.	32	19 44	1.822	+1.049	6.1	9.70	0

A curious fact is brought to light by the magnitude curves for plates 778–783 drawn from the data in Table V, namely, that although the adopted magnitudes afford a consistent representation of the measures of plates 784–791 as shown by the residuals in Table IV, they do not represent satisfactorily the scale readings for plates 778–783. This is clearly shown by the residuals in Table VIII which are the deviations for these plates of the adopted magnitudes from those corresponding to the observed scale readings, the latter being read directly from the curves. These residuals are clearly systematic, not in the sense that would indicate an error of scale, but as though the comparison star magnitudes were affected with large accidental errors. Most of the stars, for example, show deviations of the same sign and of the same order of magnitude for all of the plates of this group upon which they were measured. The difficulty is not with the comparison stars, however, but with the plates of this particular group for which the error depending on the distances of the stars from the axis of the instrument was undoubtedly abnormal. It has been found that comparatively small temperature deformations of the mirror are sufficient to modify radically the character and amount of the correction required to compensate this error. The mirror is known to have been in an abnormal condition during the early part of the night when the plates in question were exposed, but as it is impossible to determine the distance error for each plate separately, the correction for normal figure was applied. The systematic deviations in Table VIII represent the uncertainty of such a method of procedure. The comparison star magnitudes were derived from plates made after the mirror had become normal, or nearly so. That they are not also affected by an uncertainty in the distance error is shown by forming a similar table of deviations for the results obtained with the 32-inch diaphragm on plates 784, 790, 791, and 792. With this aperture the distance error is vanishingly small for the extent of field covered. The comparison of the adopted magnitudes with the scale readings for the diaphragm exposures should therefore reveal any uncertainty that may have entered through the full aperture exposures on the plates from which the magnitudes were derived. The resulting deviations are wholly accidental, however, from which we may conclude that the comparison star magnitudes are unaffected by a residual distance error.

The whole question of the determination of the proper correction to be applied for distance error is one that has given much trouble in photometric work with the 60-inch reflector. With care, however, the difficulties can largely be avoided, as will appear from a detailed discussion to be published in a later paper.

In the present case the matter is complicated by the fact that there is available but a single bright comparison star. Owing to the absence of other stars of similar brightness the magnitude curves were naturally drawn *through* the points corresponding to the bright star. For this reason no residuals appear in Table VIII for this object. The brighter magnitudes of the variables may therefore be slightly in error in such a way as to affect systematically the results for a plate; but only plates 782 and 783 can be so affected, and the smoothness of the resulting light-curve indicates that the errors must be small. The images of the variable itself are unaffected, as they were always near the axis. The results for normal light are also unaffected by residual distance error, for they were derived from plates made when the figure of the mirror was satisfactory, and depend almost exclusively upon exposures with diaphragms of 32, 14, and 9 inches for all of which the distance correction is negligible.

TABLE VII

Reduction to Mean Exposure

Unit=0.1 Scale Interval=0.35 Mag.

Exposure	778	779	780	781	782	783	784	Means	
								s	Mag.
1.....	0	+2	...	+1	−4	+1	0	0.00	0.00
2.....	+1	...	+1	+1	−2	+3	+1	+0.08	+0.03
3.....	−1	−3	+3	+2	−3	+1	+3	+0.03	+0.01
4.....	+2	−2	+4	−1	−1	+3	0	+0.07	+0.02
5.....	+2	−1	−1	−1	+2	−7	−2	−0.11	−0.04
6.....	+2	0	−1	−3	+1	+2	0	+0.01	0.00
7.....	−1	+1	−2	+1	+2	−4	−1	−0.06	−0.02
8.....	−1	+1	−1	−3	0	+3	−1	−0.03	−0.01
9.....	−3	+1	−1	+2	+3	0	−2	0.00	0.00

TABLE VIII

Comparison Star Residuals

Star No.	778	779	780	781	782	783
1..........	....	....	....	....	....	....
2..........	−10	....	....	....	−23	−43
3..........	+36	....	....	....	0	0
4..........	+18	....	....	+20	+50	+43
5..........	− 2	....	....	−17	−15	−24
6..........	0	....	....	+17	....	....
7..........	+ 5	....	....	−12	....	....
8..........	+ 3	+ 6	+ 6	0	....	....
9..........	−10	+ 1	0	− 1	....	....
10..........	+13	+25	+19	+10	....	....
11..........	−22	−20	−20	−31	....	....
12..........	....	−16	−20	−16	....	....
13..........	....	+ 7	+ 6	+ 8	....	....
14..........	....	− 7	− 7	− 4	....	....

From a comparison of the times corresponding to a series of points on the steepest parts of the two branches of the light-curve the observed time of minimum is found to be $18^{h}\ 33^{m}$ G.M.T. As the reduction to the sun is $+1^{m}$,[1] the observed heliocentric minimum is 1912, August 7, $18^h 34^m$ G.M.T. with an uncertainty of 1^m. The minimum calculated from the original elements given above is $18^h 45^m.2$. The correction to the ephemeris is therefore $-11^m.2$. As 917 periods separate the present minimum from the epoch of the elements, the correction to the period is $-0^s.74$ or $-0^d.000009$. The revised elements are therefore

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[1] *Laws Observatory Bulletin*, No. 9, 135, 1907.

$$\text{Min.} = \text{J.D. } 2417026.682 + 2^d.831070\ E \quad \text{G.M.T.}$$

with an uncertainty in the period of $0^s.07$.

From the last three plates listed in Table VI, all of which were exposed during normal light, the maximum brightness of the variable was found to be 9.70 mags. The result of the Laws Observatory visual observations, referred to the system of the Potsdam *Durchmusterung*, was 9.98 mags.,[1] which was obtained by a reference to stars 9950 and 10372 of the *Generalkatalog* whose colors are respectively GW and WG−. The spectrum of the variable is apparently of the A-type;[2] and though its exact classification is doubtful, there can be no question but that it belongs among the white stars. It is not likely therefore that the reference of the variable to the *P.D.* stars is greatly affected by color difference. The mean reduction of the two Potsdam stars to the system of the Harvard Observatory is −0.22 mag.[3] The normal brightness derived from the visual observations, referred to the Harvard system, is therefore 9.76 mags. The determination of this quantity depends upon the measurement with the equalizing wedge photometer of intervals of 2.9 and 2.4 mags.,[4] and the close agreement of the visual and photographic results indicates a satisfactory accordance of the visual and photographic magnitude scales. Further data bearing on this point will appear later.

The phase values in the fifth column of Table VI were obtained by subtracting the observed time of geocentric minimum from the epochs of observation in column four. Assuming that the interval of constant minimum light falls within the phase limits $\pm 0^d.030$, there are available for the determination of the minimum brightness sixteen observations. The mean result is 13.50 mags. The inclusion of the first three observations is questionable, though the symmetry of the curve indicates that they fall within the period of constant light. If excluded, the minimum brightness becomes 13.53 mags.

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[1] *Op. cit.*, p. 128.

[2] *Annals Harvard College Observatory*, **56**, 189.

[3] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **17**, XXXIV, 1907.

[4] *Laws Observatory Bulletin*, No. 9, 128, 1907.

The ordinate of the visual light-curve for normal light, referred to the star *B.D.*+62°1639, is +0.34 mag.[1] As the normal brightness has been found to be 9.70 mags., the visual curve may be reduced to the zero point of the Polar Sequence by adding to its ordinates the constant 9.36 mags. The values given in Table I were derived in this manner, and the corresponding curve is shown with the photographic observations in Fig. 2. The values of the deviations in hundredths of a magnitude appear in the last column of Table VI. Within the phase limits $\pm 0^{d}.030$ for which the visual curve is lacking, the residuals are referred to the minimum photographic brightness.

It will be noted that for two magnitudes below normal light the photographic variation is closely represented by the visual light-curve. From this point on, the photographic variation is greater than the visual, the divergence amounting to about three-tenths of a magnitude in an interval of a magnitude and a half. If it be assumed that the photographic and visual range of variation are necessarily the same, this would imply a relative error in the photographic and visual scales of 20 per cent; but in view of their close agreement for the first two magnitudes of the light-change and for the intervals measured in determining the normal brightness referred to above this is unlikely, and the evidence, as far as it goes, indicates that the photographic range is really greater than the visual. For the greater part of the curve the variation is rapid, and the last magnitude of the change requires but twenty minutes.

The value of the range of variation given by the photographic observations is 3.80 mags. As to the reliability of this result, the following may be remarked: The uncertainty of the values for both maximum and minimum brightness of the variable as indicated by the deviations of the individual values from the means is only 0.02 mag. The actual uncertainty for maximum light is considerably in excess of this, however, owing to the fact that there is but one comparison star at all comparable in brightness with the variable. Any error in the brightness of this star will therefore enter practically to its full value into the calculated brightness of the variable. The comparison star residuals in Table IV indicate that the uncertainty for the object in question, No. 1, is ±0.04 mag., which includes the effect of scale error, but not that of the zero point. For minimum light on the other hand the course of the magnitude curves from which the brightness of the variable was read is determined by several comparison stars, so that, disregarding again the error of the zero point, the uncertainty entering through the comparison stars into the minimum brightness is comparatively small. From a consideration of these points it seems likely that the observed range in brightness is correct within a tenth of a magnitude, unless there is an unsuspected error affecting similarly all of the separate determinations of the magnitude scale.

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[1] *Op. cit.*, p. 136.

So far as the amount of variation is concerned *RR Draconis* stands third among the 134 variables listed in *Harvard Annals*, **56**, 187. It is exceeded only by *RW Tauri*, for which the range is 3.9 mags., and by *RZ Aurigae*, to which a photographic amplitude of 5.0 mags. is assigned, though the visual variation is much less, the results by various observers being 2.1[1] and 1.7 mags.[2]

Though it is not at present desirable to enter upon a detailed investigation of the system, certain points may be noted. The observed eclipse is, of course, that of a bright object by one relatively faint, for such is always the character of the eclipse corresponding to a principal minimum. Moreover, the data are sufficient to show that the eclipse is total and not annular—that the darker star is the larger of the two.

If λ_p and λ_s represent the light received during the principal and secondary minima, respectively, and if k be the ratio of the diameter of the smaller star to that of the larger, we have[1] when the principal minimum is the total eclipse,

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[1] Hartwig, *Astronomische Nachrichten*, **178**, 395, 1908.

[2] Pračka, *ibid.*, **185**, 291, 1910.

$$\lambda_s = 1 - k^2 \lambda_p \tag{1}$$

and when the principal minimum is the annular eclipse

$$\lambda_s = \frac{1-\lambda_p}{k^2} \tag{2}$$

Corresponding to the range 3.80 mags. $\lambda_p = 0.03$, whence (1) and (2) become, respectively,

$$\lambda_s = 1 - 0.03 k^2 \text{ (Prin. Min. is Total Ecl.)} \tag{3}$$

$$\lambda_s = \frac{0.97}{k^2} \text{ (Prin. Min. is Annular Ecl.)} \tag{4}$$

subject to the condition $\lambda_s < 1$. Equation (3) is obviously possible, but in order that (4) may exist we must have $k^2 > 0.97$.

From the general relations for a circular orbit of inclination i

$$\frac{(1-k)^2}{(1+k)^2} = \frac{\cot^2 i + \sin^2 \theta_2}{\cot^2 i + \sin^2 \theta_1} \tag{5}$$

in which θ_1 and θ_2 are the longitudes of the smaller star, counted from minimum, corresponding to the first and second contacts of eclipse, that is, to the beginning of the variation and of the constant minimum phase. Since i is necessarily nearly 90° we may write (5) in the form

$$\frac{1-k}{1+k} = \frac{\sin \theta_2}{\sin \theta_1} \tag{6}$$

and derive an approximation for k from

$$k = \frac{\sin \theta_1 - \sin \theta_2}{\sin \theta_1 + \sin \theta_2} \tag{7}$$

This value will in general be too large; for since $\sin \theta_1 > \sin \theta_2$, the true value of $(1-k)/(1+k)$ from (5) will be larger than that given by (6), and hence the true k will be *less* than the approximation from (7). If therefore the value of k from (7) is such that $k^2 < 0.97$, there is no possibility of the principal minimum corresponding to annular eclipse. Such in fact is the case, for the phase values of the first and second contacts are $0^{\mathrm{d}}.22$ and $0^{\mathrm{d}}.03$, respectively. From these we find from (7) for the maximum value of k, 0.76, and of k^2, 0.58, which is far below the limit corresponding to the association of the annular eclipse with the principal minimum. The estimation of the times of contact is naturally very uncertain, but even if we assume such extreme values for the phases as $0^{\mathrm{d}}.30$ and $0^{\mathrm{d}}.015$ the result for the maximum k^2 is 0.81, which is still safely below the limit. Such phase values are, however, clearly excluded by the observational data, and we can only conclude that the eclipse corresponding to the principal minimum is total. The system is therefore remarkable in that the smaller of the two stars is the brighter.

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[1] Russell, "On the Determination of the Orbital Elements of Eclipsing Variable Stars," *Astrophysical Journal*, **35**, 323, 1912.

This result strengthens the conclusion stated above as to the probability of the photographic range being in excess of the visual. In fact with such a system the photographic range must almost certainly be greater than the visual, for, since the fainter body is the larger, the chances are very strongly in favor of its being redder than the brighter. During normal brightness the light is preponderatingly that of the brighter star, which is of the A-type, and the photographic and visual magnitudes will be sensibly the same. During the principal minima the A-type star is wholly eclipsed by its larger and redder companion. As the latter is fainter photographically than visually, the photographic range of variation must be greater than the visual. Moreover, for a large part of the light-curve there will be no appreciable difference between the photographic and the visual magnitudes, for the relatively small amount of light emitted by the darker star will not form any considerable portion of the total received until the eclipse is well under way.

Finally it may be noted that equation (3) shows that the light during the secondary minimum can scarcely be less than 0.98, which corresponds to an amplitude of only 0.02 mag.

MOUNT WILSON SOLAR OBSERVATORY
September 6, 1912

ON DARKENING AT THE LIMB IN ECLIPSING VARIABLES. II

By HENRY NORRIS RUSSELL AND HARLOW SHAPLEY

3. *Annular and partial eclipses.*—The discussion of annular and partial eclipses of stars whose disks appear to be darkened toward the limb has involved more labor in computation than any other part of the theory of eclipsing variables. The results may, however, be given in relatively small compass, and the actual reduction of the observations of any star, with the aid of the tables here published, is almost as simple as on the assumption of disks of uniform brightness.

The light-changes due to the eclipse—partial or annular—of a disk completely darkened toward the edge by a smaller one have been computed by the method previously discussed,[1] supplemented by other graphical methods of integration. The percentage of light obscured increases steadily during the partial phases; and after the second contact it continues to increase, as the eclipsing disk moves toward the brighter central region of the other, so that computations had to be made for the annular as well as for the partial phases. On plotting the resulting light-curves, it was at once evident that there was no discontinuity in the curve on passing from the partial to the annular phases. This is illustrated by the curves in Fig. 1, which are drawn accurately to scale, and which show the variations in stellar magnitude due to central annular eclipses of a star, completely darkened toward the limb according to the "cosine law" already assumed, by a smaller one of one-ninth the brightness and k times the radius. The points on the curves corresponding to the beginning and end of the annular phase are indicated by short vertical lines. It is clear that there can be no hope at all of determining the duration of the annular phase from the observed curve. Such a light-curve, in fact, would undoubtedly be taken as evidence that the eclipse was partial. The small dots on one-half of the light-curve represent points on curves computed for partial eclipses of uniform disks, with the same depth of both primary and secondary minima, and so determined as to lie as close as possible to the curves previously computed. The agreement of points and curve is as good as might be expected from the best observations, the only sensible difference being that the "uniform" brightness gives, as usual, a shorter duration for the extreme phases near first contact. It is of interest to compare the elements derived from these light-curves on one hypothesis with those used in computing them on the other, as is done in the accompanying table. While the larger of the two "darkened" stars is the brighter, the smaller of the two "uniform" disks which give a similar light-curve is the more luminous. It is easy to show that this will always be true. Let L_1 be the light of the larger of the two "darkened" stars, and $1-L_1$ that of the smaller, and let k_d be the ratio of their radii. The loss of light at the total eclipse is $1-L_1$, and at central annular eclipse βL_1, where $\beta=1-(1-k_d^2)^{\frac{3}{2}}$. Unless $1-L_1$ is small,

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[1] *Astrophysical Journal*, 36, 239, 1912.

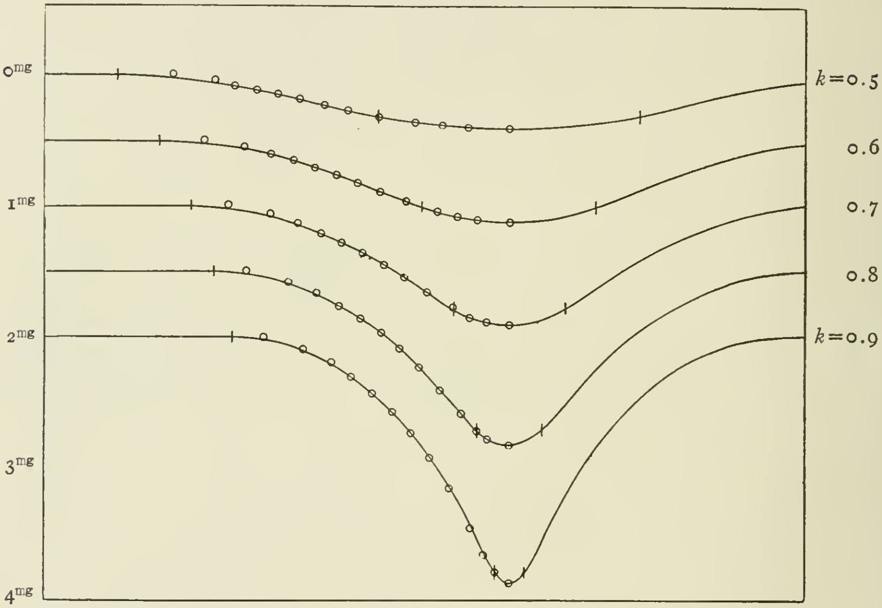

FIG. 1.—Light-curves due to annular eclipses of a star darkened toward the limb

the existence of a constant phase at the corresponding minimum will be detected. The case at present under discussion can only arise when the eclipse at principal minimum is annular. If we try to represent these minima by partial eclipses of magnitude a_0 of uniform disks, the ratio of whose radii is k_u, we must have, if we assume that the smaller star undergoes eclipse at principal minimum, $k_u^2a_0=k_u^2\beta L_1+(1-L_1)$; on the opposite hypothesis, $k_u^2a_0=\beta L_1+k_u^2(1-L_1)$. Now k_u^2, a_0, and L_1 must all be positive and less than unity. It is easy to show that this demands that a_0 in the first case, and $k_u^2a_0$ in the second shall not be less than β. Actual computation of the values of k_u and a_0 consistent with the values of $\chi(k_u, a_0, \frac{1}{4})$, which best represent the form of the various light-curves, shows that the second condition can never be satisfied, while it is always possible to satisfy the first, provided that L_1 lies between unity and a certain limiting value determined by the fact that a_0 must not exceed unity. A summary of the results is as follows:

ELEMENTS ASSUMED FOR "DARKENED" STARS					ELEMENTS DERIVED FROM THE LIGHT-CURVE ASSUMING "UNIFORM" DISKS						
k_d	r_{1d}	r_{2d}	L_1	L_2	k_u	a_0	r_{1u}	r_{2u}	$\cos i_u$	L_1	L_2
0.90	0.111	0.100	0.900	0.100	0.83	0.97	0.104	0.087	0.024	0.103	0.897
.80	.125	.100	.900	.100	.87	.84	.107	.094	.035	.119	.881
.70	.142	.100	.900	.100	.78	.73	.120	.094	.052	.137	.863
.60	.167	.100	.900	.100	.51	.83	.155	.078	.100	.120	.880
.50	.200	.100	.900	.100	.38	1.00	.186	.072	.114	.100	.900

k_d	β	$\chi(k_u, a_0, \frac{1}{4})$	LIMITING VALUES OF		
			k_u	a_0	$1-L_1$
1.00	1.000	2.085	0.844	1.000	0.000
0.90	0.917	2.037	0.861	0.917	0.000
			0.818	1.000	0.128
0.80	0.784	1.972	0.925	0.784	0.000
			0.776	1.000	0.246
0.70	0.636	1.850	0.880	0.636	0.000
			0.685	1.000	0.243
0.60	0.488	1.714	0.585	0.488	0.000
			0.554	1.000	0.184
0.50	0.350	1.588	0.000	0.37	0.000
			0.387	1.00	0.103

The principal interest of this table is the last column, which shows that, unless the "darkened" stars are of decidedly unequal brightness, it is impossible to get a satisfactory representation of the light-curve of the annular eclipse and the depth of the total eclipse on the assumption of uniformly illuminated disks. For example, if the stars are equal in radius ($k=1$) and eclipse one another centrally, the sum of the depths of the two minima will be equal to the whole light of the system. This condition is alone sufficient to prove that the stars are of equal radius; but on the hypothesis of uniform disks the value of $\chi(k_u, a_0, \frac{1}{4})$ must be 2.46, while for "darkened" stars it is 2.09—that is, the form of the light-curve will be quite different. Such systems, if carefully observed, may prove to be of great interest in the study of the actual existence and amount of darkening toward the limb.

The light-curves for a non-central eclipse of any pair of stars, "darkened" or "uniform," may be derived from that for central eclipse by diminishing all the values of $\sin^2\theta$ by a suitable constant. The considerations just developed for central annular eclipses therefore hold in general regarding the representation of light-curves, really due to eclipses of "darkened" stars, on the assumption of stars of uniform brightness. With a few exceptions (as noted above), a very close representation of the light-curve can be obtained, and very accurate observations would be necessary to distinguish between the two hypotheses.

4. *Determination of the elements in the case of annular and partial eclipses.*—As annular and partial eclipses of stars completely darkened at the limb cannot be distinguished by inspection of the light-curve, they must be discussed together. The same functional relation may be employed in the discussion of these eclipses as has been used in the previous papers. The quantity α, which represents the obscuration of light at any given phase, in terms of a standard value, might be defined here in more than one way. We will choose as unit the loss of light at the moment of internal tangency and, for clearness' sake, call the obscuration measured in these terms α''. α'' is therefore less than unity during the partial phases, and exceeds unity during the annular phases, reaching a maximum

for central eclipse, for which we may call its value $1+x$, x being a function of k. Having thus defined a'', we may proceed to determine the function $p(k, a'')$ which defines the distance of centers, and hence the functions $\psi(k, a_1'')$ and $\chi(k, a_0'', n)$ exactly as in the previous cases,[1] the only difference being that, in the tables, values of a'' greater than unity appear. For the p- and ψ-functions tabular values are given for intervals of $0.2x$ between $a_0''=1$ and $a_0''=1+x$. For the χ-functions this is not necessary, as computation shows that linear interpolation between the values for $a_0''=1$ and $a_0''=1+x$ gives the intermediate values with sufficient accuracy. As the empirical linear relations between the various χ-functions for different values of n, which existed in the case of uniform disks, no longer hold good here, three of these functions have been tabulated, for $n=0$, $n=\frac{1}{4}$, and $n=\frac{3}{4}$. The tabular intervals in a_0'', though very unequal, suffice for the drawing of a series of curves from which the values of the function may in all cases be taken.

Just as in the case of uniform disks, it is impracticable to derive the elements of the system from the light-curve of one of the minima alone, for this would make the determination of the critical data depend on quantities too small to be accurately found by observation. We must use the observed depths of both minima, and the form of the light-curve of the deeper one.

The relation connecting the depths of the minima with k and a_0 is now more complicated than in the case of uniform disks, for if the stars are of unequal radii the light-curves of the two minima (even if expressed in light-intensities rather than in magnitudes) are not similar in form; and one may show a constant phase when the other does not, even though the orbit is circular. If a_0 denotes the fraction of the light of the smaller star which is lost at the greatest phase of its eclipse behind the larger, we may call the fraction of the light of the larger star which is lost at the corresponding phase during the other eclipse, $a_0Q(k, a_0)$—thus defining a new Q-function of k and a_0. This function may be immediately computed from the data of the previous tables, and it is given in Table V. For uniform disks it reduces to k^2. If, as previously, $1-\lambda_1$

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[1] *Astrophysical Journal*, 35, 321 and 326, 1912, or see below pp. 405, 406.

and $1-\lambda_2$ denote the losses of light at the two minima, and L_1, L_2 the light-emissions of the two stars, we have

$$\left.\begin{aligned} 1-\lambda_1&=\alpha_0 L_2\\ 1-\lambda_2&=\alpha_0 L_1 Q(k,\ \alpha_0)\end{aligned}\right\}\qquad (1)$$

whence, (since $L_1+L_2=1$)

$$\alpha_0=1-\lambda_1+\frac{1-\lambda_2}{Q(k,\ \alpha_0)}\qquad (2)$$

or

$$Q(k,\ \alpha_0)=\frac{1-\lambda_2}{\alpha_0-(1-\lambda_1)}\qquad (3)$$

When $k=1$, $Q=1$, and $\alpha_0=1-\lambda_1+1-\lambda_2$ (as in the case of uniform disks)—which is the smallest value it can have consistently with the given data. For suitable values of α_0 between this limit and unity (e.g., the tabular values) we may compute $Q(k,\ \alpha_0)$ by (3)

TABLE I*y*

VALUES OF $p(k,\ \alpha'')$ FOR ANNULAR ECLIPSES OF DISKS COMPLETELY DARKENED TOWARD THE EDGE

α''	$k=1.0$	$k=0.9$	$k=0.8$	$k=0.7$	$k=0.6$	$k=0.5$	$k=0.4$
0.00....	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000
0.02....	0.796	0.786	0.778	0.772	0.768	0.765	0.762
0.05....	0.685	0.660	0.644	0.635	0.628	0.624	0.620
0.10....	0.543	0.511	0.495	0.485	0.479	0.473	0.468
0.15....	0.428	0.390	0.370	0.357	0.350	0.344	0.338
0.20....	0.328	0.287	0.263	0.250	0.240	0.232	0.226
0.25....	0.240	0.195	0.168	0.150	0.142	0.134	0.127
0.30....	0.158	0.109	+0.079	+0.060	+0.049	+0.042	+0.036
0.35....	0.080	+0.029	−0.003	−0.024	−0.036	−0.043	−0.048
0.40....	+0.004	−0.048	0.083	0.104	0.117	0.124	0.129
0.50....	−0.143	0.200	0.237	0.257	0.272	0.280	0.285
0.60....	0.284	0.341	0.379	0.401	0.413	0.421	0.427
0.70....	0.424	0.482	0.521	0.542	0.553	0.561	0.567
0.80....	0.570	0.627	0.663	0.682	0.693	0.700	0.706
0.85....	0.650	0.704	0.736	0.754	0.763	0.770	0.776
0.90....	0.737	0.787	0.814	0.828	0.835	0.842	0.847
0.95....	0.837	0.880	0.899	0.907	0.913	0.918	0.922
0.98....	0.910	0.943	0.955	0.959	0.962	0.964	0.967
1.00....	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000
$1+0.2x$..	−1.000	−1.014	−1.030	−1.047	−1.070	−1.100	−1.145
$1+0.4x$..	1.000	1.030	1.062	1.099	1.147	1.216	1.312
$1+0.6x$..	1.000	1.048	1.100	1.163	1.241	1.355	1.516
$1+0.8x$..	1.000	1.069	1.146	1.246	1.361	1.533	1.780
$1+x$....	−1.000	−1.111	−1.250	−1.429	−1.667	−2.000	−2.500
x.......	0.000	0.015	0.047	0.084	0.143	0.220	0.322

TABLE II*y*

VALUES OF $\psi(k, \alpha_1'')$ FOR ANNULAR ECLIPSES OF DISKS COMPLETELY DARKENED TOWARD THE EDGE

α_1''	$k=1.0$	$k=0.9$	$k=0.8$	$k=0.7$	$k=0.6$	$k=0.5$	$k=0.4$
0.00....	+7.865	+7.920	+7.570	+6.961	+6.333	+5.648	+5.082
0.02....	6.115	6.161	5.900	5.445	4.979	4.472	4.027
0.05....	5.248	5.213	4.977	4.605	4.224	3.813	3.474
0.10....	4.215	4.181	4.021	3.747	3.472	3.152	2.876
0.15....	3.440	3.403	3.278	3.073	2.860	2.606	2.410
0.20....	2.825	2.790	2.692	2.530	2.360	2.160	2.003
0.25....	2.307	2.280	2.194	2.065	1.945	1.791	1.665
0.30....	1.858	1.835	1.772	1.668	1.574	1.455	1.369
0.35....	1.474	1.448	1.400	1.320	1.247	1.160	1.098
0.40....	1.115	1.099	1.061	1.007	0.953	0.891	0.849
0.45....	0.794	0.777	0.750	0.714	0.676	0.636	0.608
0.50....	0.504	0.484	0.470	0.454	0.432	0.404	0.388
0.55....	+0.240	+0.231	+0.227	+0.217	+0.209	+0.196	+0.188
0.60....	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.65....	−0.217	−0.211	−02.06	−0.200	−0.194	−0.187	−0.182
0.70....	0.411	0.406	0.397	0.388	0.376	0.367	0.358
0.75....	0.585	0.580	0.572	0.556	0.546	0.535	0.526
0.80....	0.740	0.735	0.723	0.715	0.706	0.697	0.689
0.85....	0.879	0.876	0.872	0.863	0.856	0.850	0.847
0.90....	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.95....	1.097	1.106	1.118	1.128	1.141	1.148	1.154
0.98....	1.138	1.158	1.181	1.202	1.223	1.232	1.234
1.00....	−1.156	−1.190	−1.224	−1.255	−1.284	−1.296	−1.308
$1+0.2x$...........		−1.196	−1.249	−1.309	−1.383	−1.460	−1.570
$1+0.4x$...........		1.201	1.273	1.362	1.483	1.630	1.845
$1+0.6x$...........		1.207	1.295	1.416	1.583	1.804	2.126
$1+0.8x$...........		1.211	1.316	1.470	1.683	1.976	2.414
$1+x$..............		−1.215	−1.335	−1.519	−1.790	−2.164	−2.746

and determine from Table V the value of k consistent with the observational data and each assumed value of α_0. We may then find from Table III*x* the values of the χ-functions corresponding to these pairs of values of k and α_0, and by comparison with the observed data (given by the equation $\chi(k, \alpha_0, n)=\frac{\sin^2\theta(n)}{\sin^2\theta(\frac{1}{2})}\Big)$, we may determine what value of α_0 (with its corresponding value of k) gives the best representation of the observed curve, on the hypothesis that the principal minimum corresponds to the eclipse of the smaller star by the larger, whose depth is $1-\lambda_1$.

If, however, we assume that the principal minimum corresponds to the eclipse of the larger star by the smaller (of depth $1-\lambda_2$), we must express the maximum obscuration in terms of the unit

TABLE III*x*

VALUES OF THE χ-FUNCTIONS FOR TOTAL ECLIPSES OF DISKS COMPLETELY DARKENED TOWARD THE EDGE

$\chi(k, \alpha_0, \frac{3}{4})$

k	$\alpha_0=1.0$	$\alpha_0=0.98$	$\alpha_0=0.95$	$\alpha_0=0.9$	$\alpha_0=0.8$	$\alpha_0=0.6$	$\alpha_0=0.4$	$\alpha_0=0.2$	$\alpha_0=0.0$
1.00....	0.344	0.358	0.370	0.381	0.399	0.427	0.438	0.448	0.457
0.90....	0.410	0.406	0.408	0.410	0.418	0.434	0.442	0.450	0.457
0.80....	0.460	0.443	0.439	0.435	0.435	0.441	0.446	0.452	0.457
0.70....	0.499	0.470	0.461	0.455	0.450	0.448	0.450	0.454	0.457
0.60....	0.531	0.495	0.483	0.474	0.464	0.455	0.453	0.455	0.457
0.50....	0.560	0.522	0.504	0.492	0.476	0.460	0.456	0.457	0.457
0.40....	0.585	0.544	0.524	0.508	0.486	0.465	0.459	0.458	0.457

$\chi(k, \alpha_0, \frac{1}{4})$

k	$\alpha_0=1.0$	$\alpha_0=0.98$	$\alpha_0=0.95$	$\alpha_0=0.9$	$\alpha_0=0.8$	$\alpha_0=0.6$	$\alpha_0=0.4$	$\alpha_0=0.2$	$\alpha_0=0.0$
1.00....	2.086	2.065	2.026	1.977	1.922	1.854	1.814	1.768	1.707
0.90....	1.923	1.926	1.912	1.891	1.866	1.824	1.794	1.759	1.707
0.80....	1.789	1.818	1.822	1.819	1.814	1.800	1.778	1.750	1.707
0.70....	1.694	1.736	1.749	1.759	1.767	1.777	1.759	1.739	1.707
0.60....	1.615	1.665	1.689	1.705	1.732	1.753	1.740	1.727	1.707
0.50....	1.550	1.609	1.632	1.652	1.690	1.726	1.720	1.715	1.707
0.40....	1.498	1.558	1.578	1.610	1.653	1.691	1.699	1.703	1.707

$\chi(k, \alpha_0, 0)$

k	$\alpha_0=1.0$	$\alpha_0=0.98$	$\alpha_0=0.95$	$\alpha_0=0.9$	$\alpha_0=0.8$	$\alpha_0=0.6$	$\alpha_0=0.4$	$\alpha_0=0.2$	$\alpha_0=0.0$
1.00....	5.44	5.30	5.16	4.95	4.64	4.24	3.94	3.64	3.41
0.90....	4.64	4.61	4.58	4.48	4.32	4.06	3.84	3.60	3.41
0.80....	4.02	4.12	4.12	4.09	4.04	3.91	3.75	3.55	3.41
0.70....	3.58	3.72	3.76	3.77	3.80	3.75	3.63	3.50	3.41
0.60....	3.20	3.38	3.46	3.50	3.58	3.60	3.52	3.46	3.41
0.50....	2.90	3.10	3.19	3.26	3.37	3.46	3.44	3.42	3.41
0.40....	2.66	2.82	2.94	3.05	3.19	3.32	3.35	3.38	3.41

previously chosen for our tables—namely, the loss of light at internal tangency. At this moment in the other eclipse the smaller star is completely hidden, and $\alpha_0=1$. Hence, by (1), $1-\lambda_2=L_1Q(k, 1)$—which is the unit to be used in measuring the obscuration α_0''. We have therefore

$$\alpha_0''Q(k, 1)=\alpha_0 Q(k, \alpha_0). \tag{4}$$

For each pair of values of k, α_0 previously found we may find α_0'' by (4), and then take the values of the χ-functions from Table III*y*, and find as before what system of values of k, α_0, α_0'' gives the best approximation to the observed values of these functions. Which of these cases corresponds to the principal minimum must be determined by trial. Usually only one of the two assumptions

TABLE III*y*

VALUES OF THE χ-FUNCTIONS FOR ANNULAR ECLIPSES OF DISKS COMPLETELY DARKENED TOWARD THE EDGE

$\chi(k, \alpha_0'', \frac{3}{4})$

k	$\alpha_0''=1+x$	$\alpha_0''=1.0$	$\alpha_0''=0.95$	$\alpha_0''=0.9$	$\alpha_0''=0.8$	$\alpha_0''=0.6$	$\alpha_0''=0.4$	$\alpha_0''=0.2$	$\alpha_0''=0.0$
1.00....	0.344	0.344	0.370	0.381	0.399	0.427	0.438	0.448	0.457
0.90....	0.361	0.365	0.378	0.386	0.401	0.422	0.434	0.446	0.457
0.80....	0.388	0.385	0.390	0.396	0.405	0.423	0.435	0.446	0.457
0.70....	0.430	0.409	0.407	0.411	0.415	0.425	0.437	0.447	0.457
0.60....	0.477	0.430	0.428	0.426	0.427	0.430	0.441	0.448	0.457
0.50....	0.526	0.448	0.445	0.442	0.440	0.435	0.444	0.450	0.457
0.40....	0.579	0.462	0.457	0.454	0.448	0.444	0.446	0.452	0.457

$\chi(k, \alpha_0'', \frac{1}{4})$

k	$\alpha_0''=1+x$	$\alpha_0''=1.0$	$\alpha_0''=0.95$	$\alpha_0''=0.9$	$\alpha_0''=0.8$	$\alpha_0''=0.6$	$\alpha_0''=0.4$	$\alpha_0''=0.2$	$\alpha_0''=0.0$
1.00....	2.086	2.086	2.026	1.977	1.922	1.854	1.814	1.768	1.707
0.90....	2.086	2.073	2.026	1.983	1.925	1.855	1.814	1.768	1.707
0.80....	2.038	2.021	1.991	1.964	1.915	1.850	1.808	1.765	1.707
0.70....	1.923	1.941	1.936	1.924	1.883	1.840	1.800	1.760	1.707
0.60....	1.812	1.877	1.883	1.874	1.843	1.815	1.787	1.751	1.707
0.50....	1.685	1.814	1.820	1.814	1.801	1.788	1.770	1.743	1.707
0.40....	1.566	1.760	1.767	1.762	1.760	1.756	1.748	1.732	1.707

$\chi(k, \alpha_0'', 0)$

k	$\alpha_0''=1+x$	$\alpha_0''=1.0$	$\alpha_0''=0.95$	$\alpha_0''=0.9$	$\alpha_0''=0.8$	$\alpha_0''=0.6$	$\alpha_0''=0.4$	$\alpha_0''=0.2$	$\alpha_0''=0.0$
1.00....	5.44	5.44	5.16	4.95	4.64	4.24	3.94	3.64	3.41
0.90....	5.50	5.44	5.22	5.02	4.71	4.31	4.03	3.70	3.41
0.80....	5.29	5.19	5.04	4.90	4.65	4.27	3.99	3.69	3.41
0.70....	4.77	4.82	4.74	4.64	4.46	4.17	3.92	3.66	3.41
0.60....	4.25	4.44	4.41	4.35	4.24	4.01	3.83	3.60	3.41
0.50....	3.68	4.08	4.09	4.06	4.00	3.88	3.74	3.55	3.41
0.40....	3.11	3.77	3.79	3.78	3.75	3.71	3.65	3.51	3.41

gives the computed values of the χ-functions in the neighborhood of the observed values.

The question whether a given eclipse is annular or partial can arise only when the larger of the two stars is considerably the brighter, and the smaller one is in front of it during the principal minimum, for otherwise the minimum at which total eclipse is possible will be deep enough to make it certain whether or not it shows a constant phase. When this minimum is so shallow that this question cannot be definitely decided, the procedure is similar to that just described for the case of a partial eclipse. If the principal eclipse is annular, the secondary must be total. We may therefore set $\alpha_0=1$ in (3) and find the value of $Q(k, \alpha_0)$ (remembering that $1-\lambda_1$ is now the loss of light at the secondary minimum).

The last line but one of Table V gives the values of this function for internal contact of the disks, and the last line its values for central eclipse, when the loss of light is $1+x$ times that at internal contact. In either of these cases the value of k corresponding to a given Q may be found, and the χ-functions taken out of Table IIIy, the other argument being $a_0''=1$ for grazing annular eclipse, and $a_0''=1+x$ for central annular eclipse.

TABLE V

Values of $Q(k, a_0)$ for Partial Eclipses of Disks Completely Darkened toward the Edge

a_0	$k=1.0$	$k=0.9$	$k=0.8$	$k=0.7$	$k=0.6$	$k=0.5$	$k=0.4$
0.00	1.000	0.769	0.573	0.410	0.279	0.177	0.102
0.10	1.000	0.777	0.585	0.426	0.292	0.188	0.108
0.20	1.000	0.786	0.596	0.438	0.302	0.195	0.113
0.30	1.000	0.796	0.605	0.446	0.310	0.200	0.116
0.40	1.000	0.805	0.614	0.453	0.316	0.205	0.119
0.50	1.000	0.814	0.624	0.461	0.321	0.210	0.123
0.60	1.000	0.823	0.634	0.470	0.328	0.215	0.126
0.70	1.000	0.832	0.646	0.483	0.339	0.222	0.131
0.80	1.000	0.844	0.661	0.499	0.352	0.232	0.137
0.90	1.000	0.861	0.685	0.522	0.371	0.247	0.147
0.95	1.000	0.872	0.704	0.538	0.384	0.256	0.153
0.98	1.000	0.880	0.717	0.553	0.398	0.267	0.160
0.99	1.000	0.890	0.728	0.563	0.405	0.273	0.164
1.00	1.000	0.904	0.750	0.587	0.427	0.289	0.175
$1+x$	1.000	0.917	0.784	0.636	0.488	0.351	0.230

For eclipses whose maximum phase lies between these limits, with $a_0''=1+mx$ (where m is any fraction), the values of the Q- and χ-functions can be found by linear interpolation between those given in the tables for 1 and $1+x$, and the same procedure then followed. Comparison of the χ-functions so computed with those derived from the observed curve will show whether the curve can be explained by an annular eclipse of darkened stars, and, if so, whether this eclipse is grazing, central, or intermediate between these limits. When for the eclipse so computed k exceeds 0.8 (approximately), the solution is unique, i.e., there exists no other pair of values of k, a_0 which will lead to the same depths of minima and the same values of χ for the principal minimum; but for smaller values of k there are two solutions, one giving an annular, or large partial, eclipse, the other giving a smaller partial eclipse with stars more nearly equal. In some cases the difference in the computed curves for secondary minimum could distinguish which of these solutions is correct; in others the ambiguity must remain.

The condition that two solutions may be possible may also be put in the form $1-\lambda_2<0.78\lambda_1$—that is, the loss of light at principal minimum must be less than 78 per cent of the light remaining at the secondary. When both eclipses are shallow (say less than $0^m.4$), and nearly equal in depth, the solution becomes almost indeterminate, mainly because the observed values of χ cannot be read off with any precision from the light-curve. If the relative brightness of the components is known from spectrographic measures, the solution remains definite for even the smallest eclipses.

When the best values of k and α_0 or α_0'' have been determined, it is easy to fix upon such a value of $\sin^2\theta(\frac{1}{2})$ that the computed values of $\sin^2\theta$ for the beginning and end of eclipse, and for the moments when the loss of light is $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of the maximum, agree very closely with those derived from the observed curve. Unless the latter is very precisely known, it is unnecessary to go farther, and these computed points may be joined by a free-hand curve. If greater accuracy, however, is desired, the values of L_1 and L_2 may be computed by (1), and then values of $\sin^2\theta$ corresponding to a loss of light of $\alpha_1 L_2$ for the eclipse of the smaller star by the larger, or of $\alpha_1'' L_1 Q(k, 1)$ for the other eclipse be determined from the observed light-curve in the usual fashion. We then have for the values 0.0, 0.1, etc., of α_1 (or α_1''), as in former cases, $\sin^2\theta=A+B\psi(k, \alpha_1)$, where A and B are constants, and the function ψ is taken from Table II*x* if the larger star is in front, or from Table II*y* if it is behind the other. The fact that $\alpha_1=\alpha_0$ when $\theta=0$ gives the relation $A+B\psi(k, \alpha_0)=0$. It is easy to determine A and B—graphically or otherwise—so that the series of equations thus formed and in particular the last one are as accurately represented as possible (with consideration of their varying weights), and a precise light-curve may thus be determined.

When once the value of θ for the beginning or end of eclipse, which we may call θ', has been computed, the remaining elements may be found at once from the equations

$$\left.\begin{aligned} r_1^2(1+k)^2 &= \cos^2 i \cos^2 \theta' + \sin^2 \theta' \\ \cos^2 i &= r_1^2 \{1+kp(k, \alpha_0)\}, \end{aligned}\right\} \quad (5)^*$$

where $p(k, \alpha_0)$ is to be taken always from Table Ix, except in the case of annular eclipse, when we must substitute $p(k, \alpha_0'')$ from Table Iy. When the light-curve of the principal minimum has been computed, that of the secondary may be found, if the orbit is circular, by computing the χ-functions for this eclipse. As the depth and duration of this minimum are already known, three points on each branch of the light-curve may now be computed—which will almost always be sufficient. This method fails if the principal eclipse is annular, and the secondary total, in which case the constants A and B of the light-curve may be found with the aid of Table IIax, and then the light-curve itself by Table IIx, as described above.

The problem of determining the elements of the system when the stars are completely darkened toward the limb is now solved for a circular orbit. For the modifications necessary when the orbit is elliptical reference may be made to a preceding paper (*Astrophysical Journal*, **36**, 54–60) especially to the method described on p. 58, bearing in mind that the equation

$$\frac{1-\lambda_1}{\alpha_1} + \frac{1-\lambda_2}{\alpha_2 Q(k, \alpha_2)} = 1$$

must be substituted for equation (33) of that discussion. The determination of $e \sin \omega$ from the light-curve is always a difficult one and incapable of great precision.

When the elements of the elliptical orbit are known from spectroscopic observations (as is now or soon will be the case for the brighter eclipsing variables), it is best to go back to first principles, using the equation for the distance of centers at any time

$$r^2\{\cos^2 i + \sin^2 i \cos^2 (v+\omega)\} = r_1^2\{1+kp(k, \alpha)\}^2$$

where r denotes as usual the radius vector and v the true anomaly. With the aid of the spectroscopic data the first member of this equation at any instant, may be expressed in the form $a+b\cos^2 i$, where a and b are known. If k and α_0 are known, the second member becomes a known multiple of r_1^2, and the resulting equations may be solved for r_1^2 and $\cos^2 i$. To find k and α_0, when one of the minima is sufficiently deep to permit the determination of reliable values of χ from the light-curve, we may follow the method cited in the last paragraph, which is much simplified when $e\sin\omega$ is known. Sometimes the spectrographic data themselves make it possible to estimate the relative brightness of the two stars—that is, to find L_1 and L_2. We may then find the values of α corresponding to any observed magnitude at the two eclipses (remembering that during the eclipse when the small star of light L_2 partially obscures the other the loss of light corresponding to $\alpha=1$ is k^2L_1 for uniform disks and $L_1Q(k, 1)$ for "darkened" disks), and the problem is thus further simplified and may be satisfactorily solved, even when the eclipses are too shallow to allow a good observational determination of χ.

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* *Astrophysical Journal*, **35**, 331.

5. *Ellipticity of the stars.*—The light variations due to the changing presentation of a rotating ellipsoid, whose apparent surface brightness varies as the cosine of the angle of emission of its light, may readily be calculated. If $d\sigma$ denotes an element of area of the surface, and γ the angle between the visual ray and the normal, the apparent projected area of the element is $d\sigma\cos\gamma$, and its apparent surface brightness $J_0\cos\gamma$, where J_0 is a constant. The whole amount of the light received by the eye is therefore

$$L=J_0\int\cos^2\gamma d\sigma\,, \tag{6}$$

where the integral is to be taken over the visible half of the surface. Since the ellipsoid is symmetrical, and the integrand is everywhere positive, this integral is one-half the integral of $\cos^2\gamma d\sigma$ over the whole surface—which much simplifies the problem by removing annoying boundary conditions.

Let us now take the principal axes of the ellipsoid as our co-ordinate axes, and let l, m, n be the direction cosines of the normal to the element $d\sigma$, and λ, μ, ν those of the observer's direction. Then $\cos\gamma=l\lambda+m\mu+n\nu$. Introducing this into (6), the integral splits up at once into six parts, of which the three of the form $2\lambda\mu\int lmd\sigma$ vanish on account of the symmetry of the ellipsoid about the co-ordinate planes, so that we have

$$L=\tfrac{1}{2}J_0\left(\lambda^2\int l^2d\sigma+\mu^2\int m^2d\sigma+\nu^2\int n^2d\sigma\right),$$

where the integrals are to be taken over the whole surface of the ellipsoid. If i is the inclination of the line of sight to the equatorial plane of the ellipsoid (taken here as the xy-plane), and u the position angle of the longest axis of the ellipsoid in this plane at any time, we have

$$\lambda=\sin i\cos u\,,\quad \mu=\sin i\sin u\,,\quad \nu=\cos i\,,$$

and hence

$$L=A\sin^2 i\cos^2 u+B\sin^2 i\sin^2 u+C\cos^2 i \tag{7}$$

where $A=\tfrac{1}{2}J_0\int l^2d\sigma$, ... etc. As the ellipsoid rotates, u increases uniformly with the time. The light-curve is therefore a simple sine-curve of amplitude proportional to $\sin^2 i$. If L_0 denotes the maximum light (which we may suppose to occur when $u=90°$), we have $L_0-L=(A-B)\sin^2 i\cos^2 u$. In the case of uniform disks we derived an equation which was identical in form except that its first member was $L_0^2-L^2$.

We have now to determine the constants A, B, C. Let the semiaxes of the ellipsoid be a, b, c. The co-ordinates of any point on the surface may be represented by the equations

$$x=a\cos\theta\,,\qquad y=b\sin\theta\cos\phi\,,\qquad z=c\sin\theta\sin\phi\,,$$

where θ and ϕ are auxiliary angles. The direction cosines of the normal to the surface at this point are proportional to $\frac{x}{a^2}, \frac{y}{b^2}, \frac{z}{c^2}$: whence we find

$$l^2=\frac{\dfrac{\cos^2\theta}{a^2}}{\dfrac{\cos^2\theta}{a^2}+\dfrac{\sin^2\theta\cos^2\phi}{b^2}+\dfrac{\sin^2\theta\sin^2\phi}{c^2}}\,,\ \text{etc.}$$

If we set $a^2=b^2(1+\eta_1)$, $a^2=c^2(1+\eta_2)$ we may write

$$l=\frac{\cos\theta}{\sqrt{1+\eta_1\sin^2\theta\cos^2\phi+\eta_2\sin^2\theta\sin^2\phi}}\,.$$

$l d\sigma$, which is the projection of the element of area on the plane $x=0$, is easily found to be $bc \sin\theta \cos\theta d\theta d\phi$. We have, therefore, for the constant A

$$A=\tfrac{1}{2}J_0 bc\int_0^{\pi} d\theta \int_0^{2\pi} d\phi \frac{\cos^2\theta \sin\theta}{\sqrt{1+\eta_1 \sin^2\theta \cos^2\phi+\eta_2 \sin^2\theta \sin^2\phi}} .$$

In practice η_1 and η_2 are fairly small. We may therefore expand the radical as powers of these quantities. Integrating term by term, we find easily

$$A=\tfrac{2}{3}\pi bcJ_0\{1-\tfrac{1}{10}(\eta_1+\eta_2)+\tfrac{3}{280}(3\eta_1^2+2\eta_1\eta_2+3\eta_2^2)+\;.\;.\;.\;.\}$$

when, as above,

$$\eta_1=\frac{a^2-b^2}{b^2}, \qquad \eta_2=\frac{a^2-c^2}{c^2} .$$

If we wish to express the other constants in terms of the quantities η_1, η_2, we have

$$\frac{b^2-a^2}{a^2}=\frac{-\eta_1}{1+\eta_1}=-\eta_1+\eta_1^2+\;.\;.\;.\;.\;,$$

$$\frac{b^2-c^2}{c^2}=\frac{\eta_2-\eta_1}{1+\eta_1}=\eta_2-\eta_1+\eta_1^2-\eta_1\eta_2\;.\;.\;.\;.\;,$$

and hence

$$B=\tfrac{2}{3}\pi acJ_0\{1-\tfrac{1}{10}(\eta_2-2\eta_1)-\tfrac{1}{280}(32\eta_1^2-4\eta_1\eta_2-9\eta_2^2)\;.\;.\;.\;.\;\} .$$

Setting $b=a(1+\tfrac{1}{2}\eta_1-\tfrac{1}{8}\eta_1^2\;.\;.\;.\;.\;)$, we find

$$B=A(1+\tfrac{4}{5}\eta_1-\tfrac{16}{175}\eta_1^2+\tfrac{4}{175}\eta_1\eta_2\;.\;.\;.\;.\;) .$$

The expression for C may be found by interchanging η_1 and η_2 in this equation.

For a prolate spheroid of revolution, if b is the longest axis and ϵ the eccentricity of a meridian section, we have $\eta_1=-\epsilon^2$, $\eta_2=0$, and hence

$$B=A(1-\tfrac{4}{5}\epsilon^2-\tfrac{16}{175}\epsilon^4\;.\;.\;.\;.\;) .$$

For uniform apparent brightness we had $B=A\sqrt{1-\epsilon^2}=A(1-\tfrac{1}{2}\epsilon^2-\tfrac{1}{8}\epsilon^4\;.\;.\;.\;.\;)$. The amplitude of the light variation measured as a function of the maximum light is therefore increased by darkening in the ratio of $\tfrac{8}{5}-\tfrac{19}{175}\epsilon^2\;.\;.\;.\;.\;$, so that neglect of this influence, if it exists, will lead to decidedly too great a value for the ellipticity of the stars.

The exact calculation of the light-curve arising from the mutual eclipse of a pair of such darkened ellipsoids would be very complicated, for the contours of equal percentages of darkening on the apparent disk are not in general elliptical, nor is the brightest point of the disk coincident with its apparent center (unless the ellipsoid is viewed from the direction of one of its axes). Even in this case, the apparent brightness of the disk of a prolate spheroid, seen endwise, will fall off more rapidly toward the limb than in the case of a sphere for which the relation between the apparent brightness and the inclination of the surface to the line of sight is the same.

Neglecting all these things, we may, as an approximation, suppose that the distribution of apparent brightness on the elliptical disk may be obtained simply by "stretching" to a suitable degree, that already studied on the disk of a sphere, and transforming the circular contours of equal brightness into similar ellipses. As the apparent disks during actual eclipses are nearly circular, this will probably be a fair approximation. With this limitation we may proceed to a solution as follows: The equation of the non-eclipse portion of the intensity-curve should be of the form $L=L_0(1-Z\cos^2\theta)$, where $Z=(\frac{4}{5}\epsilon^2+\frac{16}{175}\epsilon^4\ \ldots\ldots)\sin^2 i$. By plotting L against $\cos^2\theta$, the value of Z may be found, and by subtracting the change in stellar magnitude due to ellipticity from the observed magnitudes, a "rectified" curve may be obtained just as in the case where the stellar disks are assumed to be of uniform brightness.

The rectified curve may be discussed as already described, using the tables for disks darkened at the limb, but the equations appropriate for elliptical disks—

$$\left.\begin{aligned}\psi(k,\alpha_1)&=\frac{(\sin^2\theta_1-\sin^2\theta_2)(1-z\cos^2\theta_3)}{(\sin^2\theta_2-\sin^2\theta_3)(1-z\cos^2\theta_1)},\\ \chi(k,\alpha_0,n)&=\frac{\sin^2\theta(n)(1-z\cos^2\theta(\frac{1}{2})}{\sin^2\theta(\frac{1}{2})(1-z\cos^2\theta(n)}.\end{aligned}\right\}*$$

The constant z appearing in these equations is $\epsilon^2\sin^2 i$. From the above value of Z we find with sufficient approximation $z=\frac{5}{4}Z-\frac{5}{28}Z^2$.

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* *Astrophysical Journal*, **36**, 65–66, 1912.

6. *Intermediate degrees of darkening.*—To discriminate between different degrees of darkening at the limb, we must have a precise determination of the whole light-curve (including the non-eclipse portions as well as the secondary minimum) of either (*a*) a star with a conspicuous constant phase at principal minimum (the shallower the better), or (*b*) a star in which the sum of the losses of light at the two minima (after correction for ellipticity of the stars and for the effects of their mutual radiation) is nearly equal to the whole light of the system. Many eclipsing variables satisfy the first requirement, and *RT Lacertae* appears to be an example of the second.[1] But unfortunately not one of these has yet been subjected to a sufficiently extensive study to define its whole light-curve with adequate precision. There is, therefore, no urgent reason for the computation of tables, etc., for intermediate degrees of darkening. Their construction would be a mere matter of reckoning, since the values of α, considered as a function of k and p, for any intermediate degree of darkening, may be found by linear interpolation between those for zero and complete darkening, which have already been computed. The values of p as a function of k and α, and hence those of the other functions involved in our discussions, may then be computed for any desired degree of darkening.

To do this for several degrees of darkening would involve much work, and an indirect method of solution appears to promise less labor. For uniform disks the loss of light during the progress of eclipse varies proportionally to the eclipsed area (measured as usual in terms of the area of the smaller disk). For disks of the same total light-emission, but darkened toward the limb, the loss of light is no longer proportional to the eclipsed area, but may be derived from it by multiplication by a certain factor, which depends on the relative radii of the stars, the eclipsed area, and the law of darkening. For the general law of darkening already assumed, $J=J_0(1-x+x\cos i)$, in which x measures the degree of darkening, this factor will evidently be a linear function of x, which becomes unity when $x=0$. We may therefore write,

$$\left.\begin{aligned} 1-l_1&=\alpha L_2\{1+xX(k,\alpha)\}\,,\\ 1-l_2&=k^2\alpha L_1\{1+xY(k,\alpha)\}\,,\end{aligned}\right\}\qquad(8)$$

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[1] See the light-curve by Luizet, *Bulletin Astronomique*, **27**, 309, 1910.

where the notation is our usual one, except that α here denotes the eclipsed *area* in terms of the area of the smaller star, and X and Y are two new functions. The first equation holds good when the large star eclipses the smaller, the other in the opposite case. The functions X and Y may easily be computed with the aid of Tables I, Ix, and Iy. If α, α', and α'' correspond to identical values of k and p in these three tables, then

$$1+X(k,\ \alpha)=\frac{\alpha'}{\alpha}, \qquad 1+Y(k,\ \alpha)=\frac{\alpha''Q(k,\ 1)}{\alpha k^2}.$$

If now k and L_1 (or L_2) are known, the equations (8) can be used to compute the values of l, that is, of the light-intensity, corresponding to any desired eclipsed area α, upon any desired assumption as to the degree of darkening. The values of $\sin^2\theta$ corresponding to given values of α may then be found as usual from the light-curve and may be discussed by the usual method, using the tables computed for uniform disks. The value of k so obtained must of course agree with that originally assumed if the latter is correct.

An approximate value of k for any intermediate degree of darkening may be obtained by simple interpolation between the values obtained by solutions for the cases of zero and complete darkening made as already described. When the passage from one of these solutions to the other inverts the relative sizes of the two stars, this interpolation should be performed for r_1 and r_2 separately and their ratio then taken. When one of the eclipses is total (i.e., has a constant phase) the values of L_1 and L_2 may be obtained at once. Otherwise their values, together with that of α_0, the maximum eclipsed area, may be obtained from the observed depths $1-\lambda_1$ and $1-\lambda_2$ of the two minima, with the aid of the assumed value of k. Assuming different values of α_0 the equations

$$1-\lambda_1=\alpha_0 L_2\{1+xX(k,\ \alpha_0)\}$$
$$1-\lambda_2=k^2\alpha_0 L_1\{1+xY(k,\ \alpha_0)\}$$

may be solved for L_1 and L_2, and the correct value of α_0 is that which makes $L_1+L_2=1$.

If the value of k found after a solution on the lines just discussed does not agree with the value originally assumed, a second solution may be made with a different assumed value of k, and exact agreement of the initial and final values of k obtained by the method of "false position."

This mode of solution is applicable to any desired degree of darkening. Though less elegant theoretically than a direct solution based on the construction of separate tables for each degree of darkening, it will probably prove in practice to involve much less work than the construction of such tables, unless a large number of stars had to be discussed—a contingency which at present seems remote, since there is not now a single star whose light-curve is known accurately enough to distinguish between different degrees of darkening with certainty. The values of the functions $X(k, \alpha)$ and $Y(k, \alpha)$ have been computed, and may be found in Table VI.

TABLE VI

VALUES OF $X(k, \alpha)$

α	$k=1.0$	$k=0.9$	$k=0.8$	$k=0.7$	$k=0.6$	$k=0.5$	$k=0.4$
0.10	−0.256	−0.261	−0.266	−0.270	−0.276	−0.282	−0.286
0.20	0.117	0.125	0.131	0.137	0.142	0.146	0.151
0.30	−0.020	−0.033	−0.042	−0.049	−0.056	0.062	0.068
0.40	+0.035	+0.022	+0.013	+0.005	−0.002	−0.008	−0.015
0.50	0.072	0.060	0.052	0.044	+0.038	+0.032	+0.027
0.60	0.097	0.083	0.073	0.067	0.062	0.058	0.055
0.70	0.103	0.090	0.082	0.074	0.069	0.066	0.064
0.80	0.088	0.079	0.073	0.067	0.064	0.062	0.060
0.90	0.058	0.051	0.047	0.045	0.044	0.043	0.042
0.95	0.035	0.030	0.028	0.027	0.026	0.026	0.025
0.98	+0.016	+0.015	+0.014	+0.014	+0.014	+0.013	+0.013
1.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000

VALUES OF $Y(k, \alpha)$

0.10	−0.256	−0.297	−0.341	−0.386	−0.435	−0.490	−0.552
0.20	0.117	0.154	0.191	0.228	0.277	0.332	0.399
0.30	−0.020	−0.051	0.093	0.138	0.190	0.251	0.322
0.40	+0.035	+0.019	−0.020	0.068	0.126	0.194	0.268
0.50	0.072	0.067	+0.033	−0.011	0.071	0.139	0.209
0.60	0.097	0.105	0.075	+0.031	−0.024	0.086	0.160
0.70	0.103	0.127	0.108	0.075	+0.025	−0.036	0.111
0.80	0.088	0.137	0.134	0.111	0.066	+0.010	0.064
0.90	0.058	0.130	0.150	0.140	0.107	0.057	−0.010
0.95	0.035	0.122	0.156	0.160	0.133	0.087	+0.022
0.98	+0.016	0.117	0.161	0.176	0.156	0.114	0.050
1.00	0.000	+0.116	+0.172	+0.198	+0.186	+0.152	+0.093
Central annular	0.000	+0.145	+0.225	+0.298	+0.356	+0.404	+0.437

SUMMARY OF THE THEORY OF ECLIPSING VARIABLES

The present paper completes a series which discusses in a fairly exhaustive manner the determination of the elements of eclipsing variables, and which, it is hoped, may be of use for many years to come in the practical solution of this problem. As the formulae and tables are scattered through four numbers of this *Journal*, it may be appropriate to give here a brief summary of the notation and methods used together with directions for the computer. References throughout are to volume and page of this *Journal.*

I. GENERAL NOTATION

The unit of length is the radius of the relative orbit; the unit of light is the combined light of the two stars; the unit of mass is their combined mass. The subscript 1 refers always to the larger star. The principal quantities used are defined as follows:

Spherical Stars

r_1, radius of large star
$r_2=kr_1$, radius of small star
δ, apparent distance of centers
i, inclination of the orbit
t, time from principal minimum in days
P, period of revolution in days
$\theta=\frac{2\pi t}{P}$, true longitude in orbit
$\frac{J_1}{J_2}$, ratio of surface brightness
ρ_1, ρ_2, densities of the stars

l, light at any time
L_1, light of large star
$L_2=1-L_1$, light of small star
λ_1, minimum light at eclipse where large star is in front
λ_2, minimum light, small star in front
α, loss of light at any time in terms of loss for complete eclipse
α_0, greatest loss at minimum
n, fraction of greatest loss
y, mass of larger star

Elliptical Stars

a_1, a_2, semi-major axes
b_1, b_2, semi-minor axes

ϵ, eccentricity of meridian section
$z=\epsilon^2 \sin^2 i$

Darkened Stars

x, coefficient of darkening, 36, 240

Z, defined by $z=\frac{5}{4}\,Z-\frac{5}{28}\,Z^2$

α'', loss of light at any time during an eclipse where the small star is in front, in terms of the loss at the moment of internal tangency.

II. FUNCTIONAL NOTATION AND INDEX OF TABLES

FUNCTION	USE	WHERE FOUND		
		Uniform	Darkened	
			Big Star in Front	Small Star in Front
$p(k, \alpha)$, 35, 332	Relation between loss of light and distance of centers.	Table I, 35, 333	Table Ix, 36, 243	Table Iy, 36, 390
$\Psi(k, \alpha_1)$, 35, 321	To find light-curve when L_1 and L_2 are known.	Table II, 35, 335	Table IIx, 36, 245	Table IIy 36, 391
$\Phi(k, \alpha)$, 36, 254	To find light-curve for central total eclipse.	Table IV, 36, 252	Table IVx, 36, 253	
$\phi_1(k)$, $\phi_2(k)$, 35, 336.	To find the elements when L_1 and L_2 are known	Table IIa, 35, 337	Table IIax, 36, 246	
$\chi(k, \alpha_0, \frac{1}{4})$, etc. 35, 326.	To find elements in case of partial eclipse.	Table III, 35, 337	Table IIIx, 36, 392	Table IIIy, 36, 393
$\omega_1(n)$, $\omega_2(n)$, 35, 329	To find light-curve in case of partial eclipse (highly approximate).	Table IIIa, 35, 338		
$Q(k, \alpha_0)$, 36, 389	To aid in solution of partial eclipse of completely darkened stars.		Table V 36, 394	
$X(k, \alpha)$, $Y(k, \alpha)$ 36, 402	To aid in discussion of intermediate degrees of darkening.		Table VI 36, 403	

Table A is for converting stellar magnitudes into light intensities: 35, 339.
Table B is for converting θ into $\sin\theta$: 35, 339.
Table C is for converting elements derived on the "uniform" hypothesis into "darkened" elements (highly approximate): 36, 250.

III. FUNDAMENTAL EQUATIONS FOR SPHERICAL STARS

The references to the original discussion precede the equations. θ_1 corresponds to $\alpha=\alpha_1$, θ_2 to $\alpha=0.6$, θ_3 to $\alpha=0.9$, $\theta(n)$ to $\alpha=n\alpha_0$, θ' to $\alpha=0$, and θ'' to $\alpha=1$.

35, 321 $\delta^2=\cos^2 i+\sin^2 i \sin^2 \theta$ . . . . (*a*)
35, 321 $\delta=r_1\{1+kp(k, \alpha)\}$ . . . . (*b*)
35, 321 $A=\sin^2 \theta_2,\ B=\sin^2 \theta_2-\sin^2 \theta_3,\ \sin^2 \theta_1=A+B\psi(k, \alpha_1)$ . . . . (*c*)
35, 323 $\cot^2 i=\frac{B}{\phi_2(k)}-A,\ r_1^2 \operatorname{cosec}^2 i=\frac{B}{\phi_1(k)}$ . . . . (*d*)

36, 254 $\sin^2\theta_1 = A\Phi(k, \alpha_1)$ . . . . (*e*)

35, 326 $\dfrac{\sin^2\theta(n)}{\sin^2\theta(\frac{1}{2})} = \chi(k, \alpha_0, n)$ . . . . (*f*)

35, 329 $\sin^2\theta(n) = C\omega_2(n) + D\omega_1(n),\ \chi(k, \alpha_0, \tfrac{1}{4}) = \dfrac{C}{D}$ . . . . (*g*)

35, 331 $\left.\begin{cases} \cos^2 i \cos^2\theta' + \sin^2\theta' = r_1^2(1+k)^2 \\ \cos^2 i = r_1^2\{1 + kp(k, \alpha_0)\}^2 \end{cases}\right\}$ . . . . (*h*)

FOR UNIFORM DISKS

$\alpha_0 = 1 - \lambda_1 + \dfrac{1-\lambda_2}{k^2}$ . . . . (*j*)*

$l_1 = 1 - \alpha L_2,\ 1 - \lambda_1 = \alpha_0 L_2$ . . . . (*k*)†

$l_2 = 1 - k^2(1 - l_1)\dfrac{L_1}{L_2}$ . . . . (*m*)

FOR DARKENED DISKS

$Q(k, \alpha_0) = \dfrac{(1-\lambda_2)}{\alpha_0 - (1-\lambda_1)}$ . . . . (*n*)

$l_1 = 1 - \alpha L_2,\ 1 - \lambda_1 = \alpha_0 L_2$; α_0 from Table I*x* . . . . (*p*)

$l_2 = 1 - Q(k, \alpha)\dfrac{L_1}{L_2}(1 - l_1) =$
$1 - \alpha_0'' L_1 Q(k, 1)$ . . . . (*q*)

$\alpha_0'' Q(k, 1) = \alpha_0 Q(k, \alpha_0)$ . . . . (*r*)

$\rho_1 = 0.01344\dfrac{y}{P^2 r_1^3}$, $\rho_2 = 0.01344\dfrac{1-y}{P^2 r_2^3}$ . . . . . . (*s*)‡

$\dfrac{J_1}{J_2} = \dfrac{k^2 L_1}{L_2}$ . . . . (*t*)

IV. DIRECTIONS FOR THE COMPUTER

Preliminary: Reduce the observed curve to intensity-curve, taking maximum intensity as unity. Determine the epoch of minimum, and draw the best symmetrical curve to represent it.

A) The light is constant between minima, therefore the stars are spherical.

1) There is a constant phase at principal minimum of brightness λ_1. Then $L_1 = \lambda_1$, and $L_2 = 1 - \lambda_1$.

Solution on uniform hypothesis: For tabular values of α_1 compute l_1 by (*k*), read θ from the light-curve, take $\sin\theta$ from Table B, find $\psi(k, \alpha_1)$ from (*c*), hence *k* from Table II, and other elements from (*d*) and Table II*a*. If $\cot^2 i$ comes out negative, use (*e*) and Table IV. Compute the light-curve for principal minimum from (*c*), (*k*), and Table II, and for the secondary minimum from (*m*).

For completely darkened stars substitute Tables II*x*, II*ax*, and IV*x*, for II, II*a*, and IV; and compute secondary minimum by (*q*) and Table V.

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* *Astrophysical Journal*, 35, 320.

† *Ibid.*, 318.

‡ *Ibid.*, 36, 73.

2) There is no constant phase at principal minimum, therefore the eclipses are partial (or perhaps the principal eclipse is annular, if the stars are darkened). Knowledge of the secondary minimum is essential in this case.

For uniform stars use (k) and the light-curve to find $\theta(n)$, then find C and D from (g), using Table IIIa. For assumed values of α_0 compute k from (j) and $\chi(k, \alpha_0, \frac{1}{4})$ from Table III, and find what value of α_0 gives $\chi=\frac{C}{D}$. Then compute θ' from (g) (setting $n=0$), the elements from (h), using Table I, and L_2 from (k).

For darkened stars find $\theta(n)$ as above, then $\chi(k, \alpha_0, \frac{3}{4})$, $\chi(k, \alpha_0, \frac{1}{4})$, and $\chi(k, \alpha_0, 0)$ from (f). For assumed values of α_0 find k from (n) and Table V, and α_0'' from (r). Then take the χ-functions from Table IIIx, with arguments k, α_0, if the principal minimum is assumed to correspond to λ_1, or from Table IIIy, with arguments k, α_0'', if to λ_2, and find what values of α_0 and k give the best agreement. Compute the other elements by (h), using Table Ix, and find L_2 from (p). For details regarding annular eclipses see **36**, 393 ff.

B) The light is not constant between minima, therefore the stars are elliptical. Rectify the intensity-curve: (1) for ellipticity of the stars, **36**, 64, 65 (illustration, **36**, 139 ff.); (2) for radiation-effect (if necessary), **36**, 67–69. Then discuss as under A) except that for equations (c), (d), (f), (g), (h) must be substituted, respectively,

$$\psi(k, \alpha_1)=\frac{(\sin^2\theta_1-\sin^2\theta_2)(1-z\cos^2\theta_3)}{(\sin^2\theta_2-\sin^2\theta_3)(1-z\cos^2\theta_1)} \quad \ldots\ldots \quad (c')^{*}$$

$$\left.\begin{aligned}\cos^2 i+\sin^2 i\sin^2\theta' &= a_1^2(1-z\cos^2\theta')(1+k)^2\\ \cos^2 i+\sin^2 i\sin^2\theta'' &= a_1^2(1-z\cos^2\theta'')(1-k)^2\\ \cos^2 i &= a_1^2(1-z)\{1+kp(k, \alpha_0)\}^2\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (d' \text{ and } h')^{\dagger}$$

$$\frac{\sin^2\theta(n)\{(1-z\cos^2\theta(\tfrac{1}{2})\}}{\sin^2\theta(\tfrac{1}{2})\{(1-z\cos^2\theta(n)\}}=\chi(k, \alpha_0, n) \quad \ldots\ldots\ldots \quad (f')^{\ddagger}$$

$$\frac{\sin^2\theta(n)}{1-z\cos^2\theta(n)}=C\omega_2(n)+D\omega_1(n)\,, \quad \frac{C}{D}=\chi(k, \alpha_0, \tfrac{1}{4}) \quad \ldots\ldots \quad (g')$$

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[*] *Astrophysical Journal*, **36**, 65.

[†] *Ibid.*, 66.

[‡] *Ibid.*, 66.

C) For the discussion of eccentric orbits refer to **36**, 54–60, 396–397, and the illustration, **36**, 146 ff. For the discussion of intermediate degrees of darkening refer to pp. 401–403, and Table VI of this paper.

Princeton University Observatory
October 26, 1912

REVIEWS

Bijdrage tot de Kennis van Lijnenspectra. By T. VAN LOHUIZEN. (Doctor's dissertation.) University of Amsterdam. The Hague. Printed by the "Transvalia" press, 1912. Pp. 102.

The author, in this thesis, (1) examines existing spectrum-series formulae; (2) tries adding more terms to the formula of Kayser and Runge; (3) devises a "new formula"; (4) tests this by series in *Li*, *Na*, and *Al;* (5) seeks to unite, in one connected system, all the series of an element; and (6) proposes several new series in the spectra of tin and antimony.

In the first section, he concludes that none of the formulae mentioned (Kayser and Runge, Rydberg, Ramage, Fowler and Shaw, Ritz, Mogendorff and Halm) are entirely satisfactory. In the second, he finds that the formula of Kayser and Runge is still unsatisfactory, even when another term, involving a fourth adjustable constant, is added. He accordingly starts with the formula of Rydberg, and seeks, in the third section, to find a modification of this which will be applicable to series in general. Curves are drawn on a large scale for different series, the wave-numbers of the lines being chosen as ordinates, and the successive integers as abscissae. A study of such curves leads him to the conclusion (fully developed in section 5) that the shape of the curve is the same for all series, even for the Balmer series of Hydrogen, but that the curve is shifted to different axes for different series, often with a slight rotation. He expresses this analytically by starting with the Balmer formula, and changing the axes by a translation along each one, and a rotation around the origin. The new expression so obtained involves a continued fraction, and this is reduced and simplified by assuming the rotation to be small, eventually taking the form:

$$y=b+(x-a)\gamma-\frac{N}{\left\{x-a-\frac{\gamma N}{x^2}\left(1-\frac{3a}{x}\right)\left(1+\frac{a}{x}\right)\right\}^2}$$

y is the wave-number of the series line; x is the integer, 1, 2, 3, etc.; a and b are the two translations, and γ is the angle of rotation; N is the universal series-constant.

The author does not call attention to the fact that this formula is not convergent; and the convergency of spectrum series is one of the strongest articles of faith of the spectroscopist, in spite of the fact that we cannot follow any series out to its limit. In the above geometrical scheme, *any* rotation makes the series ultimately divergent, and must therefore be thrown out of consideration.

It is true that the author does not use the formula given above, but drops the divergent part of it (the second term), the reason being given that only a few lines of any series are ever observed, and that the rotation is usually very small, especially so (as it happens) for cases like the principal series of *Na*, for which the most terms are known. With this further simplification, the formula runs:

$$y=b-\frac{N}{(x+a+\gamma y)^2}$$

The author has not noticed that this formula is not new; it was first given by Ritz, and has been extensively used by him and by Paschen.

In the fourth section, this "new" formula is tested, and found to be fairly satisfactory, though the series chosen (except *Al*) are what one might call "easy" ones; that is, their deviation from the Balmer form is not extreme. Due credit is not given here to the calculations of Birge, who showed that the Ritz formula would do for some of these same series. In the case of *Al*, a "difficult" series is studied, and the formula fails to represent it; but the conclusion is drawn that the first line is improperly classified with the others. More light is needed on this question before a decision can be reached.

Section 5 is largely occupied by a presentation of the graphical methods of study of the series curves. A convenient notation is presented for classifying the series of an element, including those deduced by the "combination-principle" of Ritz. Admirable as such a scheme is, it accounts for far more series than are observed, and fails to account for some (e.g., in *Ca*) that have been found; so that, at first sight, one is tempted to admire it more for its beauty than for its usefulness. Even in the case of the "observed" combination-series, only a few terms have been found, and we are badly in need of more observations to allay the suspicion that these may not be series after all. Mogendorff has recently presented the seductive idea that combination-series are to be regarded as summation- or difference-vibrations (analogous to difference-tones in acoustics) generated by some of the stronger lines of the spectrum. If this can be established as a general principle, we shall have a much firmer foundation to build a series system upon than is furnished by any approximate formula.

In the last section, the author attempts to group the lines of tin and antimony into series, of which nine are given for the first, twelve for the second element. The reality of some of these series may be questioned, as they are composed of lines of different physical aspects, (e.g., some reversed, others not), and their intensities are not regularly arranged. The problem is a difficult one, as the lines are in the extreme ultra-violet, and more observations are needed, not only of the wave-lengths, but of the Zeeman effect in this region. The author has misunderstood Kayser and Runge's scale of intensities, and has not hesitated to fill a gap in an "arc" series with a "spark" line. It gives rather a shock to one who feels, with Hicks, that "one gets an almost superstitious belief in the measurements of Kayser and Runge after long use of them in this kind of work," to see the author interpret one of their strong reversed lines as a close pair of weak ones. This section of the work would have been improved, and perhaps altered, if the author had made observations directly upon the spectra concerned.

It is a dangerous business to attempt to arrange lines into series by means of a formula alone. A series is an aggregation of lines which are due to the same, or to similar causes, and are affected in similar ways by outside agencies. They must show a progressive arrangement of intensities, and of physical aspect, as well as of frequencies. Since the arrangement of frequencies is exactly given by no known formula (excepting in the case of hydrogen), the physical aspect of the lines becomes all the more important, especially if we include under this head the Zeeman effect, the pressure shift, and the various changes which occur in passing from one sort of source to another, or among different parts of the same source. So far as we know, all the members of a well-behaved series change in the same manner in all these various alterations of conditions, and one cannot help hesitating before accepting a family of new series, until some physical proof of their relationship is presented.

F. A. SAUNDERS

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The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

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INDEX TO VOLUME XXXVI

SUBJECTS

PAGE

Absorption of Light, Selective, on the Moon's Surface and Lunar Petrography. *R. W. Wood* 75
  Spectrum of Tellurium Vapor and the Effect of High Temperature upon It. *E. J. Evans* 228
Actinometry, Yerkes. *J. A. Parkhurst* 169
Algol Variable *RR Draconis*. *Frederick H. Seares* 368
Alkaline Earth Sulphides, Influence of Temperature upon the Phenomena of Phosphorescence in. *Herbert E. Ives* and *M. Luckiesh* 330
Aluminium, Cadmium, and Zinc, Band Spectra of. *Emily E. Howson* 286
Angular Diameter of Stars by Means of Elliptically Polarized Light, New Means of Determining the. *S. Pokrowsky* 156
Attraction of Sun-Spots for Prominences. *Frederick Slocum* . . 265
Band Spectra of Aluminium, Cadmium, and Zinc. *Emily E. Howson* 286
Cadmium, and Zinc, Band Spectra of Aluminium. *Emily E. Howson* 286
Carbon, Deviation from Lambert's Cosine Law of Emission of Tungsten and, at Glowing Temperatures. *A. G. Worthing* . 345
Comet 1911 *c* (Brooks), Photographic Observations of. *E. E. Barnard* 1
Convergence Frequency in Spectral Series. *Fernando Sanford* . . 255
Darkening at the Limb in Eclipsing Variables. I, II. *Henry Norris Russell* and *Harlow Shapley* 239, 385
Deviation from Lambert's Cosine Law of the Emission from Tungsten and Carbon at Glowing Temperatures. *A. G. Worthing* . . 345
Eclipsing Variables. I, II. Darkening at the Limb in. *Henry Norris Russell* and *Harlow Shapley* 239, 358
  Variables *W Delphini*, *W Ursae Majoris*, and *W Crucis*, Elements of. *Henry Norris Russell* 133
  Variables *W Delphini*, *S Cancri*, *SW Cygni*, and *U Cephei*, Elements of. *Harlow Shapley* 269
Elements of the Eclipsing Variables *W Delphini*, *W Ursae Majoris*, and *W Crucis*. *Henry Norris Russell* 133
  of the Eclipsing Variables *W Delphini*, *S Cancri*, *SW Cygni*, and *U Cephei*. *Harlow Shapley* 269
Elliptically Polarized Light, New Method of Determining the Angular Diameter of Stars by Means of. *S. Pokrowsky* . . 156
Emission from Tungsten and Carbon at Glowing Temperatures, Deviation from Lambert's Cosine Law of. *A. G. Worthing* . 345

PAGE

INFLUENCE of Temperature on the Phenomena of Phosphorescence in the Alkaline Earth Sulphides. *Herbert E. Ives* and *M. Luckiesh* 330

INTEGRATED Spectrum of the Milky Way. *E. A. Fath* 362

LAMBERT'S Cosine Law of the Emission from Tungsten and Carbon at Glowing Temperatures, Deviation from. *A. G. Worthing* . 345

LIGHT, Primary Standard of. *Herbert E. Ives* 322

METALS, Synthetic Development of Radiation Laws for. *Edward P. Hyde* 89

MILKY Way, Integrated Spectrum of the. *E. A. Fath* 362

MOON'S Surface, Selective Absorption of Light on, and Lunar Petrography. *R. W. Wood* 75

NEW Method of Determining the Angular Diameter of Stars by Means of Elliptically Polarized Light. *S. Pokrowsky* 156

Nova Geminorum No. 2, Observations of the Spectrum of. *Walter S. Adams* and *Arnold Kohlschutter* 293

OBSERVATIONS of the Spectrum of *Nova Geminorum* No. 2. *Walter S. Adams* and *Arnold Kohlschutter* 293

ON the Determination of the Orbital Elements of the Eclipsing Variable Stars. II. *Henry Norris Russell* 54

PHOSPHORESCENCE in the Alkaline Earth Sulphides, Influence of Temperature upon the Phenomena of. *Herbert E. Ives* and *M. Luckiesh* 330

PHOTOGRAPHIC Observations of Comet 1911 *c* (Brooks). *E. E. Barnard* 1

PRIMARY Standard of Light. *Herbert E. Ives* 322

PROMINENCES, Attraction of Sun-Spots for. *Frederick Slocum* . . 265

RADIATION Laws for Metals, Synthetic Development of. *Edward P. Hyde* 89

REVIEWS:

P. G. Nutting. *Outlines of Applied Optics* (A. C. Lunn) . . 85

Dr. Moritz von Rohr. *Die optischen Instrumente* (Stanley C. Reese) 263

T. Van Lohuizen. *Bijdraget tot de Kennis van Lijnenspectra* (F. A. Saunders) 409

RR Draconis, Algol Variable. *Frederick H. Seares* 368

SELECTIVE Absorption of Light on the Moon's Surface and Lunar Petrography. *R. W. Wood* 75

SERIES, Spectral, Convergence Frequency in. *Fernando Sanford* . 255

SOLAR Radiation, High-Level Measurement of. *Frank W. Very* . 000

SPECTRA, Band, of Aluminium, Cadmium, and Zinc. *Emily E. Howson* 286

SPECTRAL Series, Convergence Frequency in. *Fernando Sanford* . . 255

SPECTRUM of the Milky Way, Integrated. *E. A. Fath* 362

of *Nova Geminorum* No. 2, Observations of. *Walter S. Adams* and *Arnold Kohlschutter* 293

PAGE

STANDARD of Light, Primary. *Herbert E. Ives* 322

SULPHIDES, Influence of Temperature upon the Phenomena of Phosphorescence in the Alkaline Earth. *Herbert E. Ives* and *M. Luckiesh* 330

SUN-SPOTS, Attraction of, for Prominences. *Frederick Slocum* . . 265

SYNTHETIC Development of Radiation Laws for Metals. *Edward P. Hyde* 89

TELLURIUM Vapor and the Effect of High Temperature upon It, The Absorption Spectrum of. *E. J. Evans* 228

TEMPERATURE, High, The Absorption Spectrum of Tellurium Vapor and the Effect of, upon It. *E. J. Evans* 228

  Influence of, upon the Phenomena of Phosphorescence in the Alkaline Earth Sulphides. *Herbert E. Ives* and *M. Luckiesh* . 330

TERTIARY Standards with the Plane Grating, The Testing and Selection of Standards. *Charles E. St. John* and *L. W. Ware* . . 14

TUNGSTEN and Carbon, Deviation from Lambert's Cosine Law of Emission at Glowing Temperatures. *A. G. Worthing* . . 345

VARIABLE *RR Draconis*, Algol. *Frederick H. Seares* 368

VARIABLES, Eclipsing, Darkening at the Limb in. I, II. *Henry Norris Russell* and *Harlow Shapley* 239, 385

  W Delphini, *S Cancri*, *SW Cygni*, and *U Cephei*, Elements of the. *Harlow Shapley* 269

  W Delphini, *W Ursae Majoris*, and *W Crucis*, Elements of Eclipsing. *Henry Norris Russell* 133

YERKES Actinometry. *J. A. Parkhurst* 169

ZINC, Band Spectra of Aluminium, Cadmium, and. *Emily E. Howson* 286

INDEX TO VOLUME XXXVI

AUTHORS

PAGE

Adams, Walter S., and Arnold Kohlschutter. Observations of the Spectrum of *Nova Geminorum* No. 2 293

Barnard, E. E. Photographic Observations of Comet 1911 *c* (Brooks) 1

Evans, E. J. The Absorption Spectrum of Tellurium Vapor and the Effect of High Temperature upon It 228

Fath, E. A. The Integrated Spectrum of the Milky Way . . . 362

Howson, Emily E. Band Spectra of Aluminium, Cadmium, and Zinc 286

Hyde, Edward P. The Synthetic Development of Radiation Laws for Metals 89

Ives, Herbert E. The Primary Standard of Light 322

Ives, Herbert, E., and M. Luckiesh. Influence of Temperature upon the Phenomena of Phosphorescence in the Alkaline Earth Sulphides 330

Kohlschutter, Arnold, and Walter S. Adams. Observations of the Spectrum of *Nova Geminorum* No. 2 293

Luckiesh, M., and Herbert E. Ives. Influence of Temperature upon the Phenomena of Phosphorescence in the Alkaline Earth Sulphides 330

Lunn, A. C. Review of: *Outlines of Applied Optics*, P. G. Nutting . 85

Parkhurst, J. A. Yerkes Actinometry 169

Pokrowsky, S. On a New Method of Determining the Angular Diameters of Stars by Means of Elliptically Polarized Light . 156

Reese, Stanley C. Review of: *Die optischen Instrumente*, Dr. Moritz von Rohr 263

Russell, Henry Norris. Elements of the Eclipsing Variables, *W Delphini*, *W Ursae Majoris*, and *W Crucis* 133

On the Determination of the Orbital Elements of Eclipsing Variable Stars. II 54

Russell, Henry Norris, and Harlow Shapley. On Darkening at the Limb in Eclipsing Variables. I, II 239, 385

St. John, Charles E., and L. W. Ware. Tertiary Standards with the Plane Grating, The Testing and Selection of Standards . 14

Saunders, F. A. Review of: *Bijdrage tot de Kennis van Lijnenspectra*, T. Van Lohuizen 409

Sanford, Fernando. On Convergence Frequency in Spectral Series 255

Seares, Frederick H. The Algol Variable *RR Draconis* . . . 368

PAGE

SHAPLEY, HARLOW. Elements of the Eclipsing Variables *W Delphini*, *S Cancri*, *SW Cygni*, and *U Cephei* 269

SHAPLEY, HARLOW, and HENRY NORRIS RUSSELL. On Darkening at the Limb in Eclipsing Variables. I, II 239, 385

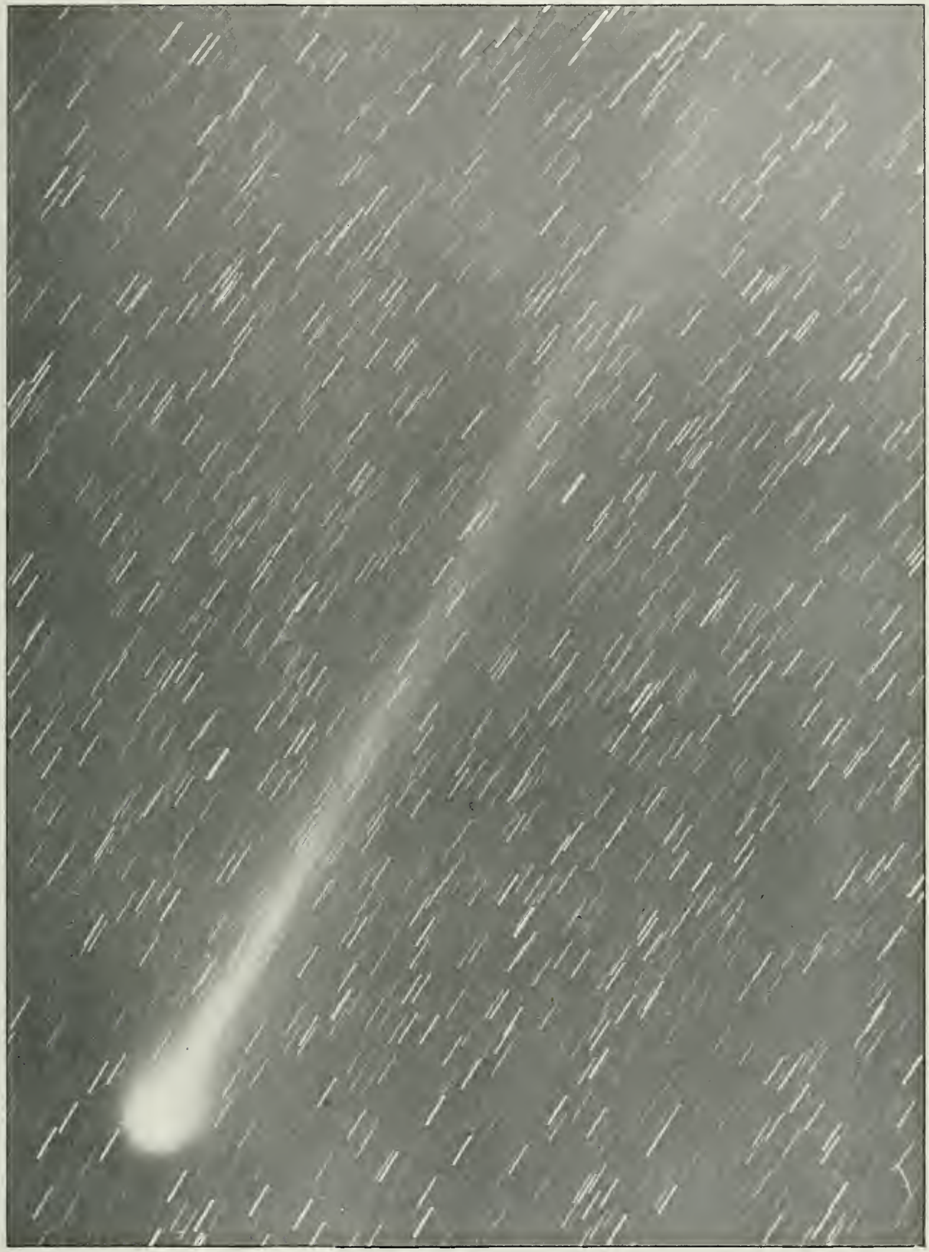
SLOCUM, FREDERICK. Attraction of Sun-Spots for Prominences . 265

WORTHING, A. G. On the Deviation from Lambert's Cosine Law of the Emission from Tungsten and Carbon at Glowing Temperatures 345

WARE, L. W., and CHARLES E. ST. JOHN. Tertiary Standards with the Plane Grating, The Testing and Selection of Standards . 14

WOOD, R. W. Selective Absorption of Light on the Moon's Surface and Lunar Petrography 75

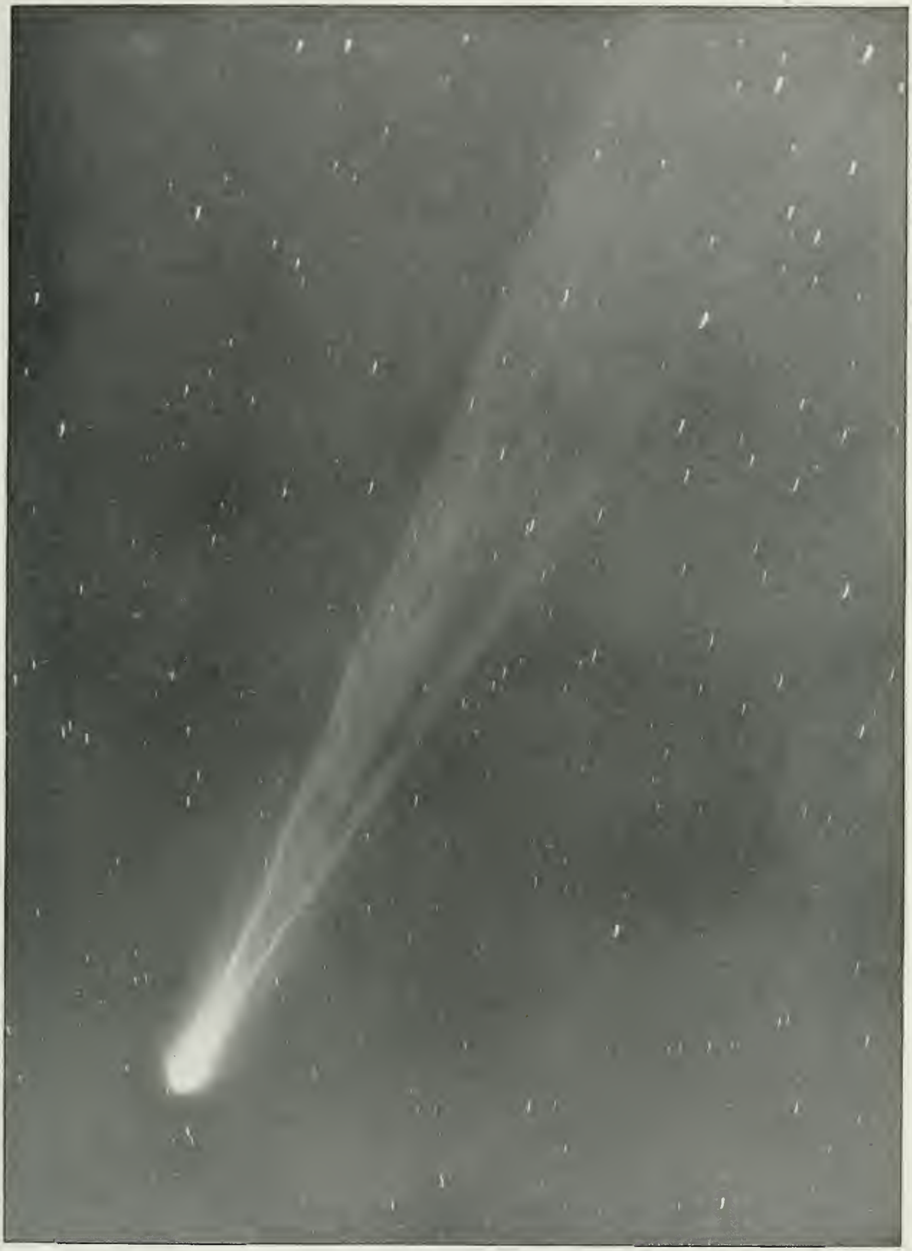
PLATE I



COMET 1911 *c* (BROOKS)

1911 September 22. $15^h\ 9^m$ G.M.T. $\alpha\ 15^h\ 25^m\ \delta + 52^\circ\ 29'$ (1855.0). Exposure $3^h\ 30^m$.
10-inch Bruce Telescope. Scale: 1 mm = $3'.4$

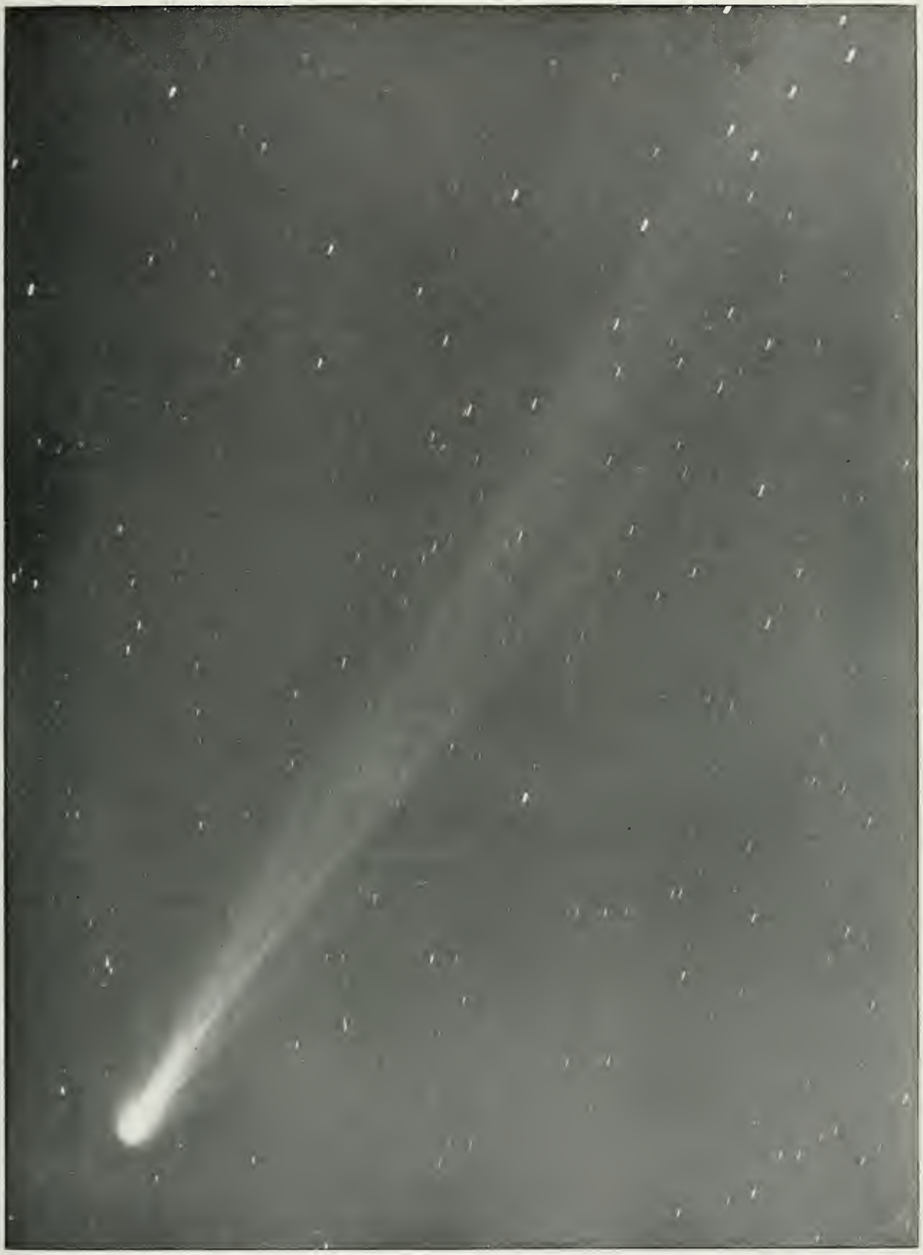
PLATE II



Comet 1911 *c* (Brooks)

1911 October 19. $22^{h}\,38^{m}$ G.M.T. $\alpha\ 12^{h}\,39^{m}\ \delta+17^{\circ}\,45'$ (1855.0). Exposure $0^{h}\,46^{m}$.
10-inch Bruce Telescope. Scale: $1\ \text{mm}=3'.2$

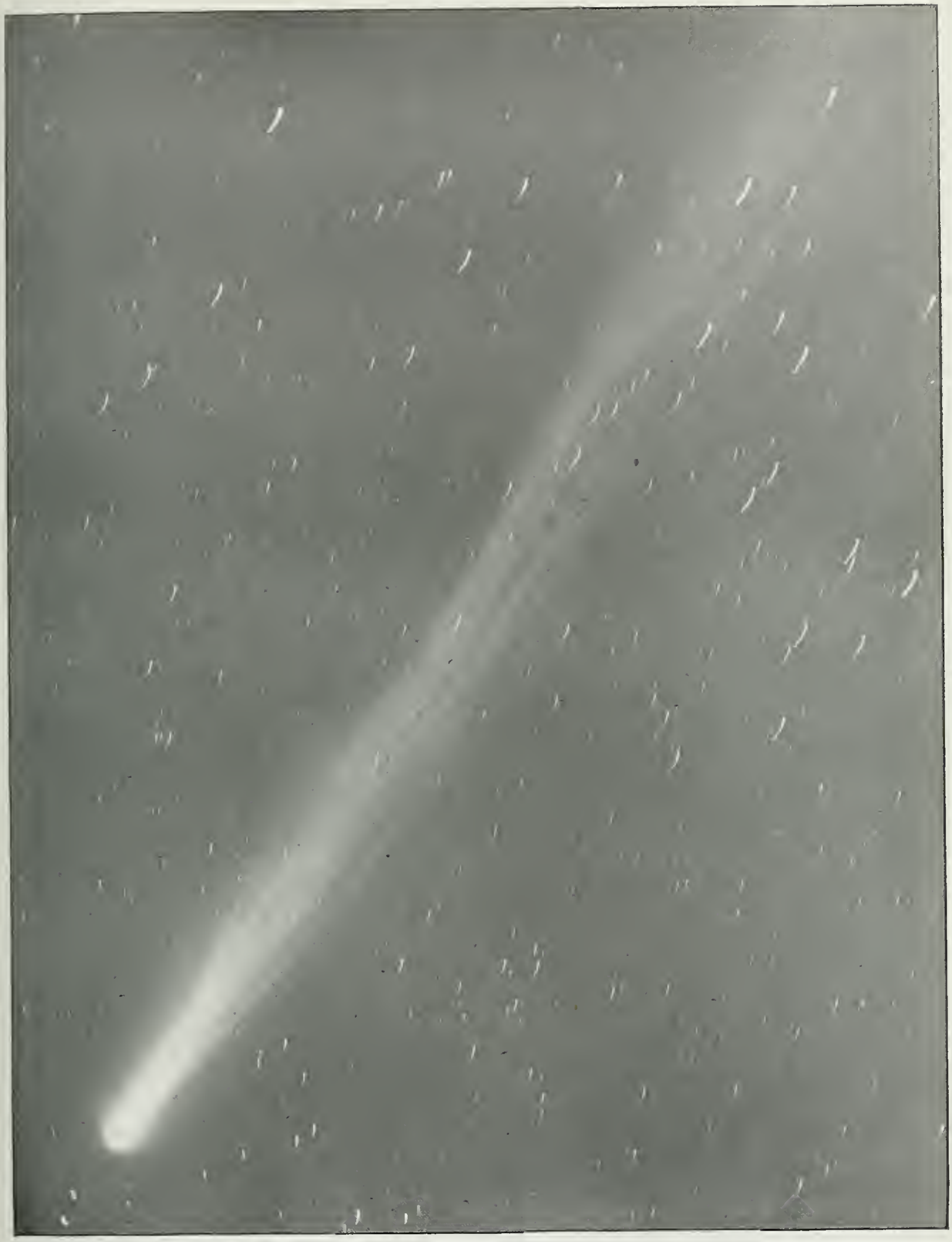
PLATE III



COMET 1911 *c* (BROOKS)

1911 October 20. $22^h\ 44^m$ G.M.T. $\alpha\ 12^h\ 38^m\ \delta + 16^\circ\ 20'$ (1855.0). Exposure $0^h\ 45^m$.
10-inch Bruce Telescope. Scale: 1 mm $= 3'.4$

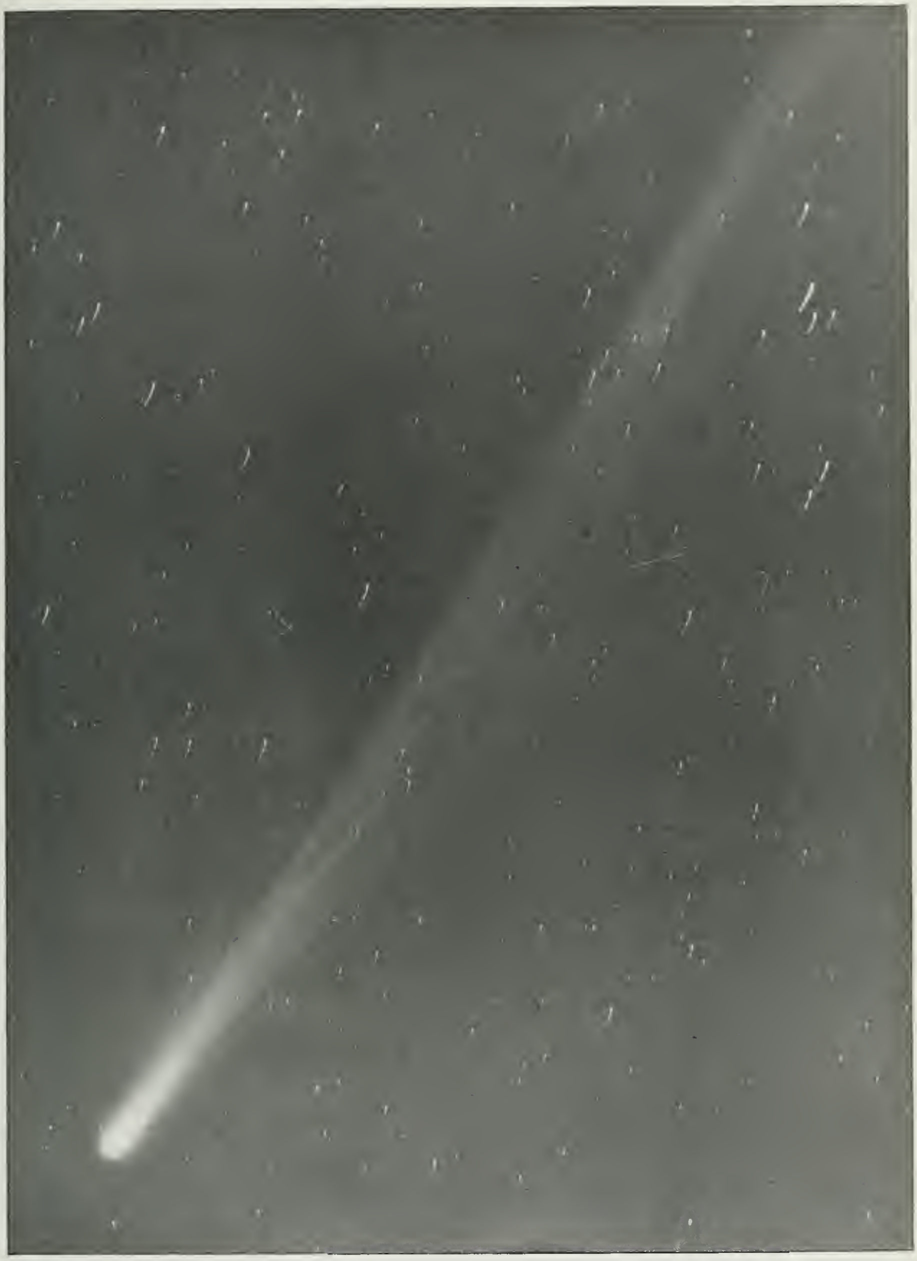
PLATE IV



Comet 1911 *c* (Brooks)

1911 October 23. $22^h\ 32^m$ G.M.T. $\alpha\ 12^h\ 33^m\ \delta + 11^h\ 35^m$ (1855.0). Exposure $1^h\ 15^m$.
10-inch Bruce Telescope. Scale: 1 mm $= 3\overset{\prime}{.}4$

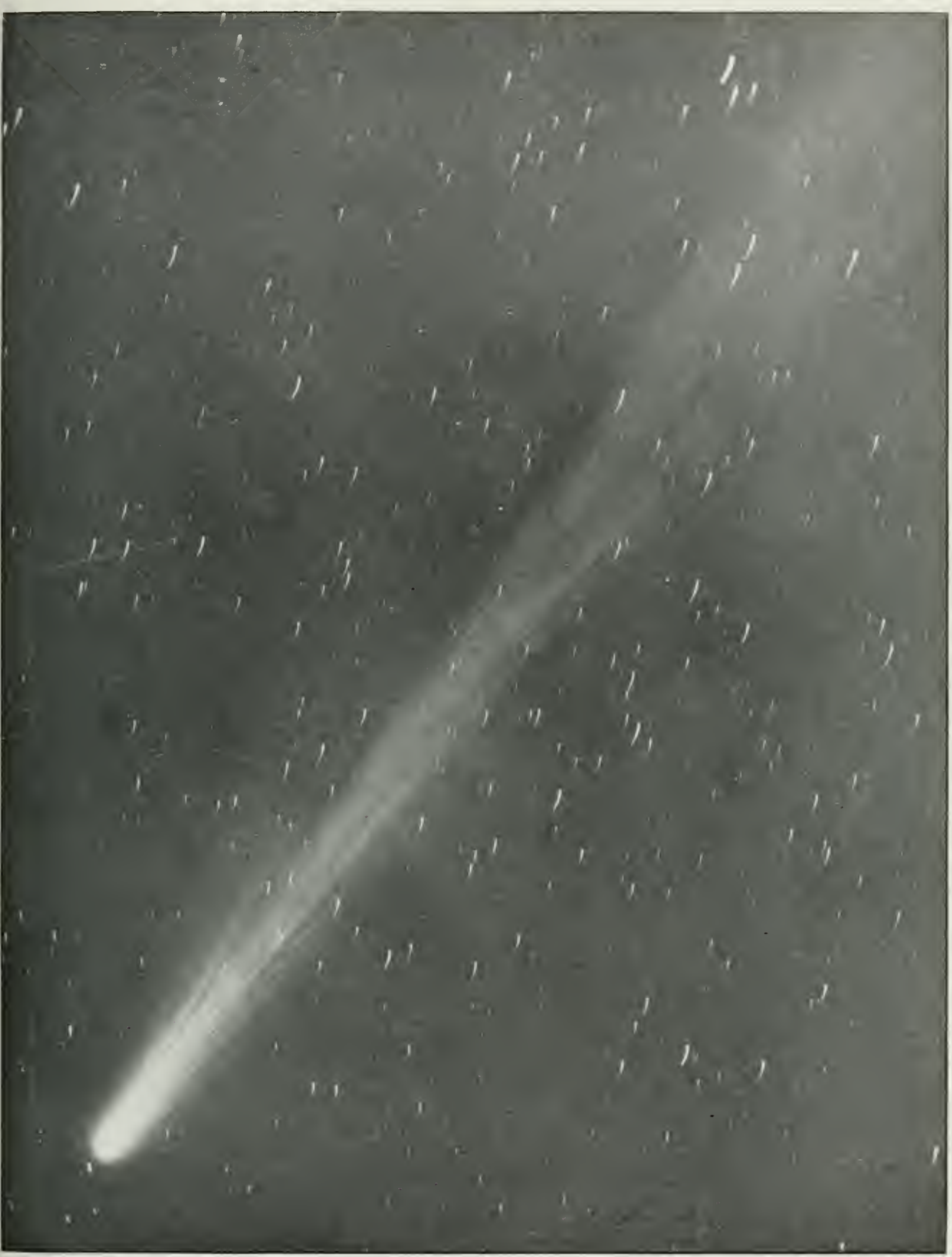
PLATE V



Comet 1911 *c* (Brooks)

1911 October 27. $22^h\ 33^m$ G.M.T. $\alpha\ 12^h\ 30^m\ \delta + 5^\circ\ 35'$ (1855.0). Exposure $1^h\ 12^m$.
10-inch Bruce Telescope. Scale: 1 mm = $3'.4$

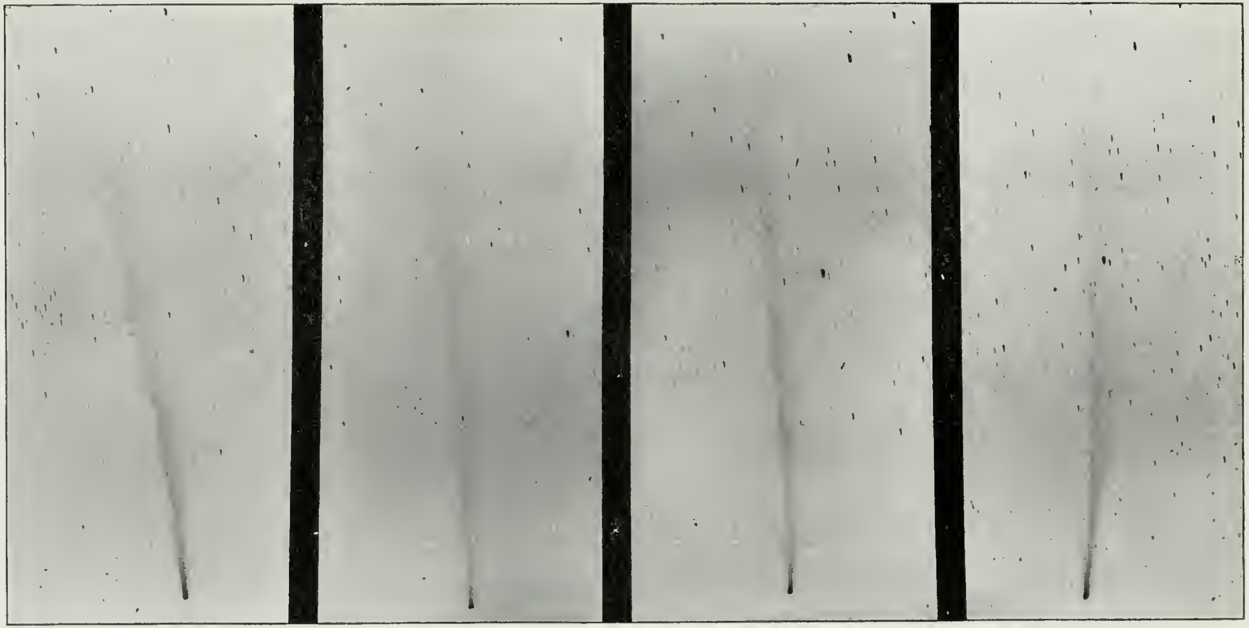
PLATE VI



Comet 1911 *c* (Brooks)

1911 October 28. $22^h\ 38^m$ G.M.T. $\alpha\ 12^h\ 30^m\ \delta +4^\circ\ 10'$ (1855.0). Exposure $1^h\ 5^m$.
10-inch Bruce Telescope. Scale: 1 mm $= 3'.4$

PLATE VII

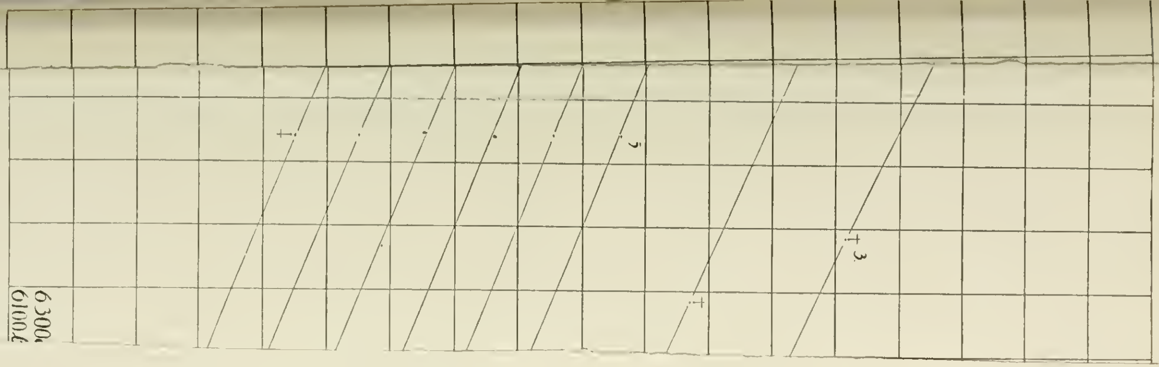


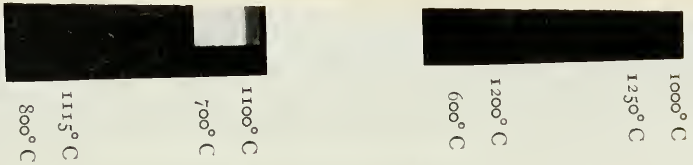
1911 October 23 — 1911 October 25 — 1911 October 27 — 1911 October 28

Comet 1911 *c* (Brooks)

With the 1.5-inch lantern lens of the Bruce Telescope. Scale: 1 mm = 24′

PLATE VIII

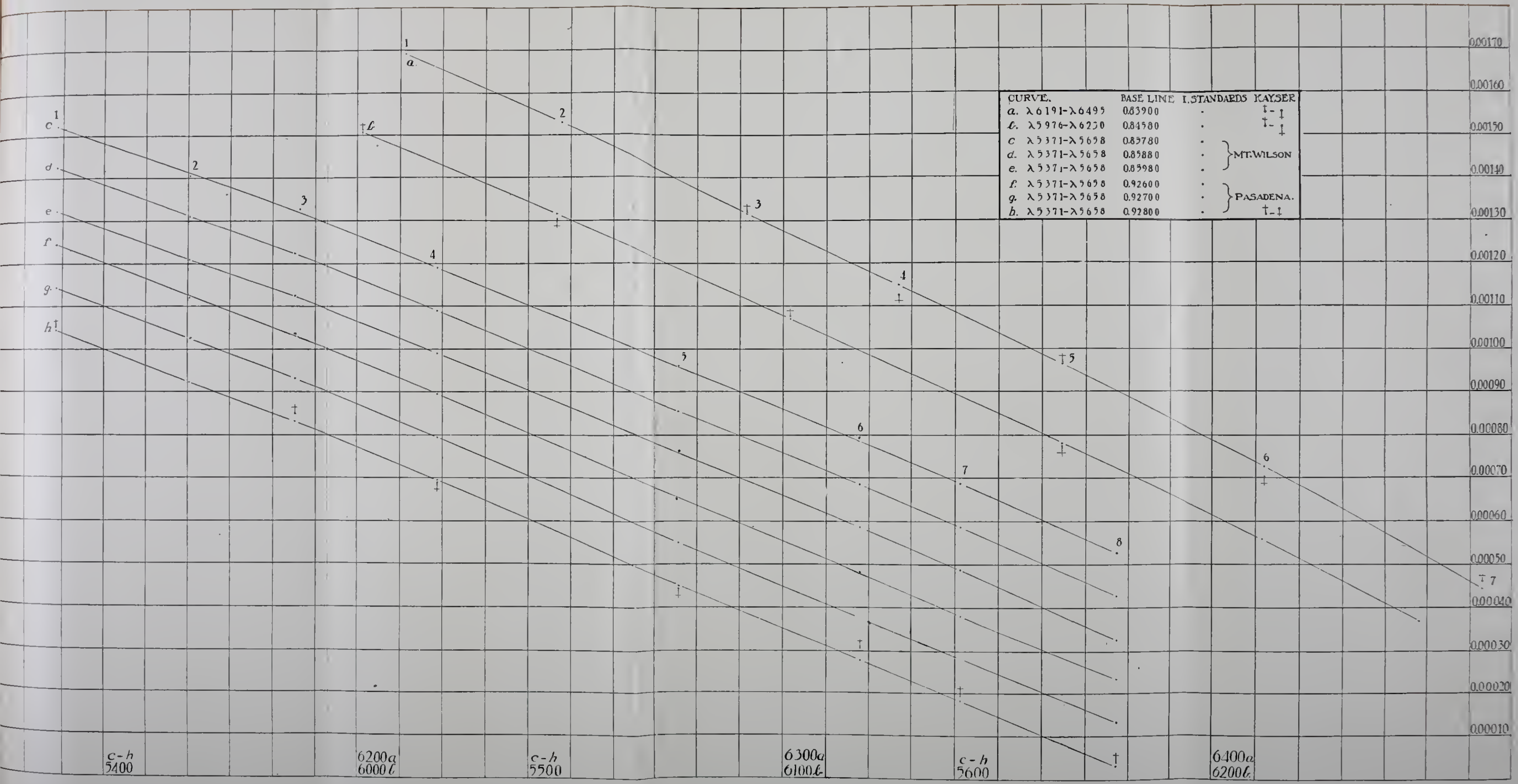




Calcium Spectroheliograms of Solar Prominences on October 8, 1910

Scale: 1 mm = 8600 km

PLATE VIII


CURVE. BASE LINE I.STANDARDS KAYSER
a. λ6191-λ6495 0.83900 · †-‡
b. λ5976-λ6230 0.84580 · †-‡
c λ5371-λ5658 0.85780 ·
d. λ5371-λ5658 0.85880 · MT. WILSON
e. λ5371-λ5658 0.85980 ·
f. λ5371-λ5658 0.92600 ·
g. λ5371-λ5658 0.92700 · PASADENA.
h. λ5371-λ5658 0.92800 · †-‡
a
b
c
d
e
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g
h
1
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3
4
5
6
7
8
0.00170
0.00160
0.00150
0.00140
0.00130
0.00120
0.00110
0.00100
0.00090
0.00080
0.00070
0.00060
0.00050
0.00040
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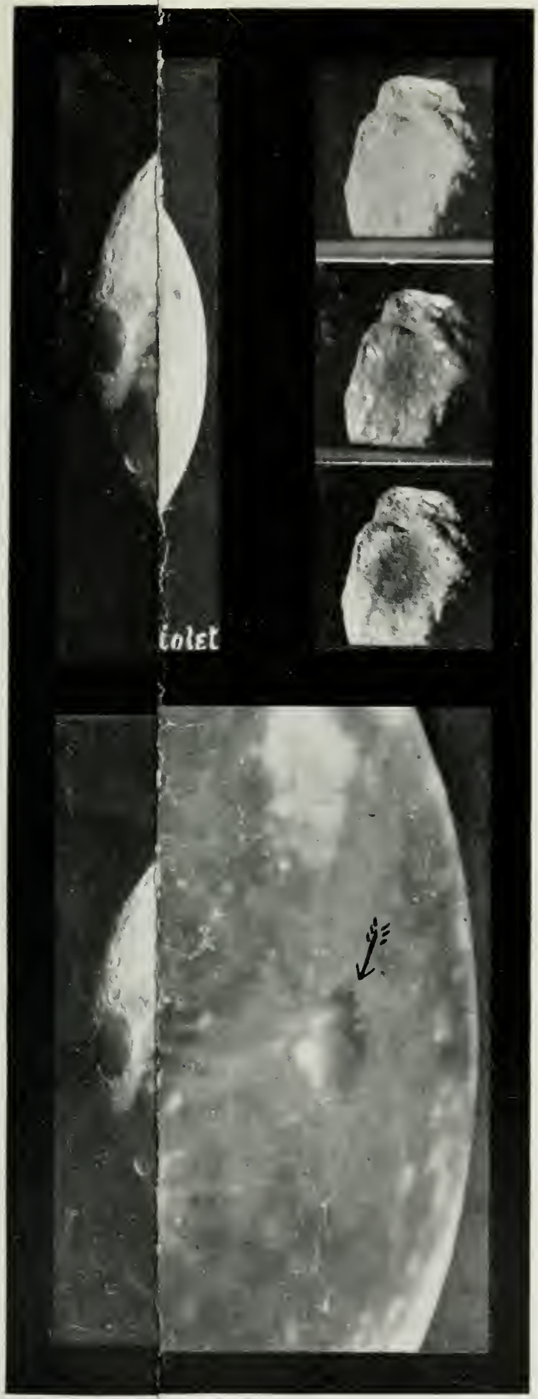
c-h 5500
6300a 6100b
c-h 5600
6400a 6200b



Yellow

Violet

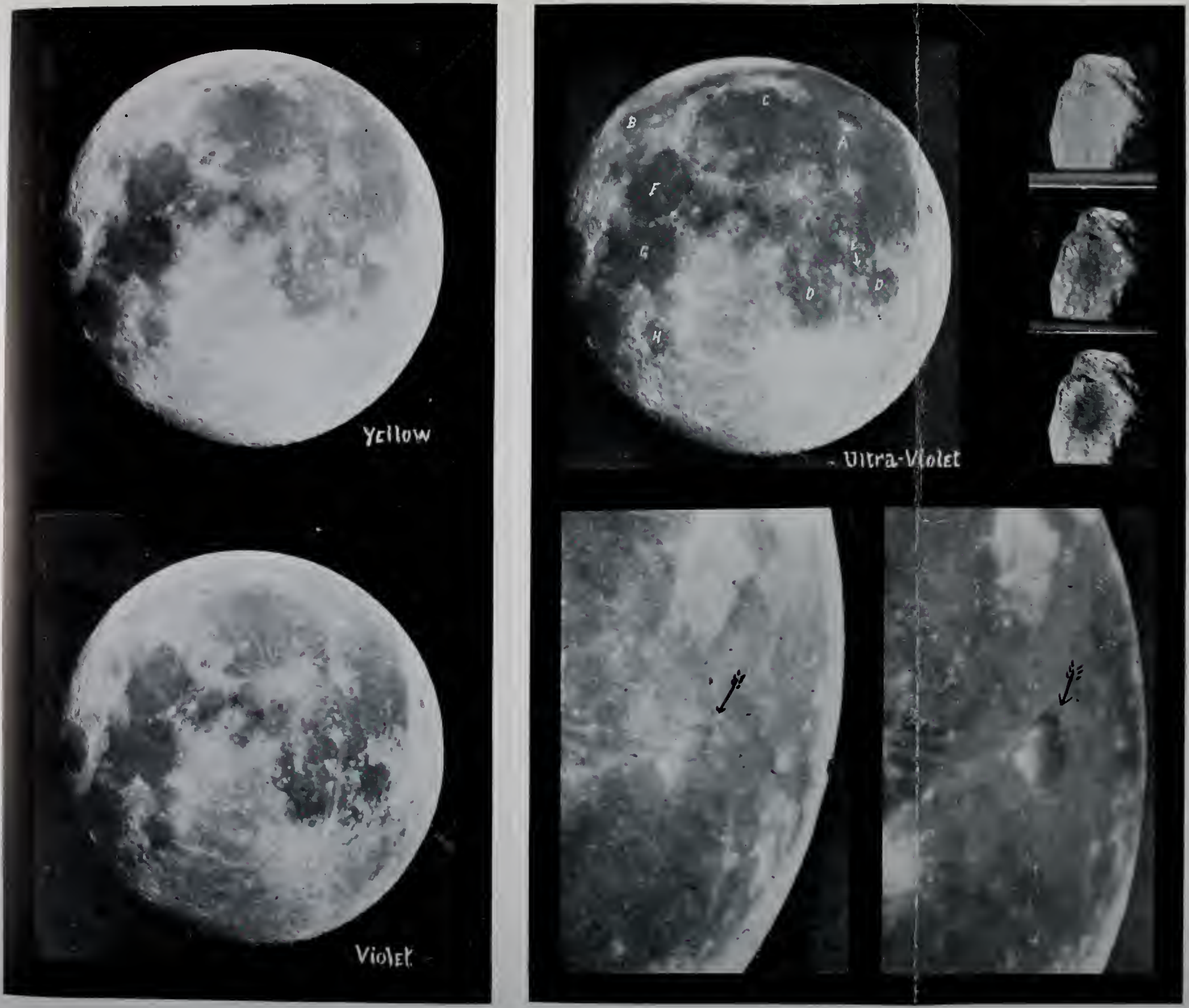
Ultra-
violet

Ultra-violet

Calcium Spectroheliograms of Solar Prominences on October 8, 1910

Scale: 1 mm = 8600 km

PLATE IX



Yellow

Violet

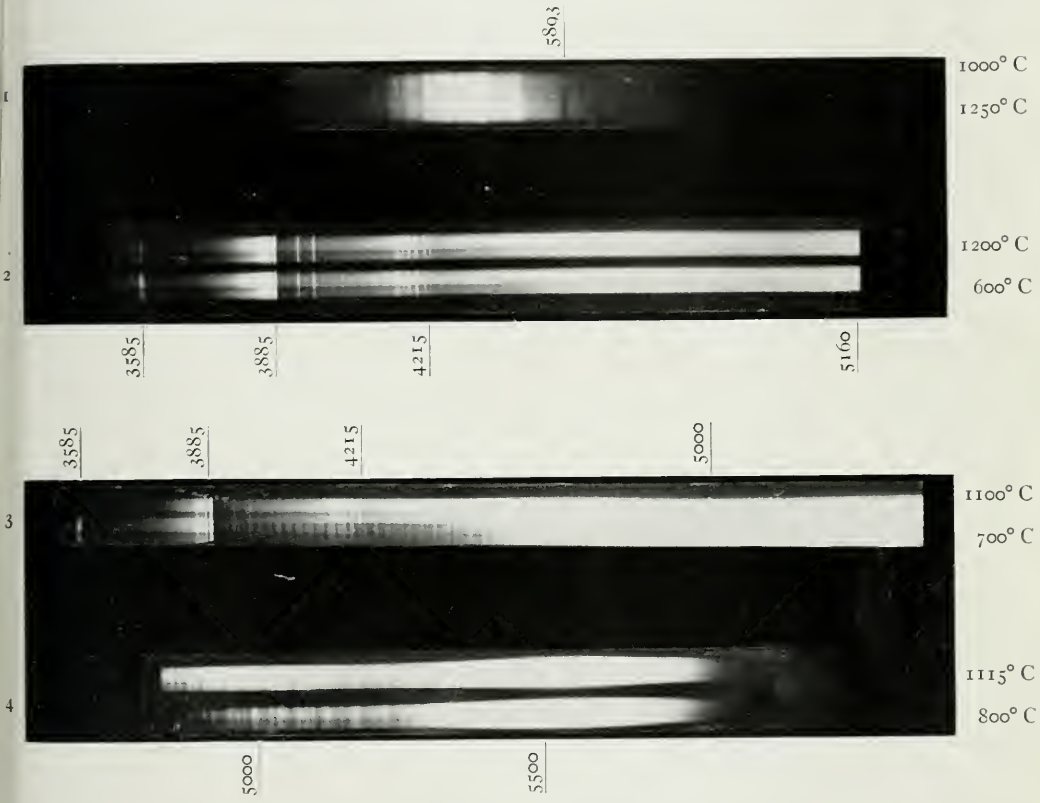
Ultra-violet

Yellow

Ultra-violet

Photographs in Light of Different Wave-Lengths

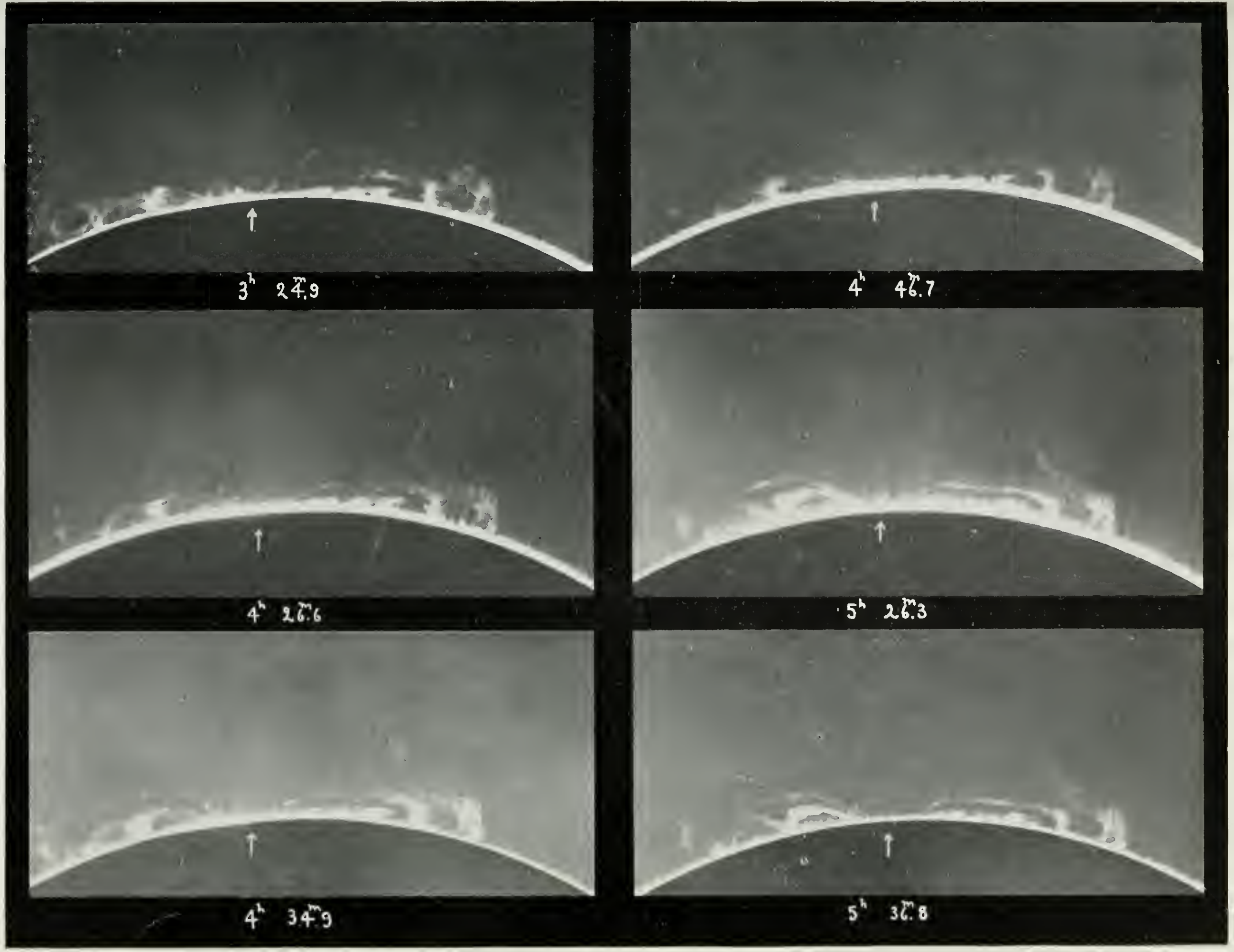
PLATE X



Absorption Spectra of Tellurium Vapor

Calcium Spectroheliograms of Solar Prominences on October 8, 1910

Scale: 1 mm = 8600 km



Calcium Spectroheliograms of Solar Prominences on October 8, 1910

Scale: 1 mm = 8600 km

PLATE XII

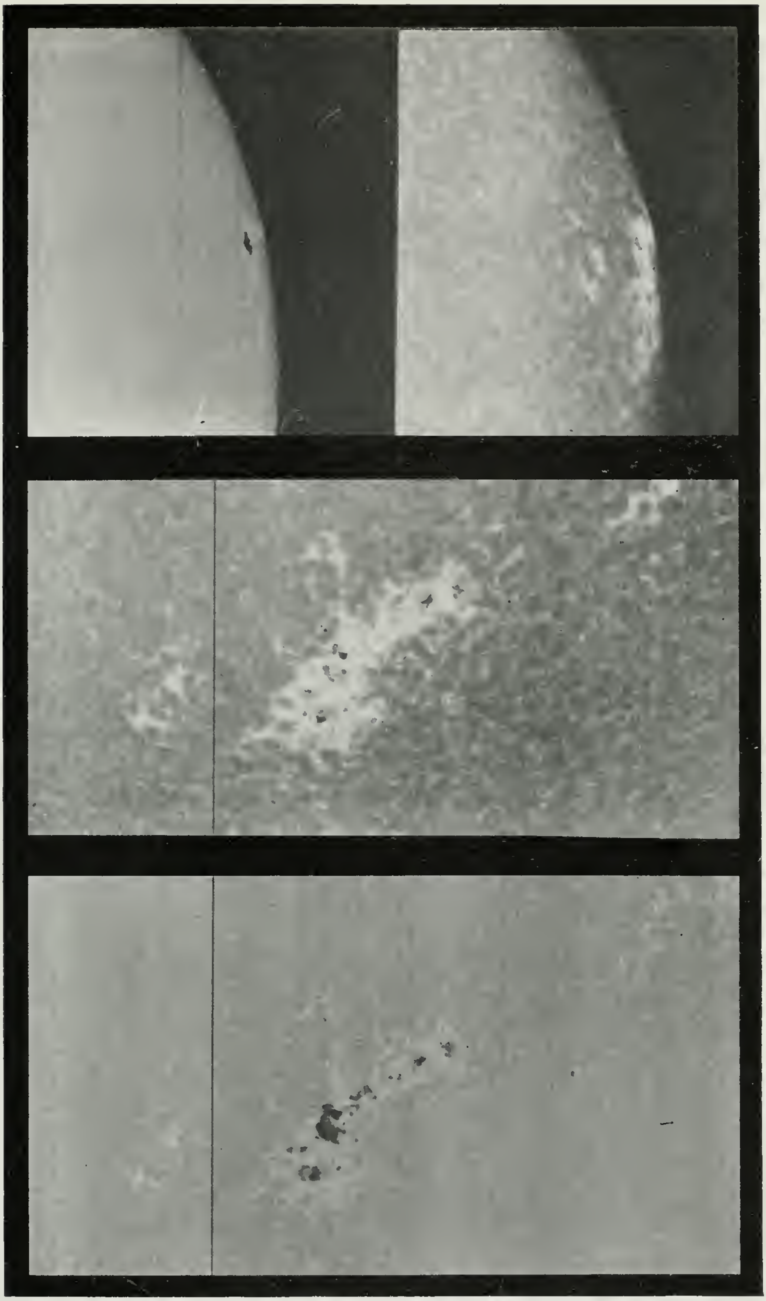


Fig. 1

Oct. 8
$3^h\ 41^m$
H_1

Fig. 2

Oct. 8
$6^h\ 17^m$
G.M.T.
H_2

Fig. 3

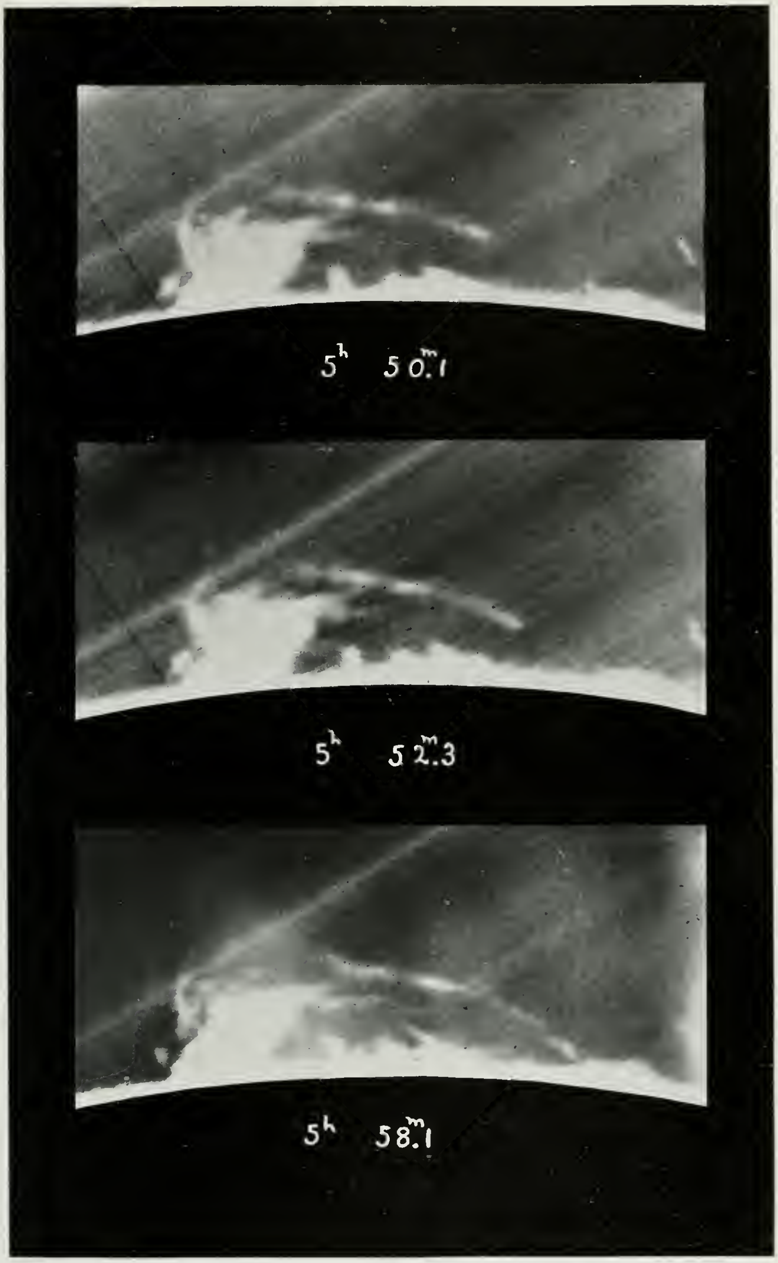
Oct. 1
$4^h\ 5^m$
H_2

Fig. 4

Oct. 1
$4^h\ 7^m$
H_1

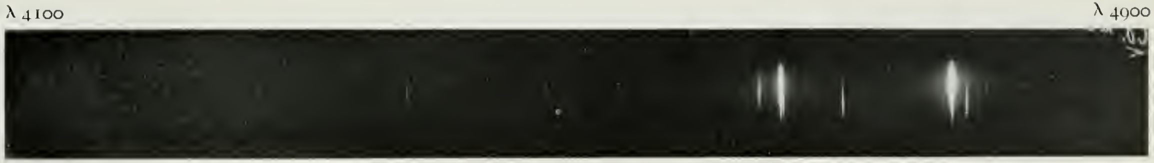
Spectroheliograms of Sun-Spots, October 1 and 8, 1910
Scale: 1 mm = 7900 km

PLATE XIII

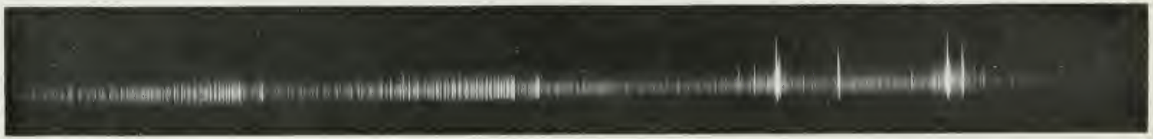


Calcium Spectroheliograms of Solar Prominences on October 22, 1910
Scale: 1 mm = 3500 km

PLATE XIV



1

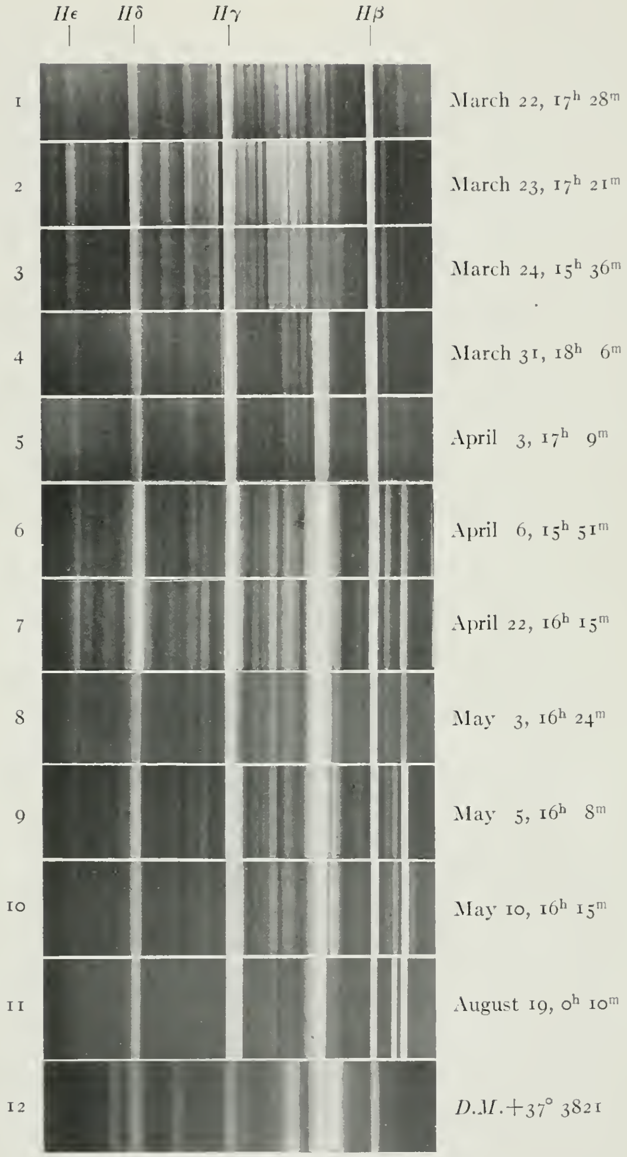


2

Fig. 1.—Cadmium arc in air

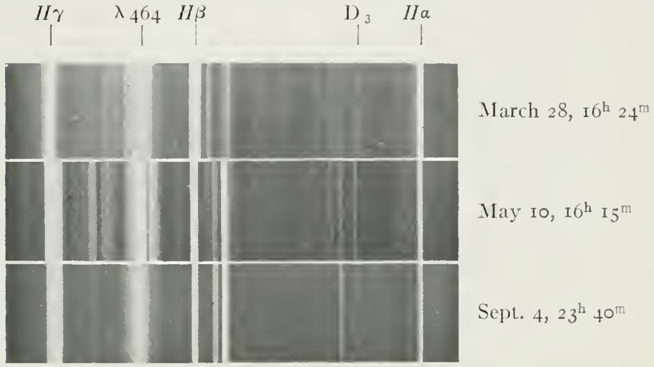
Fig. 2.—Cadmium arc *in vacuo*

PLATE XV

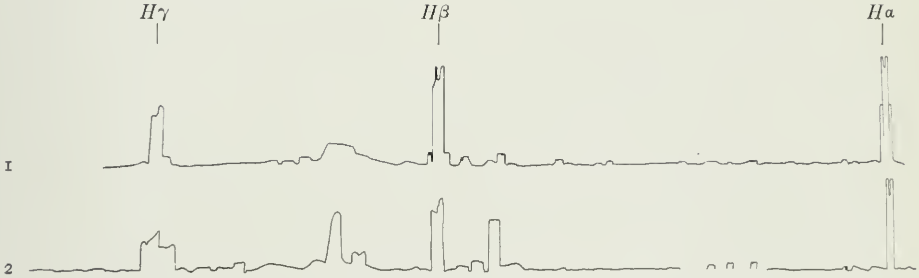

*H*ε
*H*δ
*H*γ
*H*β
1 March 22, 17^{h} 28^{m}
2 March 23, 17^{h} 21^{m}
3 March 24, 15^{h} 36^{m}
4 March 31, 18^{h} 6^{m}
5 April 3, 17^{h} 9^{m}
6 April 6, 15^{h} 51^{m}
7 April 22, 16^{h} 15^{m}
8 May 3, 16^{h} 24^{m}
9 May 5, 16^{h} 8^{m}
10 May 10, 16^{h} 15^{m}
11 August 19, 0^{h} 10^{m}
12 *D.M.*+37° 3821

1–11 Spectra of *Nova Geminorum* No. 2
12 Spectrum of Wolf-Rayet Star *D.M.*+37° 3821

PLATE XVI



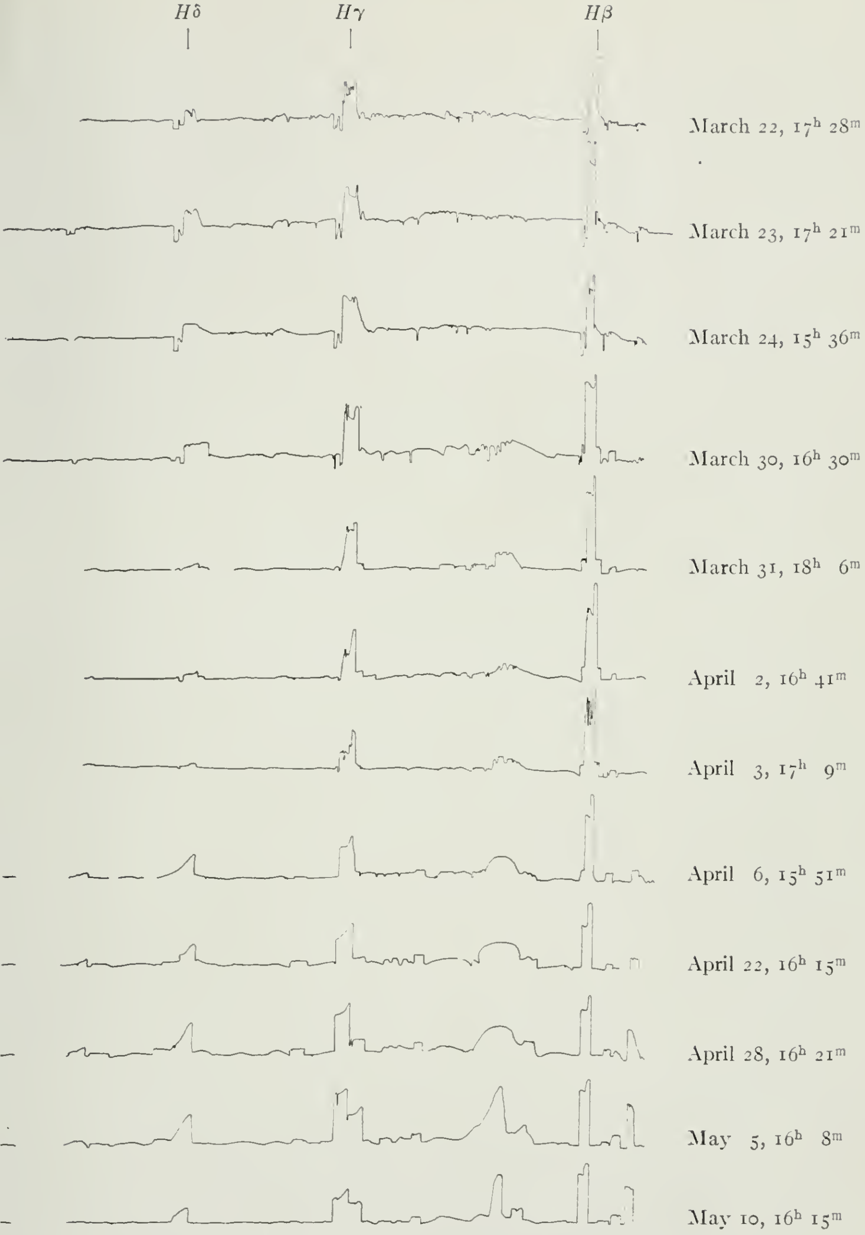
Spectra of *Nova Geminorum* No. 2



Intensity Curves of Spectra of *Nova Geminorum* No. 2

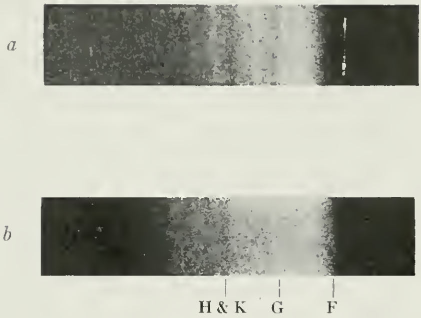
1. March 28, $16^h\ 24^m$
2. May 10, $16^h\ 15^m$

PLATE XVII



INTENSITY CURVES OF SPECTRA OF *Nova Geminorum* NO. 2

PLATE XVIII



INTEGRATED SPECTRUM OF THE MILKY WAY

a Spectrum of the Milky Way in *Sagittarius*. May and June 1911. Total exposure 65^h 13^m. Slit, 0.8 mm.

b Spectrum of sky. Slit, 0.1 mm.

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